Compressible Flow - TME085

Lecture 10

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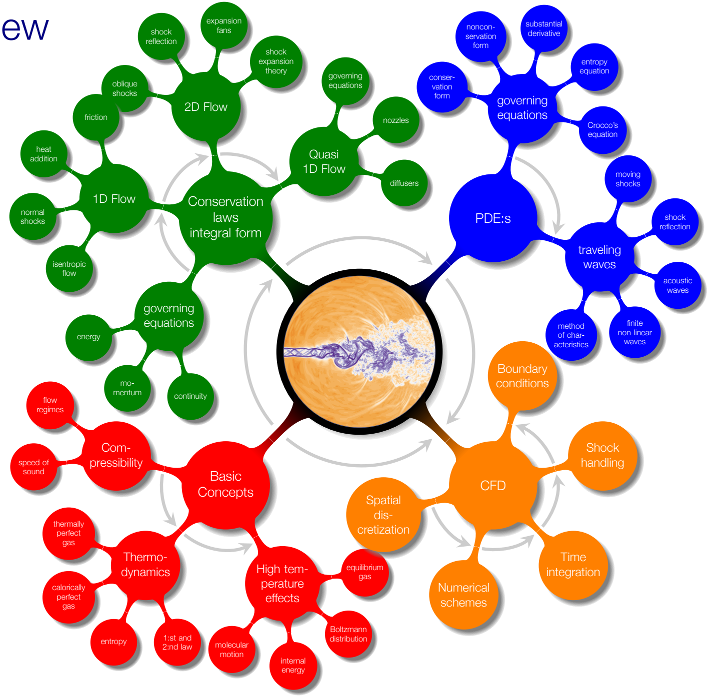
Overview



Conservation laws integral form
1D Flow
heat addition
normal shocks
isentropic flow
friction
2D Flow
oblique shocks
shock reflection
expansion fans
shock expansion theory
Quasi 1D Flow
governing equations
nozzles
diffusers
governing equations
energy
mo-mentum
continuity
PDE:s
governing equations
conser-vation form
noncon-servation form
substantial derivative
entropy equation
Crocco's equation
traveling waves
moving shocks
shock reflection
acoustic waves
finite non-linear waves
method of char-acteristics
CFD
Boundary conditions
Shock handling
Time integration
Numerical schemes
Spatial dis-cretization
Basic Concepts
Com-pressibility
flow regimes
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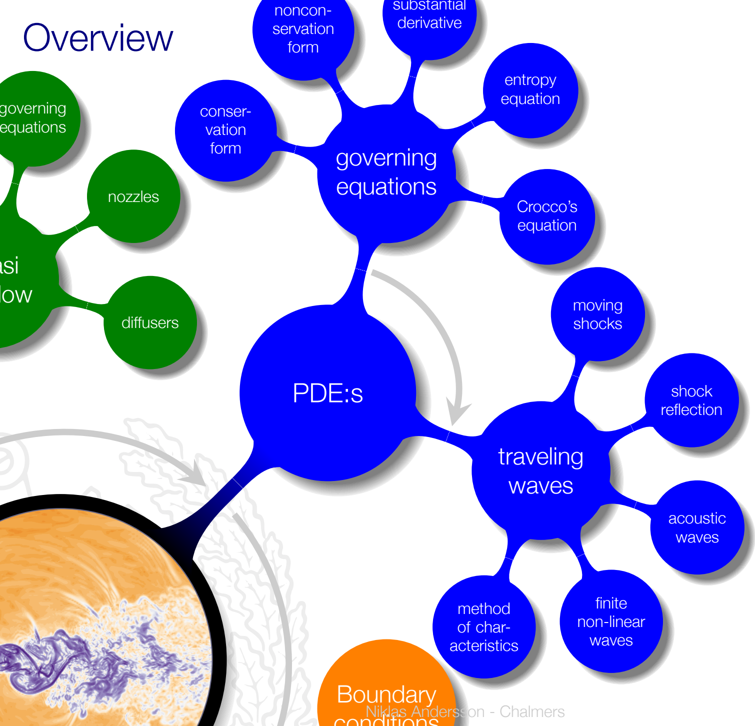
Thermo-dynamics
thermally perfect gas
calorically perfect gas
entropy
1:st and 2:nd law
High tem-perature effects
equilibrium gas
Boltzmann distribution
internal energy
molecular motion

Chapter 7
Unsteady Wave Motion

Overview



PDE:s
governing equations
conser-vation form
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substantial derivative
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method of char-acteristics
governing equations
nozzles
diffusers
Boundary

Addressed Learning Outcomes

3 Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

8 Derive (marked) and apply (all) of the presented mathematical formulae for classical gas dynamics

- j unsteady waves and discontinuities in 1D

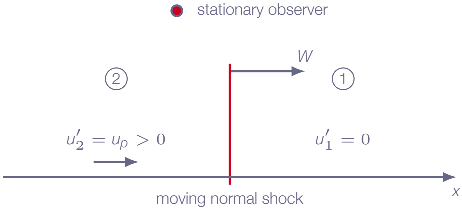
9 Solve engineering problems involving the above-mentioned phenomena (8a-8k)

12 Explain the main principles behind a modern Finite Volume CFD code and such concepts as explicit/implicit time stepping, CFL number, conservation, handling of compression shocks, and boundary conditions

what happens when a moving shock approaches a wall?

Chapter 7.2 Moving Normal Shock Waves

Moving Normal Shock Waves - Governing Equations



For stationary normal shocks we have:

$$\rho_1 u_1 = \rho_2 u_2$$
$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$$
$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2$$

With $(u_1 = W)$ and $(u_2 = W - u_p)$ we get:

$$\rho_1 W = \rho_2 (W - u_p)$$
$$\rho_1 W^2 + p_1 = \rho_2 (W - u_p)^2 + p_2$$
$$h_1 + \frac{1}{2}W^2 = h_2 + \frac{1}{2}(W - u_p)^2$$

Moving Normal Shock Waves - Relations

From the continuity equation we get:

$$u_p = W\left(1 - \frac{\rho_1}{\rho_2}\right) > 0$$

After some derivation we obtain:

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right]^{1/2}$$

Moving Normal Shock Waves - Relations

May also show that

$$\frac{\rho_2}{\rho_1} = \frac{1 + \dfrac{\gamma + 1}{\gamma - 1}\left(\dfrac{p_2}{p_1}\right)}{\dfrac{\gamma + 1}{\gamma - 1} + \dfrac{p_2}{p_1}}$$

and

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left[\frac{\dfrac{\gamma + 1}{\gamma - 1} + \dfrac{p_2}{p_1}}{1 + \dfrac{\gamma + 1}{\gamma - 1}\left(\dfrac{p_2}{p_1}\right)}\right]$$

Moving Normal Shock Waves - Relations

Induced Mach number:

$$M_p = \frac{u_p}{a_2} = \frac{u_p}{a_1}\frac{a_1}{a_2} = \frac{u_p}{a_1}\sqrt{\frac{T_1}{T_2}}$$

inserting u_p/a_1 and T_1/T_2 from relations on previous slides we get:

$$M_p = \frac{1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{\gamma-1}{\gamma+1} + \frac{p_2}{p_1}}\right]^{1/2}\left[\frac{1 + \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right)}{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right) + \left(\frac{p_2}{p_1}\right)^2}\right]^{1/2}$$

Moving Normal Shock Waves - Relations

Note that

$$\lim_{\frac{p_2}{p_1} \to \infty} M_p \to \sqrt{\frac{2}{\gamma(\gamma - 1)}}$$

for air ($\gamma = 1.4$)

$$\lim_{\frac{p_2}{p_1} \to \infty} M_p \to 1.89$$

Moving Normal Shock Waves - Relations

Note that $h_{o_1} \neq h_{o_2}$

constant total enthalpy is only valid for stationary shocks!

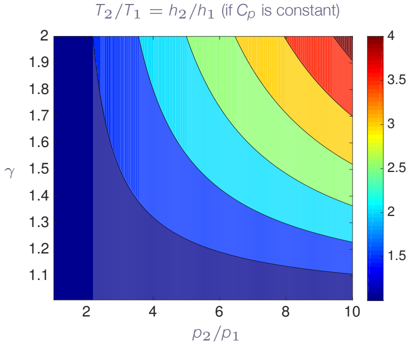
shock is uniquely defined by pressure ratio p_2/p_1

$$u_1 = 0$$

$$h_{o_1} = h_1 + \frac{1}{2}u_1^2 = h_1$$

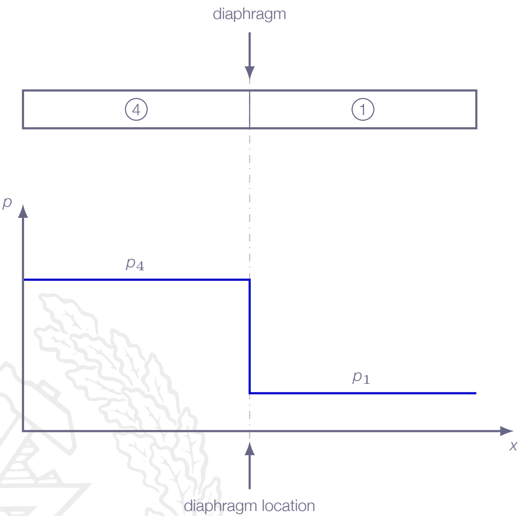
$$h_{o_2} = h_2 + \frac{1}{2}u_2^2$$

$$h_2 > h_1 \Rightarrow h_{o_2} > h_{o_1}$$



The Shock Tube

Shock Tube

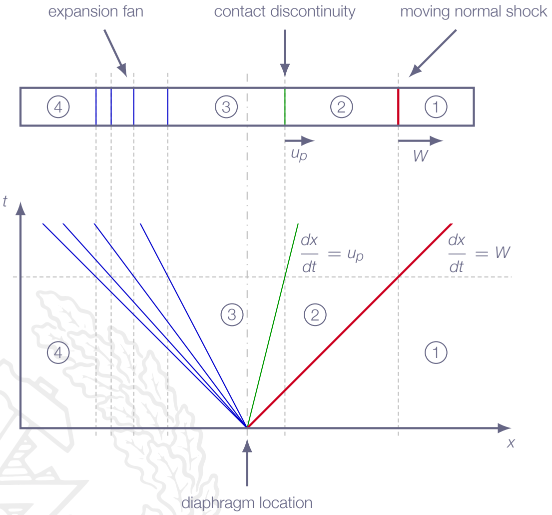


tube with closed ends
diaphragm inside, separating two different constant states
(could also be two different gases)

if diaphragm is removed suddenly (by inducing a breakdown) the two states come into contact and a flow develops

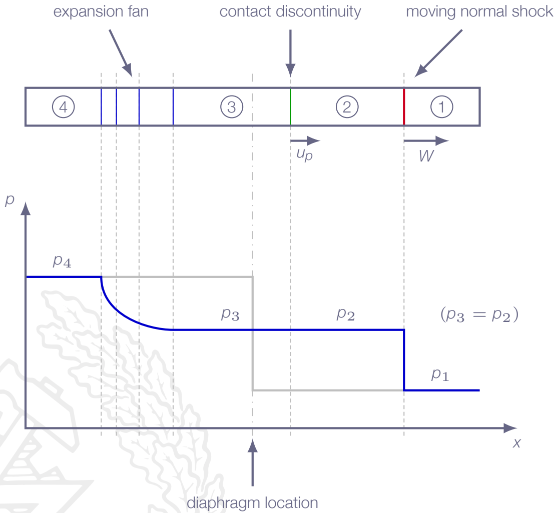
assume that $p_4 > p_1$:
state 4 is "driver" section
state 1 is "driven" section

Shock Tube



flow at some time after diaphragm breakdown

Shock Tube



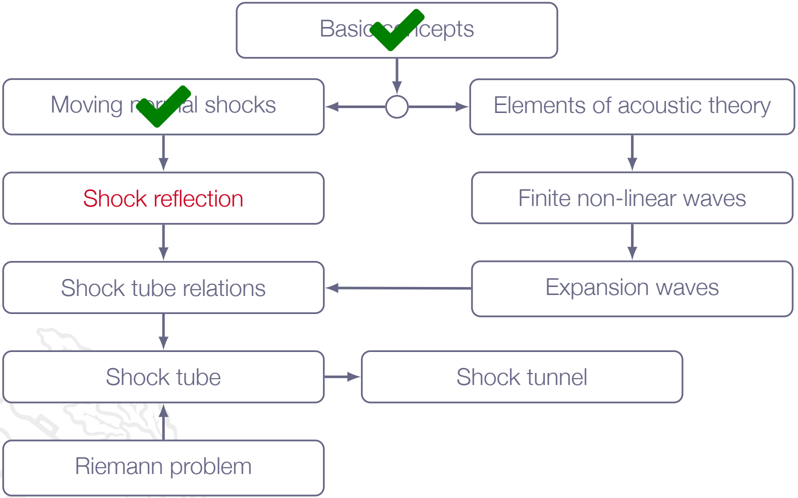
flow at some time after diaphragm breakdown

Shock Tube

- By using light gases for the driver section (*e.g.* He) and heavier gases for the driven section (*e.g.* air) the pressure p_4 required for a specific p_2/p_1 ratio is significantly reduced

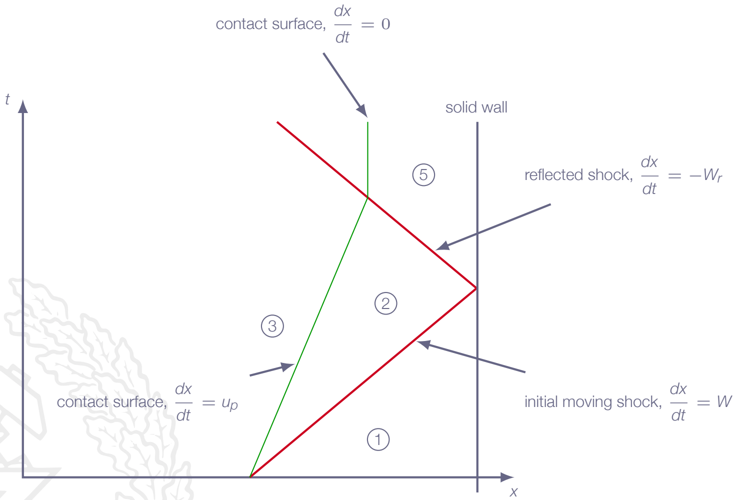
- If T_4/T_1 is increased, the pressure p_4 required for a specific p_2/p_1 is also reduced

Roadmap - Unsteady Wave Motion



Chapter 7.3
Reflected Shock Wave

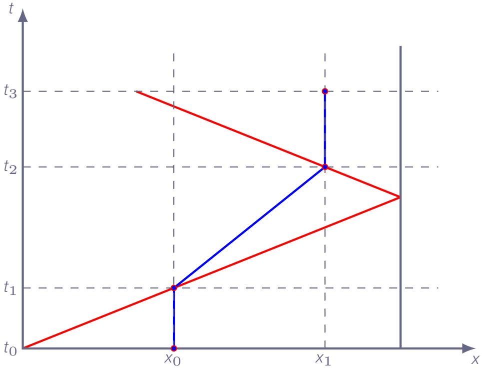
Shock Reflection



Shock Reflection - Particle Path

A fluid particle located at x_0 at time t_0 (a location ahead of the shock) will be affected by the moving shock and follow the blue path

time	location	velocity
t_0	x_0	0
t_1	x_0	u_p
t_2	x_1	u_p
t_3	x_1	0



Shock Reflection Relations

- velocity ahead of reflected shock: $W_r + u_p$
- velocity behind reflected shock: W_r

Continuity:

$$\rho_2(W_r + u_p) = \rho_5 W_r$$

Momentum:

$$p_2 + \rho_2(W_r + u_p)^2 = p_5 + \rho_5 W_r^2$$

Energy:

$$h_2 + \frac{1}{2}(W_r + u_p)^2 = h_5 + \frac{1}{2}W_r^2$$

Shock Reflection Relations

Reflected shock is determined such that $u_5 = 0$

$$\frac{M_r}{M_r^2 - 1} = \frac{M_s}{M_s^2 - 1}\sqrt{1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2}(M_s^2 - 1)\left(\gamma + \frac{1}{M_s^2}\right)}$$

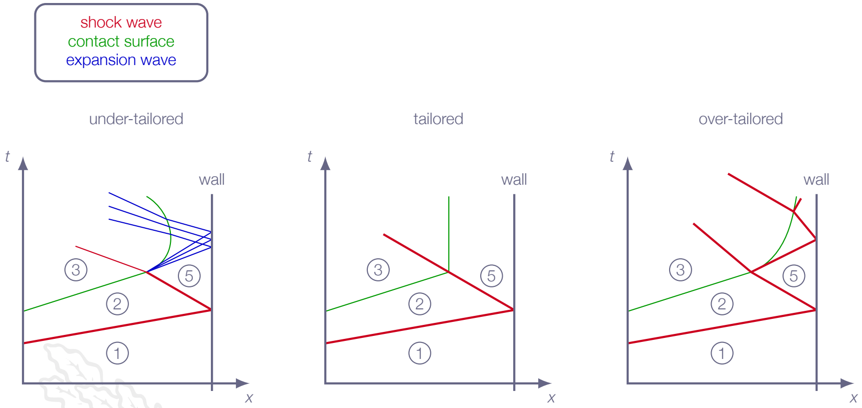
where

$$M_r = \frac{W_r + u_p}{a_2}$$

Tailored v.s. Non-Tailored Shock Reflection

- The time duration of condition 5 is determined by what happens after interaction between reflected shock and contact discontinuity
- For special choice of initial conditions (tailored case), this interaction is negligible, thus prolonging the duration of condition 5

Tailored v.s. Non-Tailored Shock Reflection



Under-tailored conditions:

Mach number of incident wave lower than in tailored conditions

Over-tailored conditions:

Mach number of incident wave higher than in tailored conditions

Shock Reflection - Example

Shock reflection in shock tube ($\gamma = 1.4$)
(Example 7.1 in Anderson)

Incident shock (given data)

p_2/p_1	10.0
M_s	2.95
T_2/T_1	2.623
p_1	1.0 [bar]
T_1	300.0 [K]

Calculated data

M_r 2.09

Table A.2

p_5/p_2	4.978
T_5/T_2	1.77

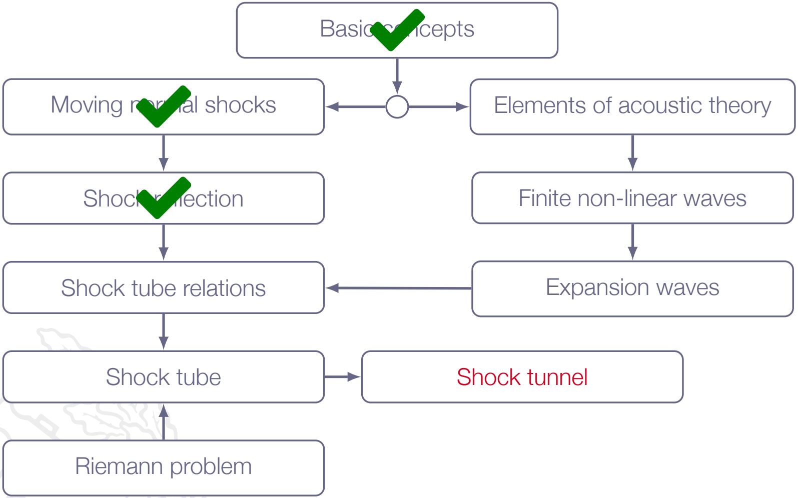
$$p_5 = \left(\frac{p_5}{p_2}\right)\left(\frac{p_2}{p_1}\right)p_1 = 49.78$$

$$T_5 = \left(\frac{T_5}{T_2}\right)\left(\frac{T_2}{T_1}\right)T_1 = 1393$$

Shock Reflection - Shock Tube

- Very high pressure and temperature conditions in a specified location with very high precision (p_5, T_5)
  - measurements of thermodynamic properties of various gases at extreme conditions, *e.g.* dissociation energies, molecular relaxation times, etc.
  - measurements of chemical reaction properties of various gas mixtures at extreme conditions

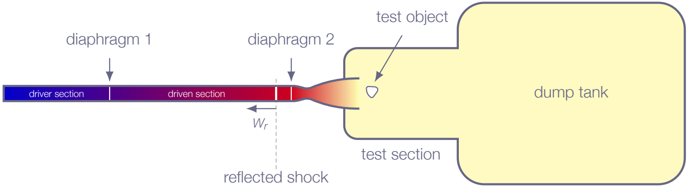
Roadmap - Unsteady Wave Motion



Shock Tunnel

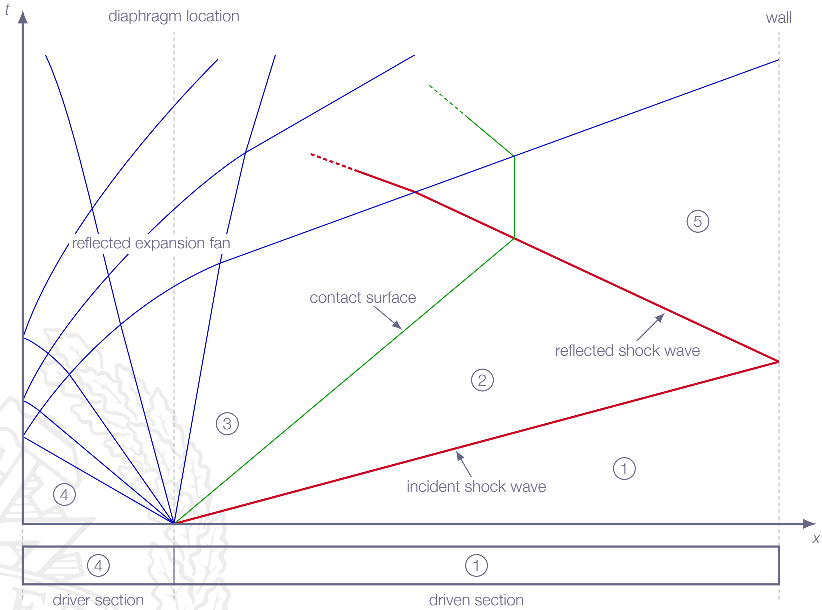
- Addition of a convergent-divergent nozzle to a shock tube configuration
- Capable of producing flow conditions which are close to those during the reentry of a space vehicles into the earth's atmosphere
  - high-enthalpy, hypersonic flows (short time)
  - real gas effects
- Example - Aachen TH2:
  - velocities up to 4 km/s
  - stagnation temperatures of several thousand degrees

Shock Tunnel



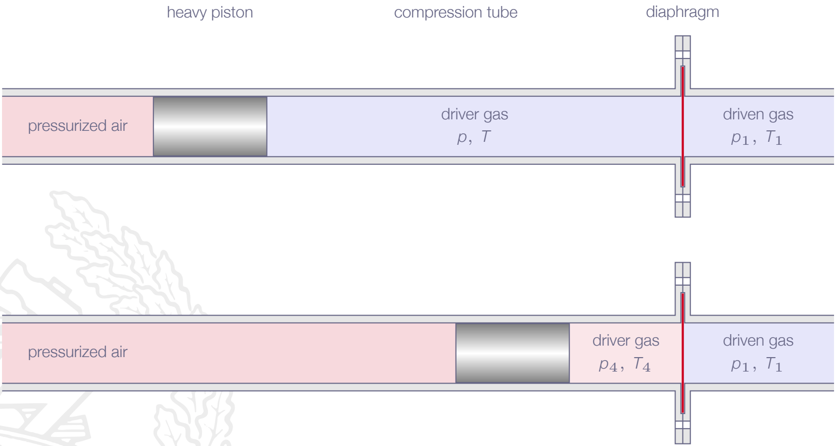
1. High pressure in region 4 (driver section)
   - diaphragm 1 burst
   - primary shock generated
2. Primary shock reaches end of shock tube
   - shock reflection
3. High pressure in region 5
   - diaphragm 2 burst
   - nozzle flow initiated
   - hypersonic flow in test section

Shock Tunnel

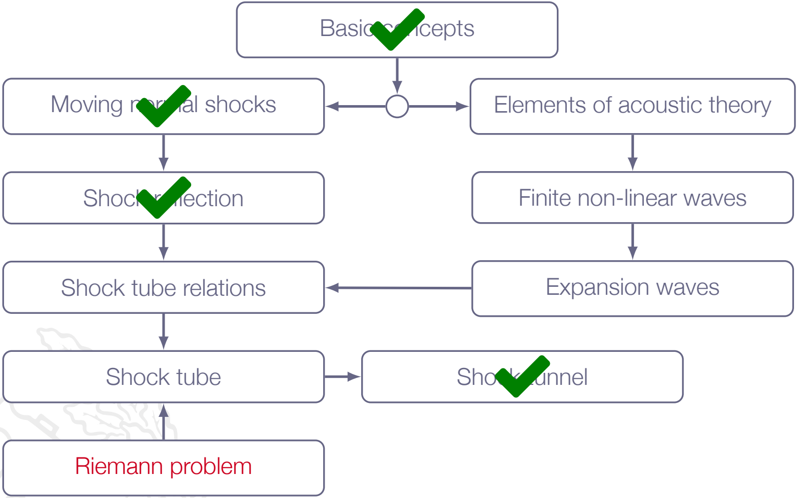


Shock Tunnel

By adding a compression tube to the shock tube a very high p_4 and T_4 may be achieved for any gas in a fairly simple manner



Roadmap - Unsteady Wave Motion



Riemann Problem

The shock tube problem is a special case of the general Riemann Problem

> *"... A Riemann problem, named after Bernhard Riemann, consists of an initial value problem composed by a conservation equation together with piecewise constant data having a single discontinuity ..."*

Wikipedia

Riemann Problem

May show that solutions to the shock tube problem have the general form:

$$
\begin{aligned}
p &= p(x/t) \\
\rho &= \rho(x/t) \\
u &= u(x/t) \\
T &= T(x/t) \\
a &= a(x/t)
\end{aligned}
$$

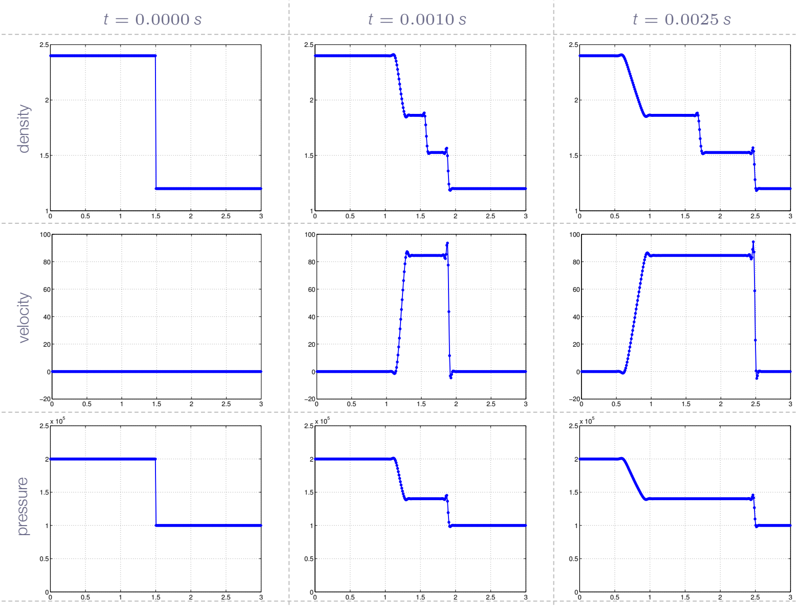
where $x = 0$ denotes the position of the initial jump between states 1 and 4

Riemann Problem - Shock Tube

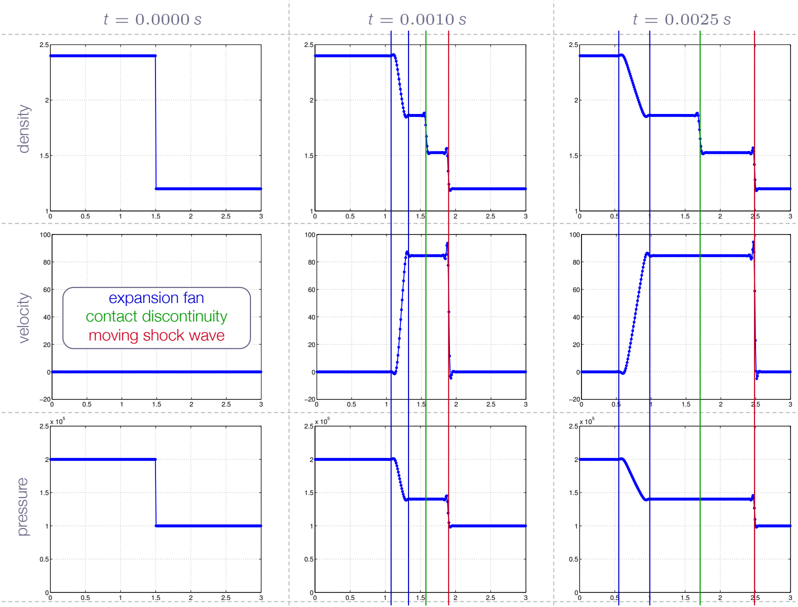
Shock tube simulation:

- left side conditions (state 4):
  - $\rho = 2.4 \, kg/m^3$
  - $u = 0.0 \, m/s$
  - $p = 2.0 \, bar$
- right side conditions (state 1):
  - $\rho = 1.2 \, kg/m^3$
  - $u = 0.0 \, m/s$
  - $p = 1.0 \, bar$
- Numerical method
  - Finite-Volume Method (FVM) solver
  - three-stage Runge-Kutta time stepping
  - third-order characteristic upwinding scheme
  - local artificial damping

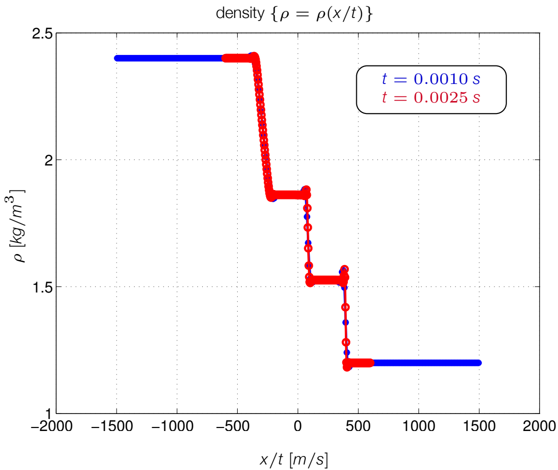
Riemann Problem - Shock Tube



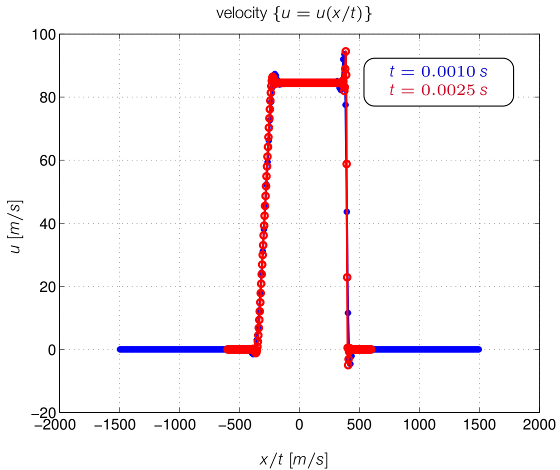
Riemann Problem - Shock Tube



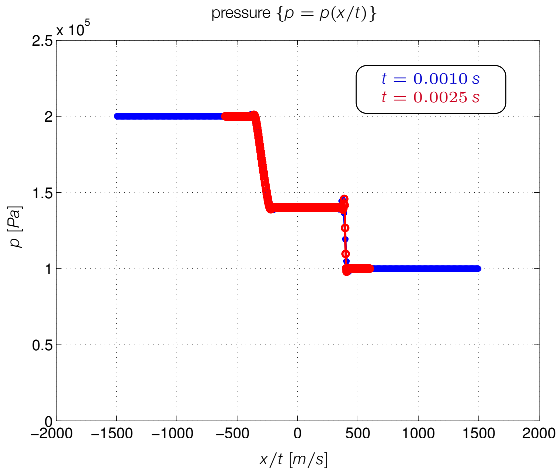
Riemann Problem - Shock Tube



Riemann Problem - Shock Tube



Riemann Problem - Shock Tube



Roadmap - Unsteady Wave Motion