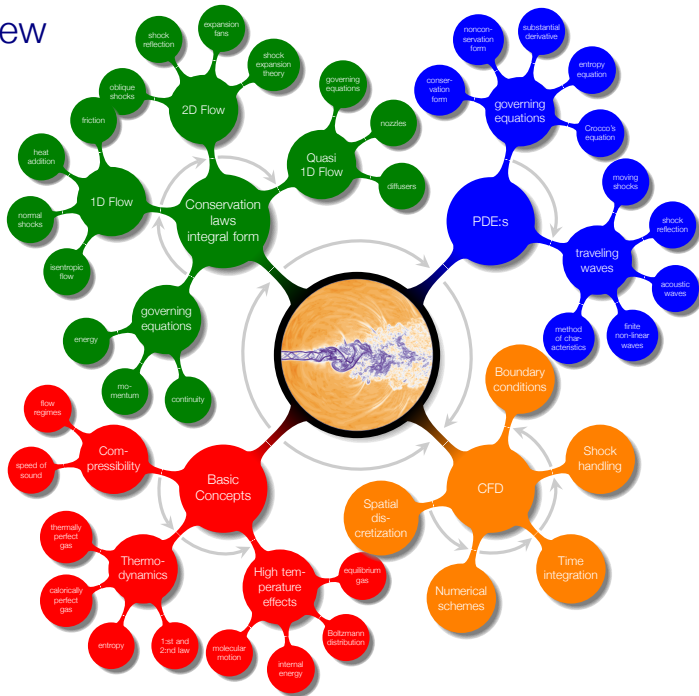# Compressible Flow - TME085

Lecture 14

Niklas Andersson

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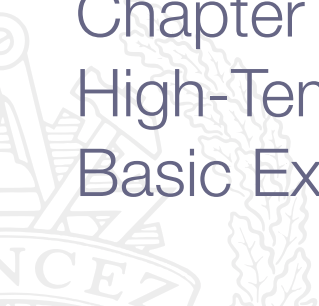Gothenburg, Sweden

niklas.andersson@chalmers.se

CHALMERS
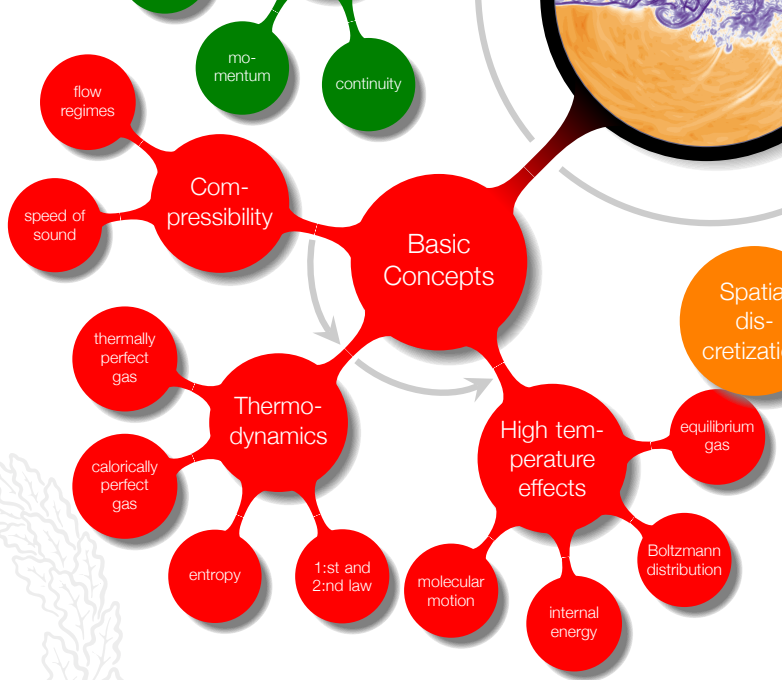UNIVERSITY OF TECHNOLOGY

Overview

Chapter 16
Properties of
High-Temperature Gases

Chapter 17
High-Temperature Flows:
Basic Examples

momentum

continuity

flow regimes

Com-pressibility

speed of sound

Basic Concepts

Spatial dis-cretization

thermally perfect gas

Thermo-dynamics

calorically perfect gas

High temperature effects

equilibrium gas

entropy

1:st and 2:nd law

molecular motion

internal energy

Boltzmann distribution

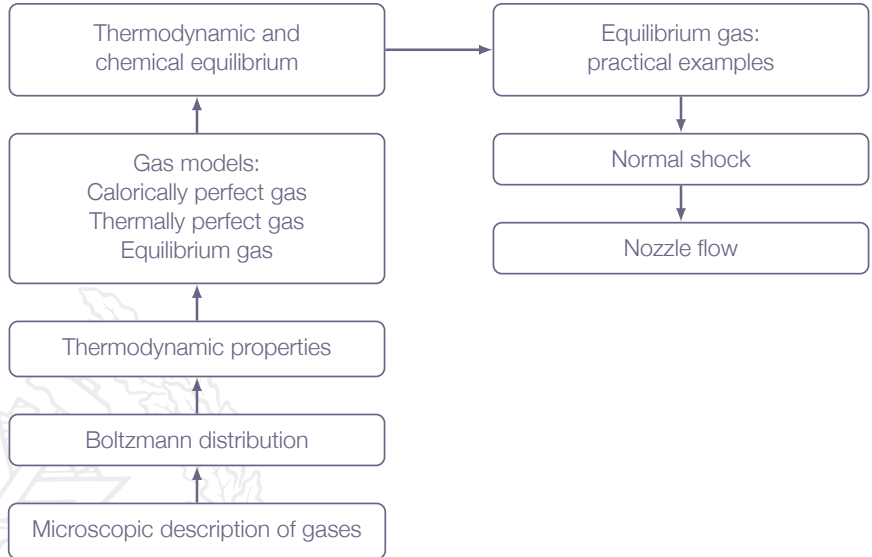# Addressed Learning Outcomes

6 Define the special cases of calorically perfect gas, thermally perfect gas and real gas and explain the implication of each of these special cases

A deep dive into the theory behind the definitions of calorically perfect gas, thermally perfect gas, and other models
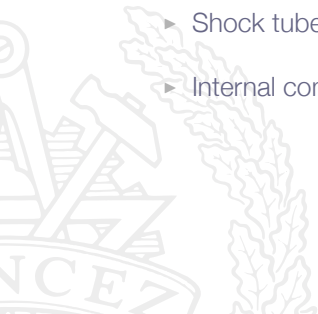
Roadmap - High Temperature Effects

Properties of High-Temperature Gases

Applications:

- Rocket nozzle flows

- Reentry vehicles

- Shock tubes / Shock tunnels

- Internal combustion engines

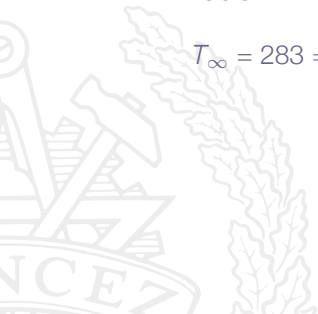# Properties of High-Temperature Gases

Example: Reentry vehicle

Mach 32.5
Air
Calorically perfect gas
$T_\infty = 283$

Table A.2 $\Rightarrow T_s/T_\infty = 206$

$T_\infty = 283 \Rightarrow T_s = 58\ 300$ K

Properties of High-Temperature Gases

Example: Reentry vehicle

Mach 32.5
Air
Calorically perfect gas
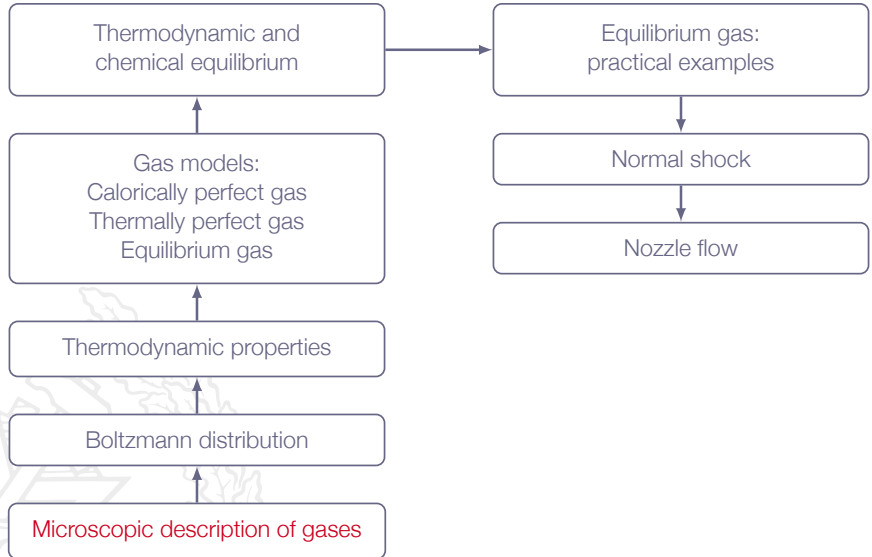$T_\infty = 283$

Table A.2 $\Rightarrow T_s/T_\infty = 206$

$T_\infty = 283 \Rightarrow T_s = 58\ 300$ K

A more correct value is $T_s = 11\ 600$ K

Something is fishy here!
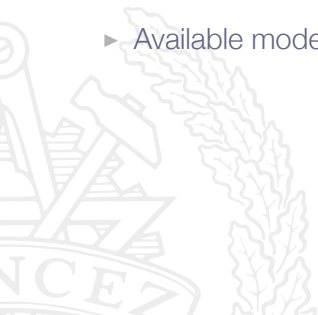
Roadmap - High Temperature Effects
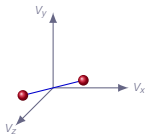
Chapter 16.2
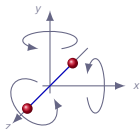# Microscopic Description of Gases

Microscopic Description of Gases

- Hard to make measurements

- Accurate, reliable theoretical models needed

- Available models do work quite well

Molecular Energy

Translational kinetic energy
thermal degrees of freedom: 3

Rotational kinetic energy
thermal degrees of freedom:
2 for diatomic gases
2 for linear polyatomic gases
3 for non-linear polyatomic gases
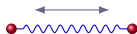
Vibrational energy
(kinetic energy + potential energy)
thermal degrees of freedom: 2

Electronic energy of electrons in orbit
(kinetic energy + potential energy)

CO_2
linear polyatomic molecule

H_2O
non-linear polyatomic molecule
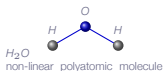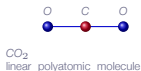
- ▶ Translational energy
- ▶ Rotational energy
  (only for molecules - not for mono-atomic gases)
- ▶ Vibrational energy
- ▶ Electronic energy

Molecular Energy

The energy for one molecule can be described by

$$\varepsilon' = \varepsilon'_{trans} + \varepsilon'_{rot} + \varepsilon'_{vib} + \varepsilon'_{el}$$

Results of quantum mechanics have shown that each energy is quantized *i.e.* they can exist only at discrete values

Not continuous! Might seem unintuitive

Molecular Energy

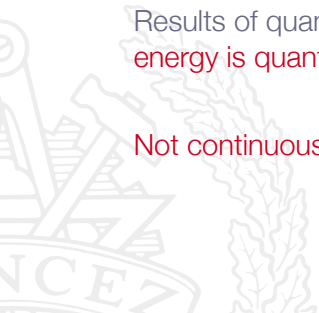The lowest quantum numbers defines the zero-point energy for each mode

- for rotational energy the zero-point energy is exactly zero

- $\varepsilon'_{0_{trans}}$ is very small but finite - *at absolute zero, molecules still moves but not much*

$$\varepsilon_{j_{trans}} = \varepsilon'_{j_{trans}} - \varepsilon'_{0_{trans}}$$

$$\varepsilon_{l_{vib}} = \varepsilon'_{l_{vib}} - \varepsilon'_{0_{vib}}$$

$$\varepsilon_{k_{rot}} = \varepsilon'_{k_{rot}}$$

$$\varepsilon_{m_{el}} = \varepsilon'_{m_{el}} - \varepsilon'_{0_{el}}$$

Energy States
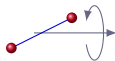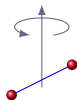
- three cases with the **same rotational energy**

- different direction of angular momentum

- quantum mechanics $\Rightarrow$ different **distinguishable states**

- a finite number of possible states for each energy level

Macrostates and Microstates

Macrostate:

- ▶ molecules collide and exchange energy $\Rightarrow$ the N_j distribution (the macrostate) will change over time

- ▶ some macrostates are more probable than other

- ▶ most probable macrostates (distribution) $\Rightarrow$ thermodynamic equilibrium

Microstate:

- ▶ same number of molecules in each energy level but different states

- ▶ the most probable macrostate is the one with the most possible microstates $\Rightarrow$ possible to find the most probable macrostate by counting microstates

Macrostates and Microstates

Macrostate I Microstate I

ε'_o : ● ● ○ ○ ○ $\qquad\qquad (N_o = 2, g_o = 5)$

ε'_1 : ● ● ● ○ ● ● $\qquad (N_1 = 5, g_1 = 6)$

ε'_2 : ● ● ● ○ ○ $\qquad\qquad (N_2 = 3, g_2 = 5)$

$\vdots$
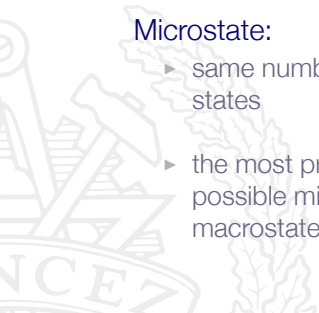
ε'_j : ○ ● ● $\qquad\qquad\qquad (N_j = 2, g_j = 3)$

Macrostates and Microstates

Macrostate I   Microstate II

$\varepsilon_o' :$ ○   ●   ○   ○   ● $(N_o = 2, g_o = 5)$

$\varepsilon_1' :$ ●   ○   ●   ●   ●   ● $(N_1 = 5, g_1 = 6)$

$\varepsilon_2' :$ ○   ○   ●   ●   ● $(N_2 = 3, g_2 = 5)$

$\vdots$

$\varepsilon_j' :$ ○   ●   ● $(N_j = 2, g_j = 3)$
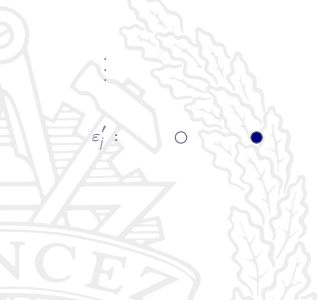
Macrostates and Microstates

Macrostate II Microstate I

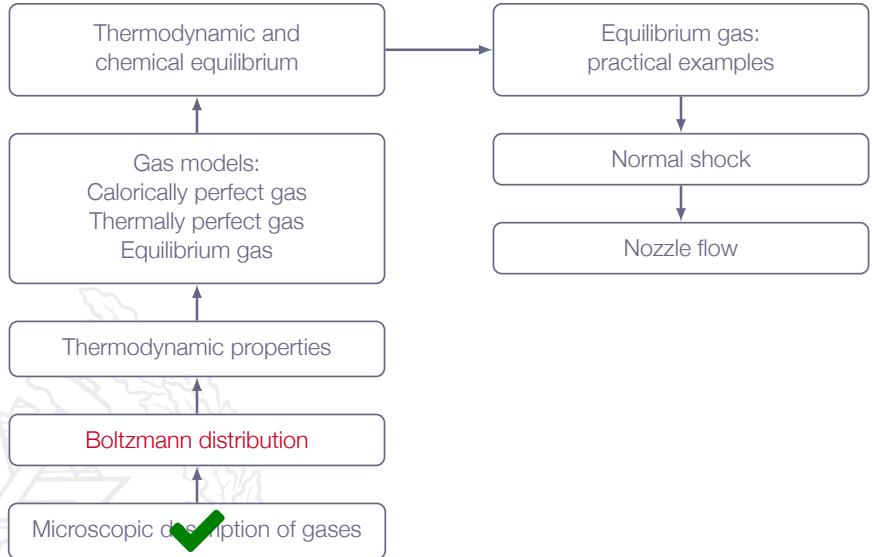|						
$\varepsilon'_o:$	○	●	○	○	○	$(N_o = 1, g_o = 5)$
$\varepsilon'_1:$	●	○	●	●	●	●	$(N_1 = 5, g_1 = 6)$
$\varepsilon'_2:$	●	○	●	●	●	$(N_2 = 4, g_2 = 5)$
$\vdots$						
$\varepsilon'_j:$	○	○	●			$(N_j = 1, g_j = 3)$

Macrostates and Microstates

$$N = \sum_j N_j$$

$$E = \sum_j \varepsilon'_j N_j$$

Roadmap - High Temperature Effects

Chapter 16.5
The Limiting Case:
Boltzmann Distribution
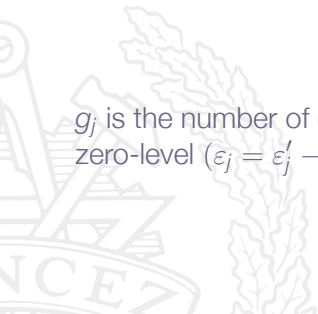
Boltzmann Distribution

The Boltzmann distribution:

$$N_j^* = N\frac{g_j \mathrm{e}^{-\varepsilon_j/kT}}{Q}$$

where $Q = f(T, V)$ is the state sum defined as

$$Q \equiv \sum_j g_j \mathrm{e}^{-\varepsilon_j/kT}$$

g_j is the number of degenerate states, ε_j is the energy above zero-level ($\varepsilon_j = \varepsilon_j' - \varepsilon_o$), and k is the Boltzmann constant

Boltzmann Distribution

The Boltzmann distribution:

$$N_j^* = N\frac{g_j \mathrm{e}^{-\varepsilon_j/kT}}{Q}$$

For molecules or atoms of a given species, quantum mechanics says that a set of well-defined energy levels ε_j exists, over which the molecules or atoms can be distributed at any given instant, and that each energy level has a certain number of energy states, g_j.

For a system of N molecules or atoms at a given T and V, N_j^* are the number of molecules or atoms in each energy level ε_j when the system is in thermodynamic equilibrium.

Boltzmann Distribution
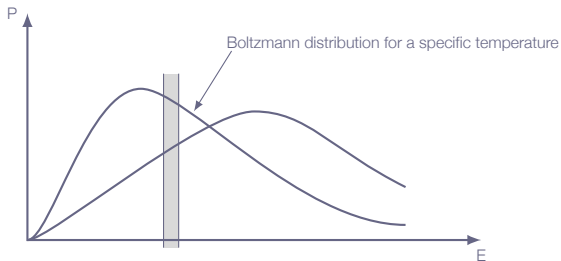
At temperatures above $\sim 5K$, molecules are distributed over many energy levels, and therefore the states are generally sparsely populated ($N_j \ll g_j$)

Higher energy levels become more populated as temperature increases

Roadmap - High Temperature Effects
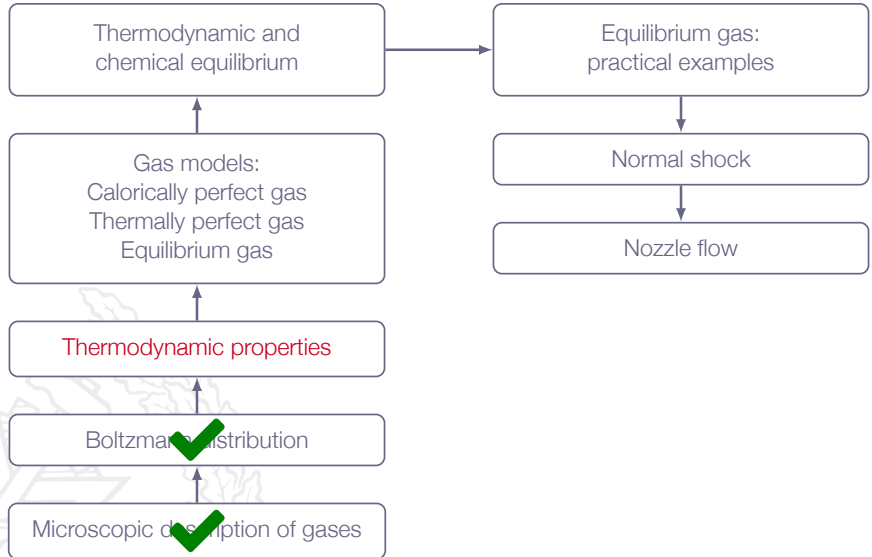
Chapter 16.6 - 16.8
Evaluation of Gas Thermodynamic Properties

Internal Energy

The internal energy is calculated as

$$E = NkT^2 \left(\frac{\partial \ln Q}{\partial T} \right)_V$$
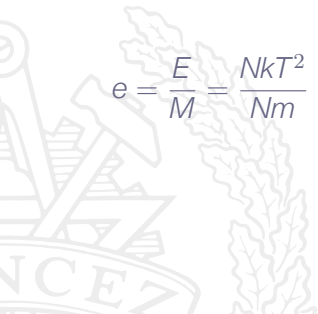
The internal energy per unit mass is obtained as

$$e = \frac{E}{M} = \frac{NkT^2}{Nm} \left(\frac{\partial \ln Q}{\partial T} \right)_V = \left\{ \frac{k}{m} = R \right\} = RT^2 \left(\frac{\partial \ln Q}{\partial T} \right)_V$$

Internal Energy - Translation

$$\varepsilon'_{trans} = \frac{h^2}{8m} \left(\frac{n_1^2}{a_1^2} + \frac{n_2^2}{a_2^2} + \frac{n_3^2}{a_3^2} \right)$$

$n_1 - n_3$	quantum numbers (1,2,3,...)
$a_1 - a_3$	linear dimensions that describes the size of the system
h	Planck's constant
m	mass of the individual molecule

$$\Rightarrow \cdots \Rightarrow$$

$$Q_{trans} = \left(\frac{2\pi mkT}{h^2} \right)^{3/2} V$$

Internal Energy - Translation

$$Q_{trans} = \left(\frac{2\pi mkT}{h^2} \right)^{3/2} V$$

$$\ln Q_{trans} = \frac{3}{2} \ln T + \frac{3}{2} \ln \frac{2\pi mk}{h^2} + \ln V \Rightarrow$$

$$\left(\frac{\partial \ln Q_{trans}}{\partial T} \right)_V = \frac{3}{2} \frac{1}{T} \Rightarrow$$

$$e_{trans} = RT^2 \left(\frac{\partial \ln Q_{trans}}{\partial T} \right)_V = RT^2 \frac{3}{2T} = \frac{3}{2} RT$$

Internal Energy - Rotation

$$\varepsilon'_{rot} = \frac{h^2}{8\pi^2 I} J(J+1)$$

> J rotational quantum number (0,1,2,...)
> I moment of inertia (tabulated for common molecules)
> h Planck's constant

$$\Rightarrow \cdots \Rightarrow$$

$$Q_{rot} = \frac{8\pi^2 I k T}{h^2}$$

Internal Energy - Rotation

$$Q_{rot} = \frac{8\pi^2 I k T}{h^2}$$

$$\ln Q_{rot} = \ln T + \ln \frac{8\pi^2 I k}{h^2} \Rightarrow$$

$$\left(\frac{\partial \ln Q_{rot}}{\partial T} \right)_V = \frac{1}{T} \Rightarrow$$

$$e_{rot} = RT^2 \left(\frac{\partial \ln Q_{rot}}{\partial T} \right)_V = RT^2 \frac{1}{T} = RT$$
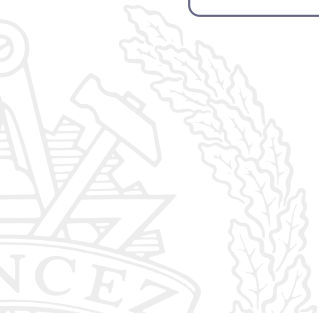
Internal Energy - Vibration

$$\varepsilon'_{vib} = h\nu \left(n + \frac{1}{2}\right)$$

> n vibrational quantum number (0,1,2,...)
> ν fundamental vibrational frequency (tabulated for common molecules)
> h Planck's constant

$$\Rightarrow \cdots \Rightarrow$$

$$Q_{vib} = \frac{1}{1 - e^{-h\nu/kT}}$$

Internal Energy - Vibration

$$Q_{vib} = \frac{1}{1 - e^{-h\nu/kT}}$$

$$\ln Q_{vib} = -\ln(1 - e^{-h\nu/kT}) \Rightarrow$$

$$\left(\frac{\partial \ln Q_{vib}}{\partial T}\right)_V = \frac{h\nu/kT^2}{e^{h\nu/kT} - 1} \Rightarrow$$

$$e_{vib} = RT^2 \left(\frac{\partial \ln Q_{vib}}{\partial T}\right)_V = RT^2 \frac{h\nu/kT^2}{e^{h\nu/kT} - 1} = \frac{h\nu/kT}{e^{h\nu/kT} - 1} RT$$

$$\lim_{T \to \infty} \frac{h\nu/kT}{e^{h\nu/kT} - 1} = 1 \Rightarrow e_{vib} \le RT$$

Specific Heat

$$e = e_{trans} + e_{rot} + e_{vib} + e_{el}$$

$$e = \frac{3}{2}RT + RT + \frac{h\nu/kT}{e^{h\nu/kT-1}}RT + e_{el}$$

$$C_v \equiv \left(\frac{\partial e}{\partial T}\right)_V$$

Specific Heat

Molecules with only translational and rotational energy

$$e = \frac{3}{2}RT + RT = \frac{5}{2}RT \Rightarrow C_v = \frac{5}{2}R$$

$$C_p = C_v + R = \frac{7}{2}R$$

$$\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4$$

Specific Heat

Mono-atomic gases with only translational and rotational energy
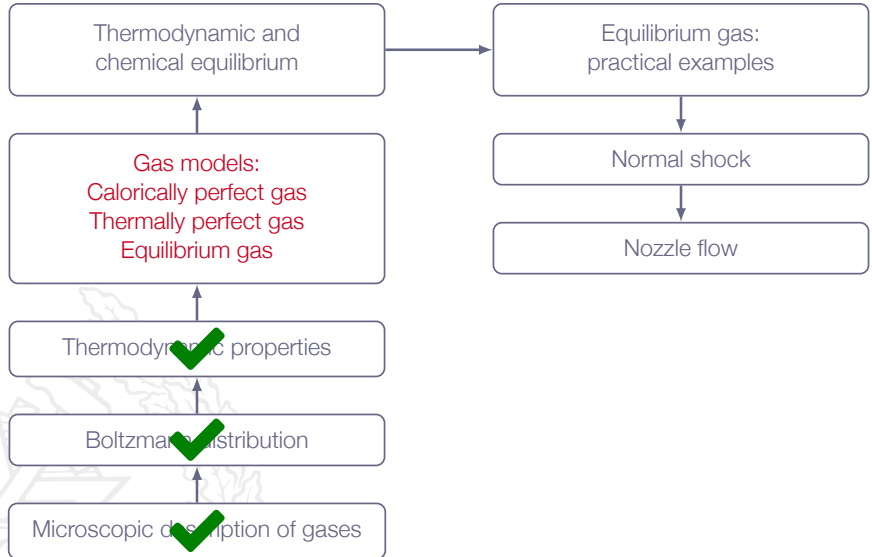
$$e = \frac{3}{2}RT \Rightarrow C_v = \frac{3}{2}R$$

$$C_p = C_v + R = \frac{5}{2}R$$

$$\gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1\frac{2}{3} \simeq 1.67$$

Roadmap - High Temperature Effects

Calorically Perfect Gas

- In general, only **translational** and **rotational** modes of molecular excitation

- Translational and rotational energy levels are sparsely populated, according to **Boltzmann distribution** (the Boltzmann limit)

- Vibrational energy levels are practically unpopulated (except for the zero level)

- Characteristic values of γ for each type of molecule, *e.g.* mono-atomic gas, di-atomic gas, tri-atomic gas, etc
    - *He*, *Ar*, *Ne*, ... - mono-atomic gases ($\gamma = 5/3$)
    - H_2, O_2, N_2, ... - di-atomic gases ($\gamma = 7/5$)
    - H_2O (gaseous), CO_2, ... - tri-atomic gases ($\gamma < 7/5$)

Calorically Perfect Gas

$$p = \rho R T \quad e = C_v T \qquad\qquad C_p - C_v = R$$
$$h = C_p T$$
$$h = e + p/\rho \qquad\qquad \gamma = C_p/C_v$$
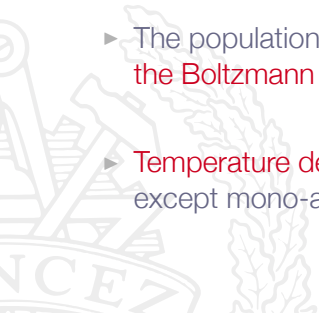
$$C_v = \frac{R}{\gamma - 1}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

γ, R, C_v, and C_p are constants

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T}$$

Thermally Perfect Gas

- In general, only translational, rotational and vibrational modes of molecular excitation

- Translational and rotational energy levels are sparsely populated, according to Boltzmann distribution (the Boltzmann limit)

- The population of the vibrational energy levels approaches the Boltzmann limit as temperature increases

- Temperature dependent values of γ for all types of molecules except mono-atomic (no vibrational modes possible)

Thermally Perfect Gas

$$p = \rho RT \quad e = e(T) \qquad C_v = de/dT \qquad C_p - C_v = R$$
$$h = h(T) \qquad C_p = dh/dT$$
$$h = e + p/\rho \qquad\qquad \gamma = C_p/C_v$$

$$C_v = \frac{R}{\gamma - 1}$$
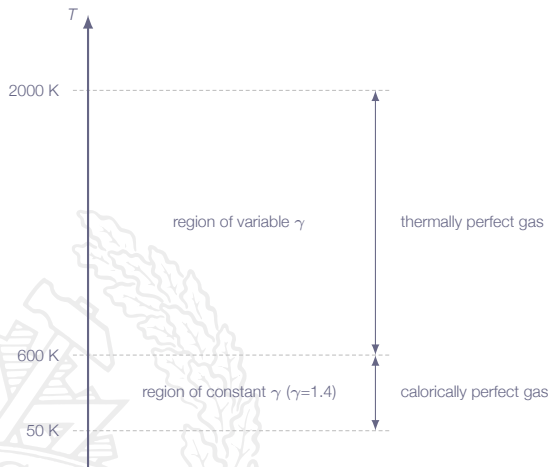
$$C_p = \frac{\gamma R}{\gamma - 1}$$

R is constant
γ, C_v, and C_p are variable (functions of T)

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT}$$

High-Temperature Effects

Example: properties of air

Thermally perfect gas:
e and *h* are non-linear functions of *T*

the temperatur range represents standard
atmospheric pressure (lower pressure gives
lower temperatures)

T

2000 K -

region of variable γ thermally perfect gas

600 K -

region of constant γ (γ=1.4)   calorically perfect gas

50 K -

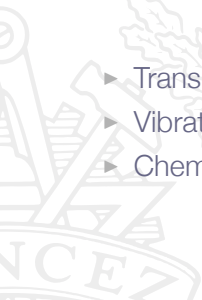# High-Temperature Effects

For cases where the vibrational energy is not negligible (high temperatures)

$$\lim_{T \to \infty} e_{vib} = RT \Rightarrow C_v = \frac{7}{2}R$$

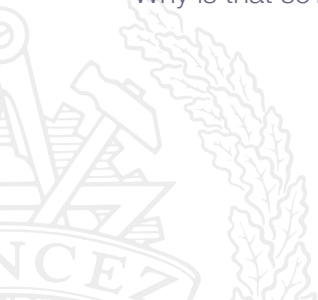However, chemical reactions and ionization will take place long before that

- Translational and rotational energy fully excited above $\sim$5 K
- Vibrational energy is non-negligible above 600 K
- Chemical reactions begin to occur above $\sim$2000 K
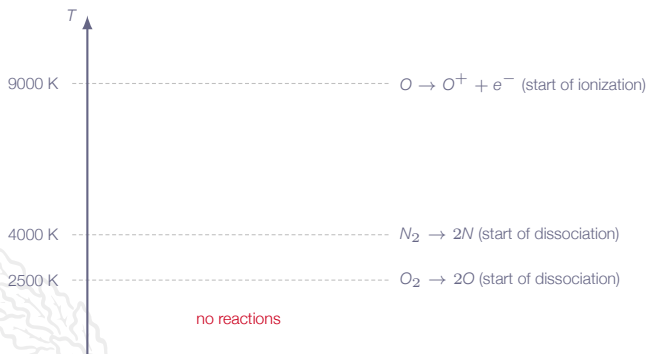
High-Temperature Effects

As temperature increase further vibrational energy becomes less important

Why is that so?

High-Temperature Effects

Example: properties of air (continued)

T

9000 K - $O \rightarrow O^+ + e^-$ (start of ionization)

4000 K - $N_2 \rightarrow 2N$ (start of dissociation)

2500 K - $O_2 \rightarrow 2O$ (start of dissociation)

no reactions

With increasing temperature, the gas becomes more and more mono-atomic which means that vibrational modes becomes less important

Equilibrium Gas

For temperatures $T > \sim 2500K$

- Air may be described as being in thermodynamic and chemical equilibrium (Equilibrium Gas)
  - reaction rates (time scales) low compared to flow time scales
  - reactions in both directions (example: $O_2 \rightleftharpoons 2O$)

- Tables must be used (Equilibrium Air Data) or special functions which have been made to fit the tabular data

Equilibrium Gas

How do we obtain a thermodynamic description?

$$p = p(R, T) \qquad e = e(\nu, T)$$

$$C_v = \left(\frac{\partial e}{\partial T} \right)_\nu$$

$$h = h(p, T)$$

$$h = e + \frac{p}{\rho}$$

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p$$

$$a_e^2 = \gamma R T \frac{1 + \dfrac{1}{p} \left(\dfrac{\partial e}{\partial \nu} \right)_T}{1 - \rho \left(\dfrac{\partial h}{\partial p} \right)_T} \qquad \gamma = \frac{C_p}{C_v} = \frac{\left(\dfrac{\partial h}{\partial T} \right)_p}{\left(\dfrac{\partial e}{\partial T} \right)_\nu} \qquad RT = \frac{p}{\rho}$$

Note: R is not a constant here *i.e.* this is not the ideal gas law

Roadmap - High Temperature Effects
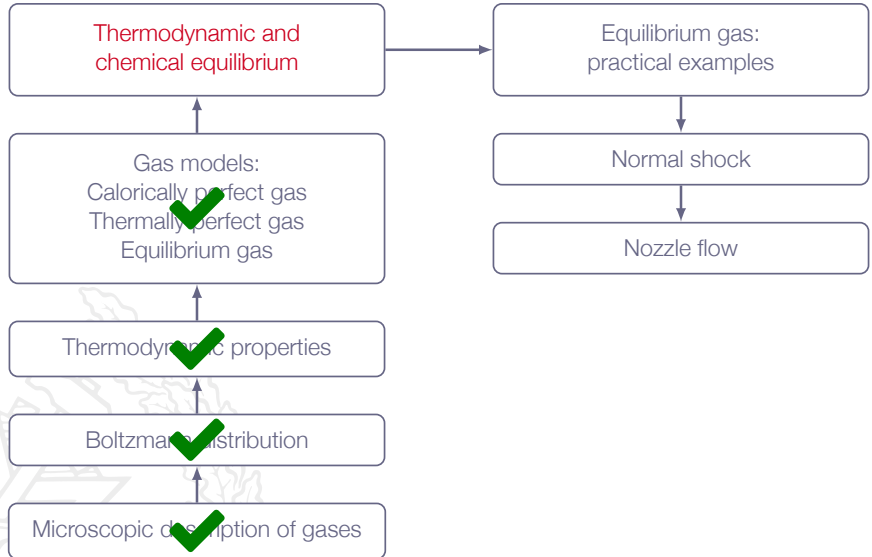
Chapter 17.1 Thermodynamic and Chemical Equilibrium

Thermodynamic Equilibrium

Molecules are distributed among their possible energy states according to the **Boltzmann distribution** (which is a **statistical equilibrium**) for the given temperature of the gas

- ▸ extremely fast process (time and length scales of the molecular processes)

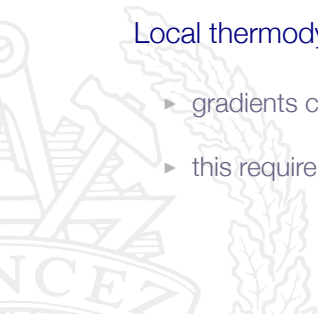- ▸ much faster than flow time scales in general (not true inside shocks)
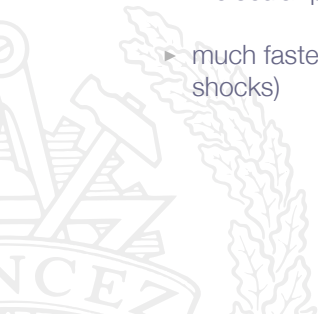
Thermodynamic Equilibrium

Global thermodynamic equilibrium:

- there are no gradients of p, T, ρ, $\mathbf{v}$, species concentrations

- "true thermodynamic equilibrium"

Local thermodynamic equilibrium:

- gradients can be neglected locally

- this requirement is fulfilled in most cases (hard not to get)

Chemical Equilibrium

Composition of gas (species concentrations) is fixed in time

- ► forward and backward rates of all chemical reactions are equal

- ► zero net reaction rates

- ► chemical reactions may be either slow or fast in comparison to flow time scale depending on the case studied
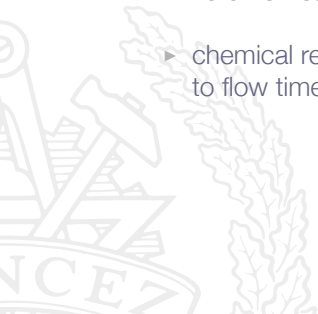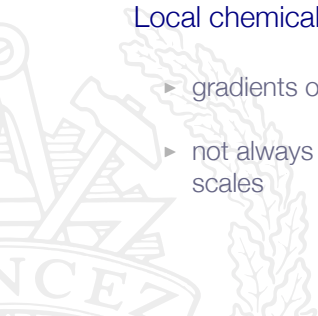
Chemical Equilibrium

Global chemical equilibrium:

- there are no gradients of species concentrations

- together with global thermodynamic equilibrium $\Rightarrow$ all gradients are zero

Local chemical equilibrium

- gradients of species concentrations can be neglected locally

- not always true - depends on reaction rates and flow time scales

Thermodynamic and Chemical Equilibrium

Most common cases:

|	Thermodynamic Equilibrium	Chemical Equilibrium	Gas Model
1	local thermodynamic equilibrium	local chemical equilibrium	equilibrium gas
2	local thermodynamic equilibrium	chemical non-equilibrium	finite rate chemistry
3	local thermodynamic equilibrium	frozen composition	frozen flow
4	thermodynamic non-equilibrium	frozen composition	vibrationally frozen flow

- ► length and time scales of flow decreases from 1 to 4
- ► Frozen composition $\Rightarrow$ no (or slow) reactions
- ► vibrationally frozen flow gives the same gas relations as calorically perfect gas!
  - ► no chemical reactions and unchanged vibrational energy
  - ► example: small nozzles with high-speed flow

Roadmap - High Temperature Effects
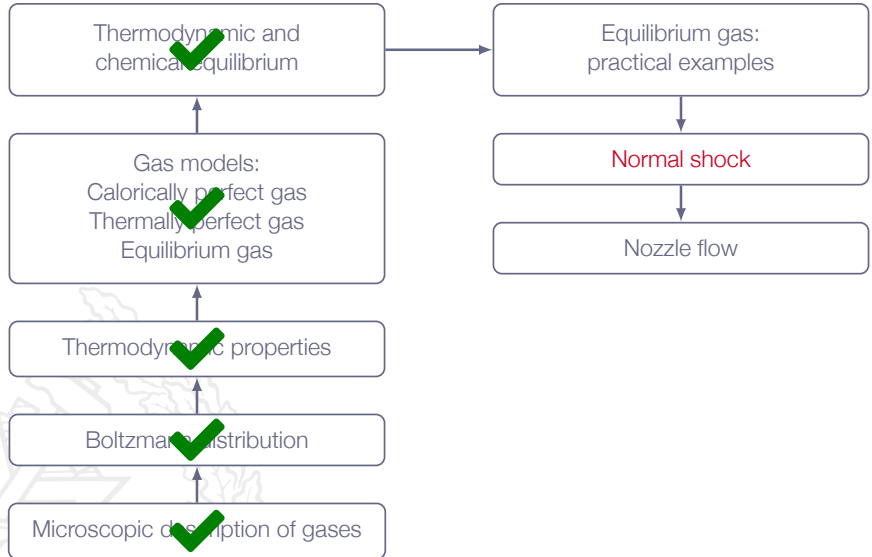
Chapter 17.2
Equilibrium Normal Shock Wave Flows
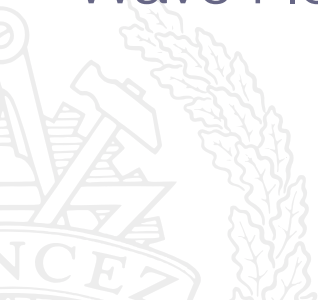
Equilibrium Normal Shock Wave Flows

Question: Is the equilibrium gas assumption OK?

Answer:

- for hypersonic flows with very little ionization in the shock region, it is a fair approximation

- not perfect, since the assumption of local thermodynamic and chemical equilibrium is not really true around the shock

- however, it gives a significant improvement compared to the calorically perfect gas assumption
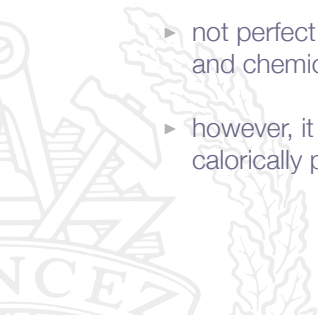
Equilibrium Normal Shock Wave Flows

Basic relations (for all gases), stationary normal shock:

$$\begin{cases} \rho_1 u_1 = \rho_2 u_2 \\[2mm] \rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2 \\[2mm] h_1 \dfrac{1}{2} u_1^2 = h_2 + \dfrac{1}{2} u_2^2 \end{cases}$$

For equilibrium gas we have:

$$\begin{cases} \rho = \rho(p, h) \\[2mm] T = T(\rho, h) \end{cases}$$

(we are free to choose any two states as independent variables)

Equilibrium Normal Shock Wave Flows

Assume that ρ_1, u_1, p_1, T_1, and h_1 are known

$$u_2 = \frac{\rho_1 u_1}{\rho_2} \Rightarrow \rho_1 u_1^2 + p_1 = \rho_2 \left(\frac{\rho_1}{\rho_2} u_1\right)^2 + p_2 \Rightarrow$$

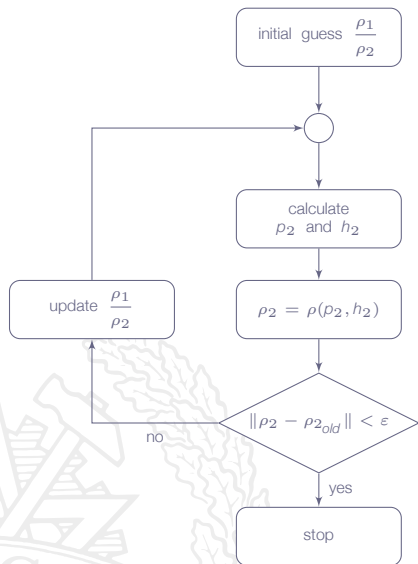$$p_2 = p_1 + \rho_1 u_1^2 \left(1 - \frac{\rho_1}{\rho_2}\right)$$

Also

$$h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2}\left(\frac{\rho_1}{\rho_2} u_1\right)^2 \Rightarrow$$

$$h_2 = h_1 + \frac{1}{2} u_1^2 \left(1 - \left(\frac{\rho_1}{\rho_2}\right)^2\right)$$

Equilibrium Normal Shock Wave Flows

when converged:

$$\left.\begin{array}{l} \rho_2 = \rho(p_2, h_2) \\ T_2 = T(\rho_2, h_2) \end{array}\right\} \Rightarrow$$

$\rho_2, u_2, p_2, T_2, h_2$ known

Equilibrium Air - Normal Shock

Tables of thermodynamic properties for different conditions are available

For a very strong shock case ($M_1 = 32$), the table below (Table 17.1) shows some typical results for equilibrium air

	calorically perfect gas ($\gamma = 1.4$)	equilibrium air
p_2/p_1	1233	1387
ρ_2/ρ_1	5.97	15.19
h_2/h_1	206.35	212.80
T_2/T_1	206.35	41.64

Equilibrium Air - Normal Shock

Analysis:

- ▶ Pressure ratio is comparable
- ▶ Density ratio differs by factor of 2.5
- ▶ Temperature ratio differs by factor of 5

Explanation:

- ▶ Using equilibrium gas means that vibration, dissociation and chemical reactions are accounted for
- ▶ The chemical reactions taking place in the shock region lead to an "absorption" of energy into chemical energy
  - ▶ drastically reducing the temperature downstream of the shock
  - ▶ this also explains the difference in density after the shock

Equilibrium Air - Normal Shock

Additional notes:

- ▶ For a normal shock in an equilibrium gas, the pressure ratio, density ratio, enthalpy ratio, temperature ratio, etc all depend on three upstream variables, *e.g.* u_1, p_1, T_1

- ▶ For a normal shock in a thermally perfect gas, the pressure ratio, density ratio, enthalpy ratio, temperature ratio, etc all depend on two upstream variables, *e.g.* M_1, T_1

- ▶ For a normal shock in a calorically perfect gas, the pressure ratio, density ratio, enthalpy ratio, temperature ratio, etc all depend on one upstream variable, *e.g.* M_1
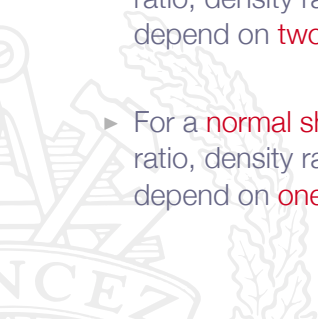
Equilibrium Gas - Detached Shock

calorically perfect gas

equilibrium gas

$M = 20$

$M = 20$

shock moves closer to body

What's the reason for the difference in predicted shock position?

Equilibrium Gas - Detached Shock

Calorically perfect gas:

- all energy ends up in translation and rotation $\Rightarrow$ increased temperature

Equilibrium gas:

- energy is absorbed by reactions $\Rightarrow$ does not contribute to the increase of gas temperature
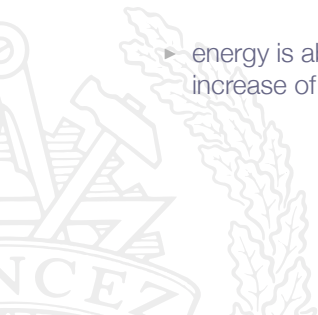
Roadmap - High Temperature Effects
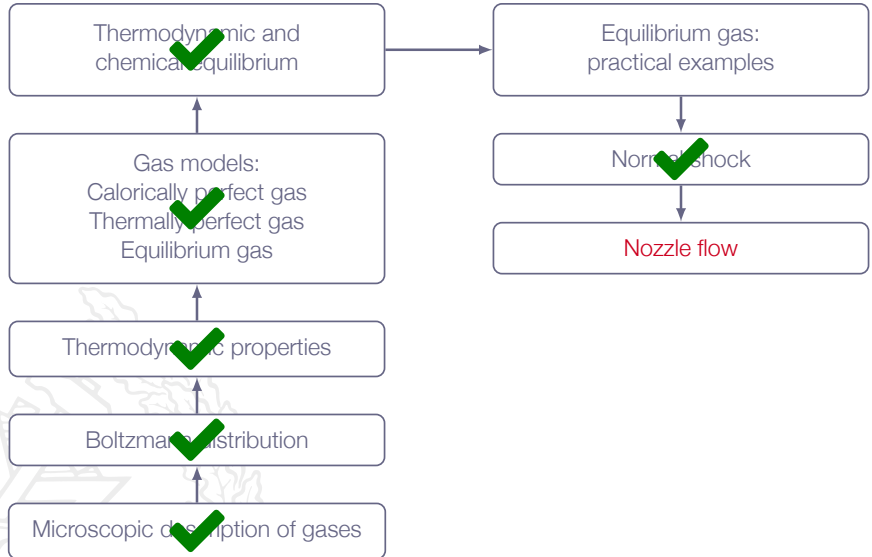
Chapter 17.3
Equilibrium
Quasi-One-Dimensional
Nozzle Flows

Equilibrium Quasi-1D Nozzle Flows

First question: Is chemically reacting gas also isentropic (for inviscid and adiabatic case)?

entropy equation: $Tds = dh - \nu dp$

Quasi-1D equations in differential form (all gases):

momentum equation: $dp = -\rho u du$

energy equation: $dh + u du = 0$
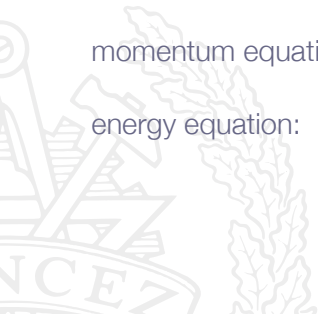
Equilibrium Quasi-1D Nozzle Flows

$$udu = -\frac{dp}{\rho} = -\nu dp$$

$$Tds = -udu - \nu dp = -udu + udu = 0 \Rightarrow$$

$$ds = 0$$

Isentropic flow!

Equilibrium Quasi-1D Nozzle Flows

Second question: Does the area-velocity relation also hold for a chemically reacting gas?

Isentropic process gives

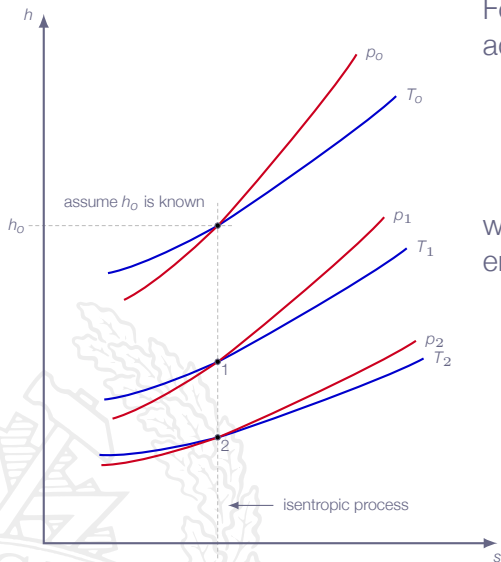$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$$

$M = 1$ at nozzle throat still holds
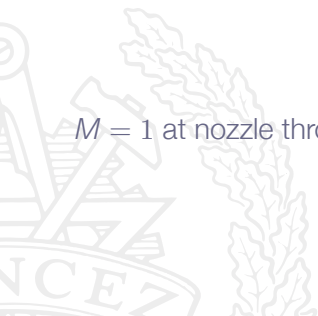
Equilibrium Quasi-1D Nozzle Flows

For general gas mixture in thermodynamic and chemical equilibrium, we may find tables or graphs describing relations between state variables.

Example: Mollier diagram

p = constant

T = constant

for any point (h, s), we may find p, T, ρ, a, ...
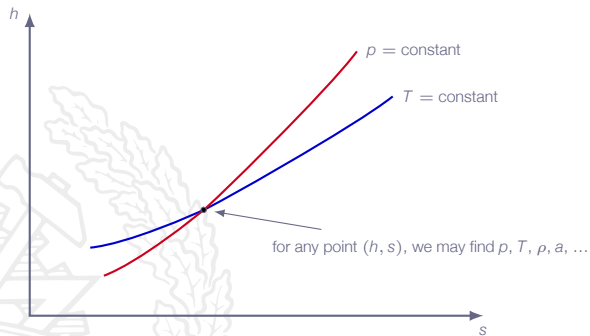
Equilibrium Quasi-1D Nozzle Flows

For steady-state inviscid adiabatic nozzle flow we have:

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 = h_o$$

where h_o is the reservoir enthalpy

Equilibrium Quasi-1D Nozzle Flows

At point 1 in Mollier diagram we have:

$$\frac{1}{2}u_1^2 = h_o - h_1 \Rightarrow u_1 = \sqrt{2(h_o - h_1)}$$

Assume that $u_1 = a_1$ (sonic conditions) gives

$$\rho_1 u_1 A_1 = \rho^* a^* A^*$$

At any point along isentropic line, we have $u = \sqrt{2(h_o - h)}$ and ρ, p, T, a etc are all given which means that ρu is given

$$\frac{A}{A^*} = \frac{\rho^* a^*}{\rho u}$$

may be computed for any point along isentropic line

Equilibrium Quasi-1D Nozzle Flows

- Equilibrium gas gives higher *T* and more thrust
- During the expansion chemical energy is released due to shifts in the equilibrium composition
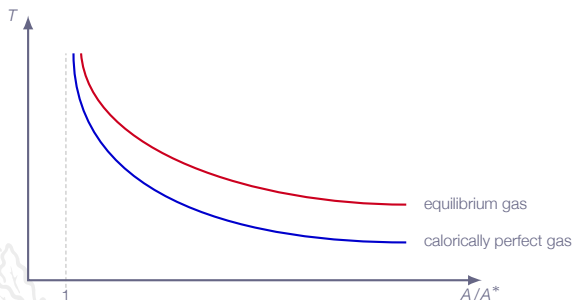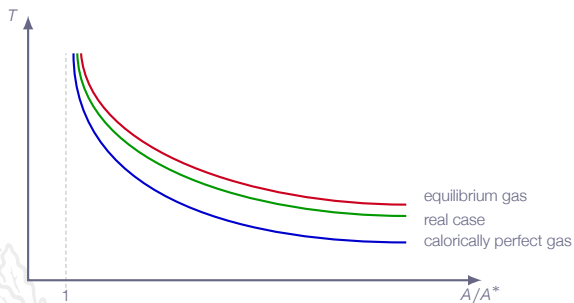
Equilibrium Quasi-1D Nozzle Flows

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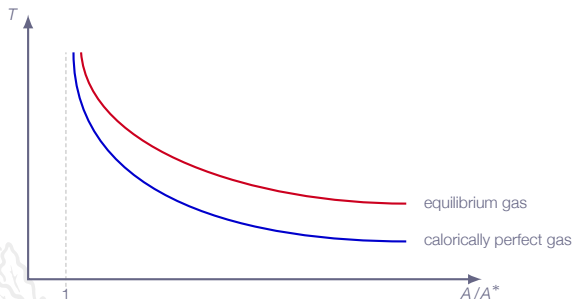
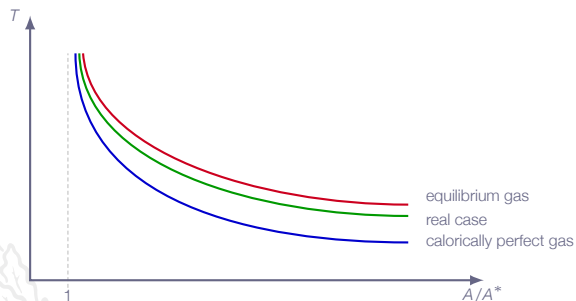- Chemical and vibrational energy transfered to translation and rotation $\Rightarrow$ increased temperature

Equilibrium Quasi-1D Nozzle Flows - Reacting Mixture

Real nozzle flow with reacting gas mixture:

Equilibrium Quasi-1D Nozzle Flows - Reacting Mixture

Real nozzle flow with reacting gas mixture:

- Space nozzle applications: $u_e \approx 4000$ m/s
- Required prediction accuracy 5 m/s

Equilibrium Quasi-1D Nozzle Flows - Reacting Mixture

Equilibrium gas:
- ► very **fast chemical reactions**
- ► local thermodynamic and chemical equilibrium

Vibrationally frozen gas:
- ► very **slow chemical reactions**
  (no chemical reactions ⇒ frozen gas)
- ► **vibrational energy** of molecules have **no time to change**
- ► **calorically perfect gas!**

Large Nozzles

High T_o, high p_o, high reactivity

Real case is close to equilibrium gas results

Example: Ariane 5 launcher, main engine (Vulcain 2)

- $H_2 + O_2 \rightarrow H_2O$ in principle, but many different radicals and reactions involved (at least $\sim$10 species, $\sim$20 reactions)
- $T_o \sim 3600\,K$, $p_o \sim 120\,bar$
- Length scale $\sim$ a few meters
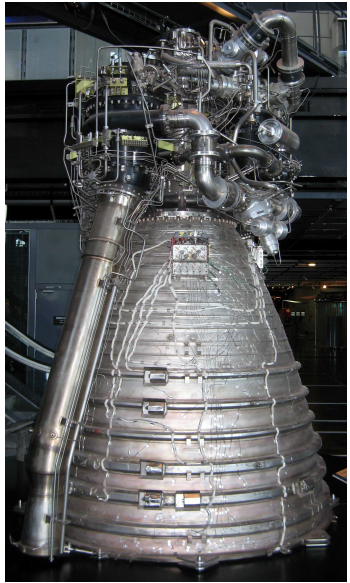- Gas mixture is quite close to equilibrium conditions all the way through the expansion
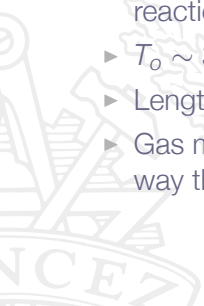
Ariane 5

Ariane 5 space launcher
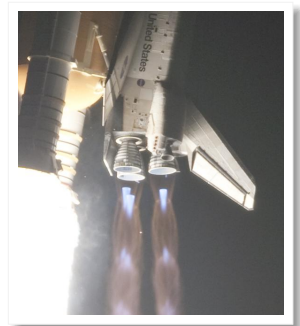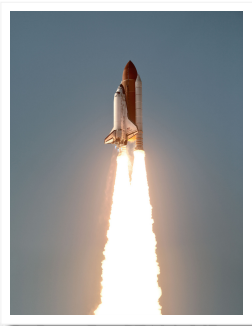
extreme high temperature and high speed flow regime

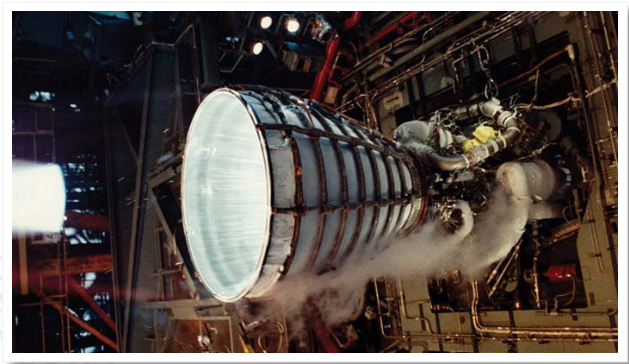# Vulcain Engine

Vulcain engine:

first stage of the Ariane 5 launcher

Space Shuttle Launcher - SSME

Space Shuttle Launcher - SSME

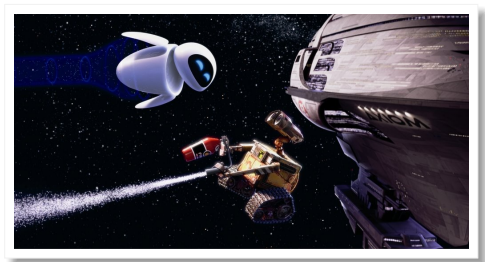# Small Nozzles

Low T_o, low p_o, lower reactivity

Real case is close to frozen flow results

Example:

Small rockets on satellites (for maneuvering, orbital adjustments, etc)

Small Nozzles

Roadmap - High Temperature Effects