Compressible Flow - TME085

Lecture 10

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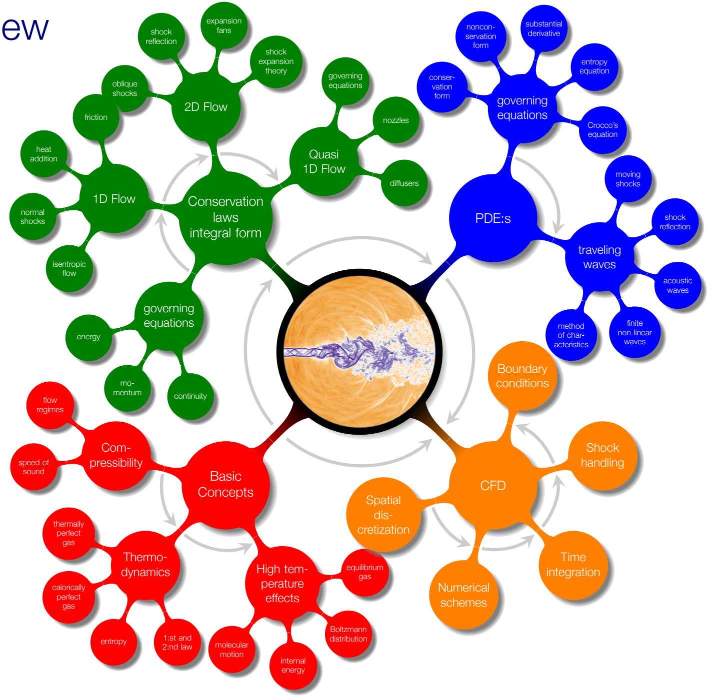
Overview


Conservation laws integral form
2D Flow
shock reflection
expansion fans
shock expansion theory
oblique shocks
friction
1D Flow
heat addition
normal shocks
isentropic flow
Quasi 1D Flow
governing equations
nozzles
diffusers
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energy
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noncon-servation form
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Crocco's equation
traveling waves
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shock reflection
acoustic waves
method of char-acteristics
finite non-linear waves
Basic Concepts
Com-pressibility
flow regimes
speed of sound
Thermo-dynamics
thermally perfect gas
calorically perfect gas
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1:st and 2:nd law
High tem-perature effects
equilibrium gas
Boltzmann distribution
molecular motion
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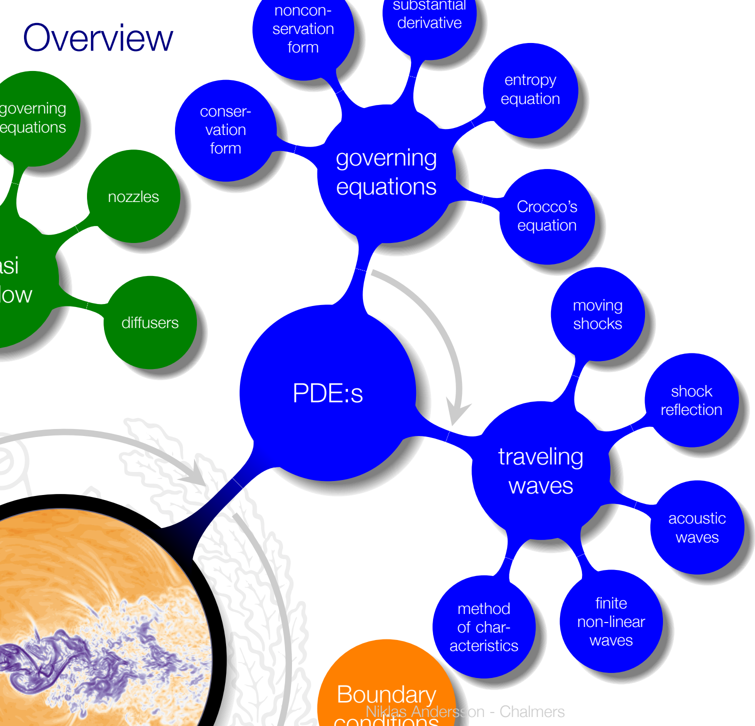
CFD
Boundary conditions
Shock handling
Spatial dis-cretization
Time integration
Numerical schemes

Chapter 7
Unsteady Wave Motion

Overview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acteristics
finite
non-linear
waves
Boundary

Addressed Learning Outcomes

3 Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

8 Derive (marked) and apply (all) of the presented mathematical formulae for classical gas dynamics

   j unsteady waves and discontinuities in 1D

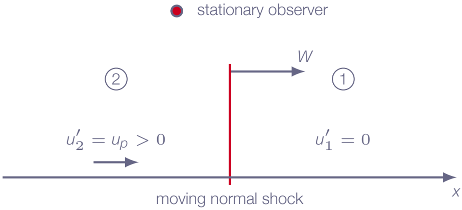
9 Solve engineering problems involving the above-mentioned phenomena (8a-8k)

12 Explain the main principles behind a modern Finite Volume CFD code and such concepts as explicit/implicit time stepping, CFL number, conservation, handling of compression shocks, and boundary conditions

what happens when a moving shock approaches a wall?

Chapter 7.2 Moving Normal Shock Waves

Moving Normal Shock Waves - Governing Equations



For stationary normal shocks we have:

$$\rho_1 u_1 = \rho_2 u_2$$
$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$$
$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2$$

With $(u_1 = W)$ and $(u_2 = W - u_p)$ we get:

$$\rho_1 W = \rho_2 (W - u_p)$$
$$\rho_1 W^2 + p_1 = \rho_2 (W - u_p)^2 + p_2$$
$$h_1 + \frac{1}{2}W^2 = h_2 + \frac{1}{2}(W - u_p)^2$$

Moving Normal Shock Waves - Relations

From the continuity equation we get:

$$u_p = W\left(1 - \frac{\rho_1}{\rho_2}\right) > 0$$

After some derivation we obtain:

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right]^{1/2}$$

Moving Normal Shock Waves - Relations

May also show that

$$\frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}$$

and

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left[\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}\right]$$

Moving Normal Shock Waves - Relations

Induced Mach number:

$$M_p = \frac{u_p}{a_2} = \frac{u_p}{a_1}\frac{a_1}{a_2} = \frac{u_p}{a_1}\sqrt{\frac{T_1}{T_2}}$$

inserting u_p/a_1 and T_1/T_2 from relations on previous slides we get:

$$M_p = \frac{1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{\gamma-1}{\gamma+1} + \frac{p_2}{p_1}}\right]^{1/2}\left[\frac{1 + \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right)}{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right) + \left(\frac{p_2}{p_1}\right)^2}\right]^{1/2}$$

Moving Normal Shock Waves - Relations

Note that

$$\lim_{\frac{p_2}{p_1} \to \infty} M_p \to \sqrt{\frac{2}{\gamma(\gamma - 1)}}$$

for air ($\gamma = 1.4$)

$$\lim_{\frac{p_2}{p_1} \to \infty} M_p \to 1.89$$

Moving Normal Shock Waves - Relations

Note that $h_{o_1} \neq h_{o_2}$

constant total enthalpy is only valid for stationary shocks!

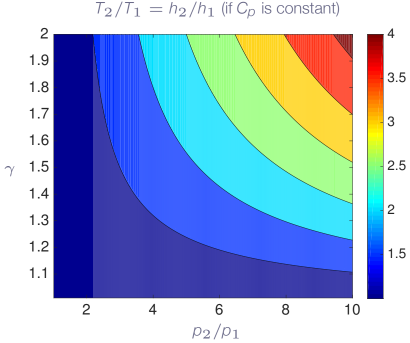
shock is uniquely defined by pressure ratio p_2/p_1

$$u_1 = 0$$

$$h_{o_1} = h_1 + \frac{1}{2}u_1^2 = h_1$$

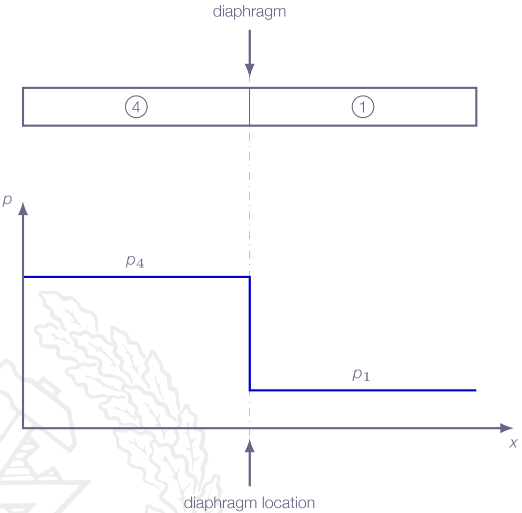
$$h_{o_2} = h_2 + \frac{1}{2}u_2^2$$

$$h_2 > h_1 \Rightarrow h_{o_2} > h_{o_1}$$



The Shock Tube

Shock Tube

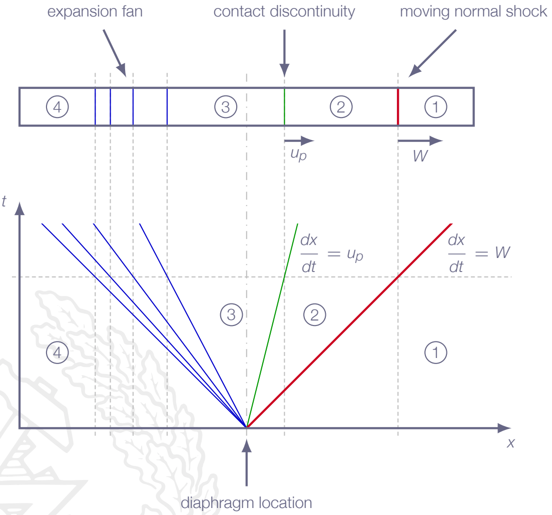


tube with closed ends
diaphragm inside, separating two different constant states
(could also be two different gases)

if diaphragm is removed suddenly (by inducing a breakdown) the two states come into contact and a flow develops

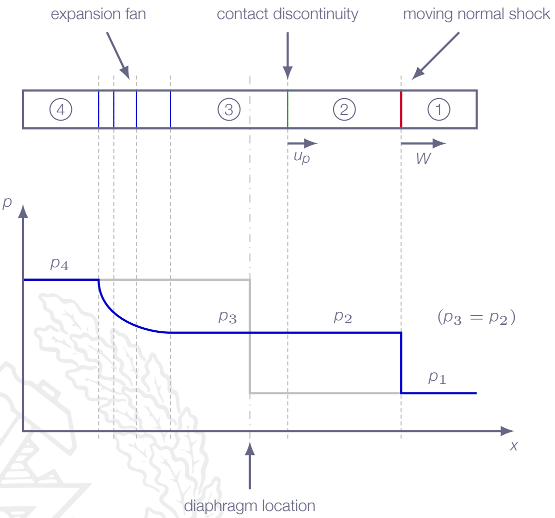
assume that $p_4 > p_1$:
state 4 is "driver" section
state 1 is "driven" section

Shock Tube



flow at some time after diaphragm breakdown

Shock Tube



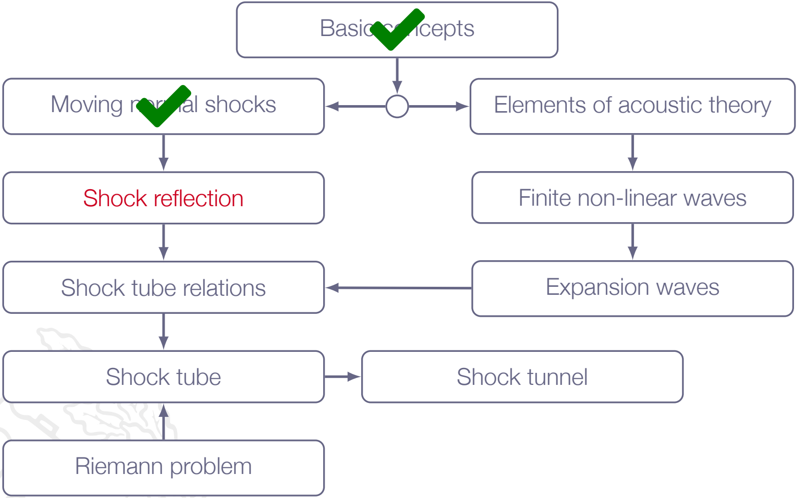
flow at some time after diaphragm breakdown

Shock Tube

- By using light gases for the driver section (*e.g.* He) and heavier gases for the driven section (*e.g.* air) the pressure p_4 required for a specific p_2/p_1 ratio is significantly reduced

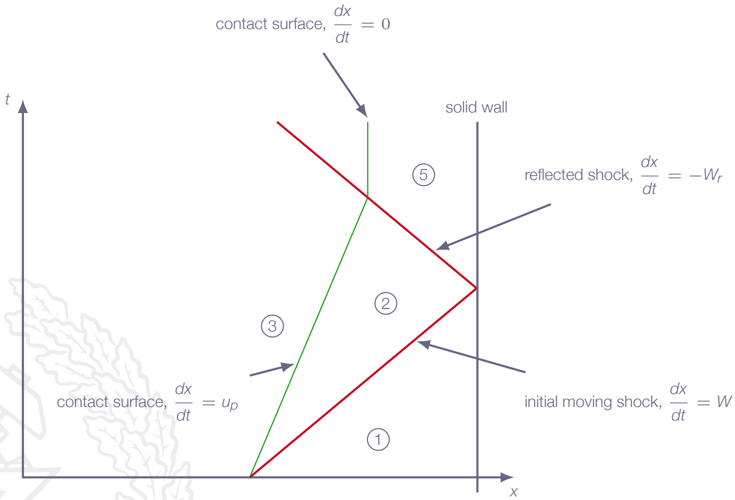
- If T_4/T_1 is increased, the pressure p_4 required for a specific p_2/p_1 is also reduced

Roadmap - Unsteady Wave Motion



Chapter 7.3
Reflected Shock Wave

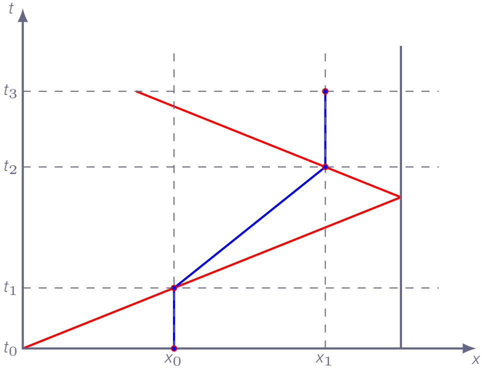
Shock Reflection



Shock Reflection - Particle Path

A fluid particle located at x_0 at time t_0 (a location ahead of the shock) will be affected by the moving shock and follow the blue path

time	location	velocity
t_0	x_0	0
t_1	x_0	u_p
t_2	x_1	u_p
t_3	x_1	0



Shock Reflection Relations

- velocity ahead of reflected shock: $W_r + u_p$
- velocity behind reflected shock: W_r

Continuity:

$$\rho_2(W_r + u_p) = \rho_5 W_r$$

Momentum:

$$p_2 + \rho_2(W_r + u_p)^2 = p_5 + \rho_5 W_r^2$$

Energy:

$$h_2 + \frac{1}{2}(W_r + u_p)^2 = h_5 + \frac{1}{2}W_r^2$$

Shock Reflection Relations

Reflected shock is determined such that $u_5 = 0$

$$\frac{M_r}{M_r^2 - 1} = \frac{M_s}{M_s^2 - 1}\sqrt{1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2}(M_s^2 - 1)\left(\gamma + \frac{1}{M_s^2}\right)}$$

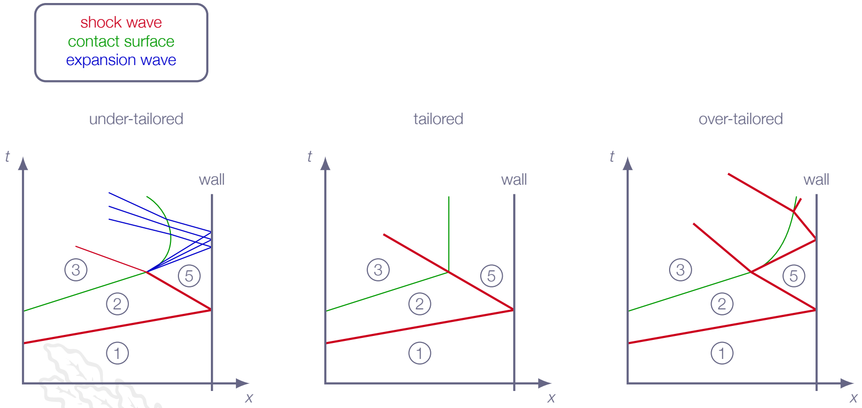
where

$$M_r = \frac{W_r + u_p}{a_2}$$

Tailored v.s. Non-Tailored Shock Reflection

- The time duration of condition 5 is determined by what happens after interaction between reflected shock and contact discontinuity
- For special choice of initial conditions (tailored case), this interaction is negligible, thus prolonging the duration of condition 5

Tailored v.s. Non-Tailored Shock Reflection



Under-tailored conditions:

Mach number of incident wave lower than in tailored conditions

Over-tailored conditions:

Mach number of incident wave higher than in tailored conditions

Shock Reflection - Example

Shock reflection in shock tube ($\gamma = 1.4$)
(Example 7.1 in Anderson)

Incident shock (given data)

p_2/p_1	10.0
M_s	2.95
T_2/T_1	2.623
p_1	1.0 [bar]
T_1	300.0 [K]

Calculated data

M_r	2.09

Table A.2

p_5/p_2	4.978
T_5/T_2	1.77

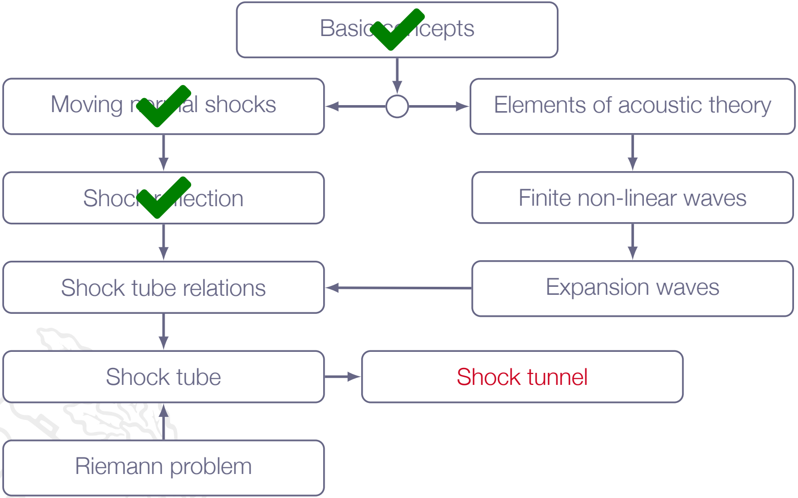
$$p_5 = \left(\frac{p_5}{p_2}\right)\left(\frac{p_2}{p_1}\right)p_1 = 49.78$$

$$T_5 = \left(\frac{T_5}{T_2}\right)\left(\frac{T_2}{T_1}\right)T_1 = 1393$$

Shock Reflection - Shock Tube

- Very high pressure and temperature conditions in a specified location with very high precision (p_5, T_5)
  - measurements of thermodynamic properties of various gases at extreme conditions, *e.g.* dissociation energies, molecular relaxation times, etc.
  - measurements of chemical reaction properties of various gas mixtures at extreme conditions

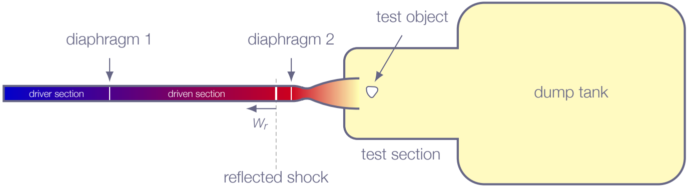
Roadmap - Unsteady Wave Motion



Shock Tunnel

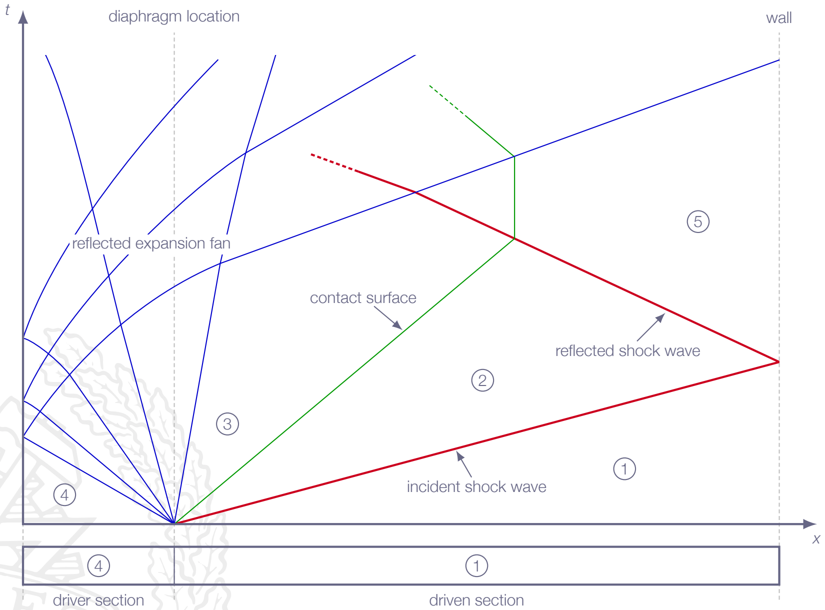
- Addition of a convergent-divergent nozzle to a shock tube configuration
- Capable of producing flow conditions which are close to those during the reentry of a space vehicles into the earth's atmosphere
  - high-enthalpy, hypersonic flows (short time)
  - real gas effects
- Example - Aachen TH2:
  - velocities up to 4 km/s
  - stagnation temperatures of several thousand degrees

Shock Tunnel



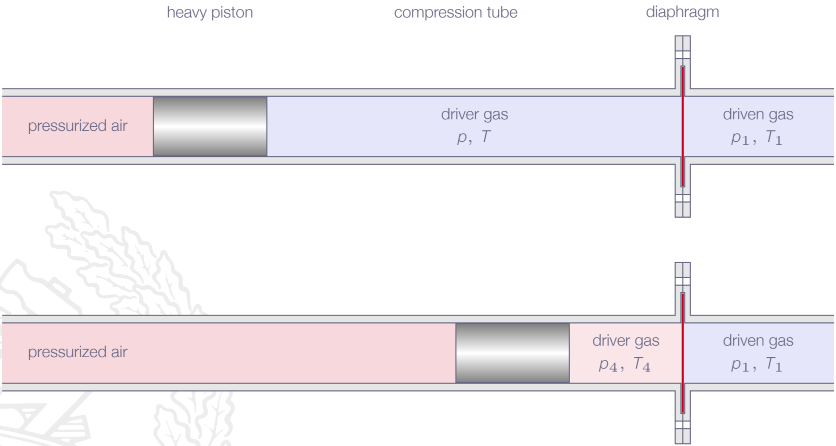
1. High pressure in region 4 (driver section)
   - diaphragm 1 burst
   - primary shock generated
2. Primary shock reaches end of shock tube
   - shock reflection
3. High pressure in region 5
   - diaphragm 2 burst
   - nozzle flow initiated
   - hypersonic flow in test section

Shock Tunnel



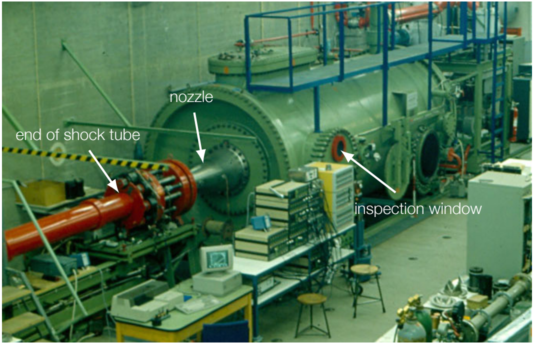
Shock Tunnel

By adding a compression tube to the shock tube a very high p_4 and T_4 may be achieved for any gas in a fairly simple manner



The Aachen Shock Tunnel - TH2

Shock tunnel built
1975



The Aachen Shock Tunnel - TH2

Shock tube specifications:

diameter	140 mm
driver section	6.0 m
driven section	15.4 m
diaphragm 1	10 mm stainless steel
diaphragm 2	copper/brass sheet
max operating (steady) pressure	1500 bar

The Aachen Shock Tunnel - TH2

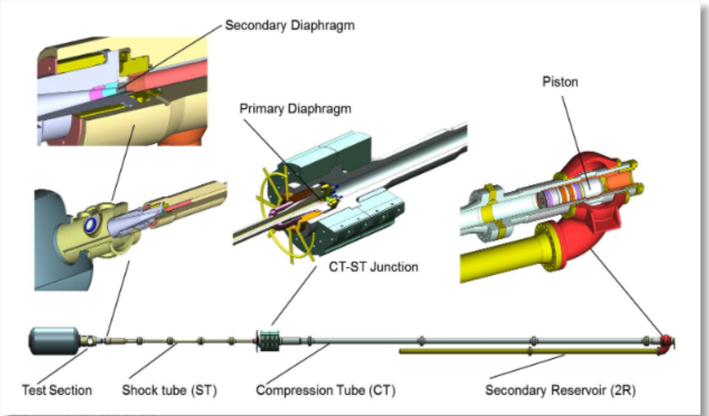
- Driver gas (usually helium):
  - 100 bar $< p_4 <$ 1500 bar
  - electrical preheating (optional) to 600 K
- Driven gas:
  - 0.1 bar $< p_1 <$ 10 bar
- Dump tank evacuated before test

The Aachen Shock Tunnel - TH2

initial conditions			shock		reservoir		free stream			
p_4 [bar]	T_4 [K]	p_1 [bar]	M_s	p_2 [bar]	p_5 [bar]	T_5 [K]	M_∞	T_∞ [K]	u_∞ [m/s]	p_∞ [mbar]
100	293	1.0	3.3	12	65	1500	7.7	125	1740	7.6
370	500	1.0	4.6	26	175	2500	7.4	250	2350	20.0
720	500	0.7	5.6	50	325	3650	6.8	460	3910	42.0
1200	500	0.6	6.8	50	560	4600	6.5	700	3400	73.0
100	293	0.9	3.4	12	65	1500	11.3	60	1780	0.6
450	500	1.2	4.9	29	225	2700	11.3	120	2480	1.5
1300	520	0.7	6.4	46	630	4600	12.1	220	3560	1.2
26	293	0.2	3.4	12	15	1500	11.4	60	1780	0.1
480	500	0.2	6.6	50	210	4600	11.0	270	3630	0.7
100	293	1.0	3.4	12	65	1500	7.7	130	1750	7.3
370	500	1.0	5.1	27	220	2700	7.3	280	2440	26.3

The Caltech Shock Tunnel - T5

Free-piston shock tunnel



The Caltech Shock Tunnel - T5

- Compression tube (CT):
  - length 30 m, diameter 300 mm
  - free piston (120 kg)
  - max piston velocity: 300 m/s
  - driven by compressed air (80 bar - 150 bar)

- Shock tube (ST):
  - length 12 m, diameter 90 mm
  - driver gas: helium + argon
  - driven gas: air
  - diaphragm 1: 7 mm stainless steel
  - p_4 max 1300 bar

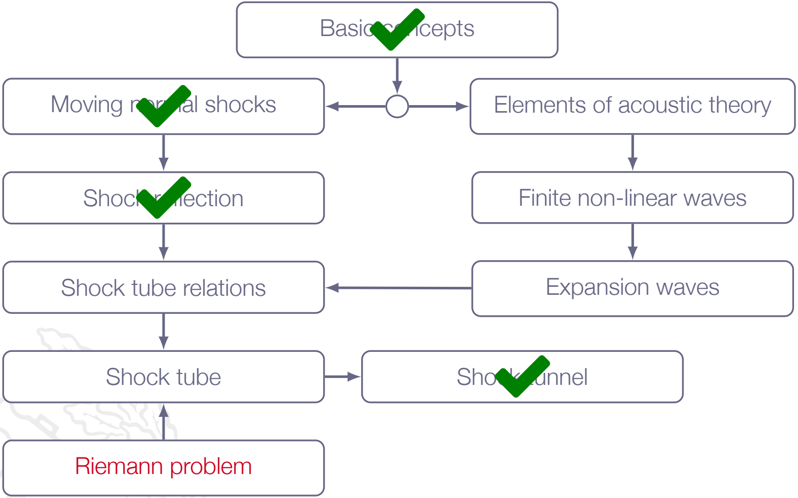
The Caltech Shock Tunnel - T5

- Reservoir conditions:
  - p_5 1000 bar
  - T_5 10000 K

- Freestream conditions (design conditions):
  - M_∞ 5.2
  - T_∞ 2000 K
  - p_∞ 0.3 bar
  - typical test time 1 ms

Other Examples of Shock Tunnels

Roadmap - Unsteady Wave Motion



Riemann Problem

The shock tube problem is a special case of the general Riemann Problem

> *"... A Riemann problem, named after Bernhard Riemann, consists of an initial value problem composed by a conservation equation together with piecewise constant data having a single discontinuity ..."*

Wikipedia

Riemann Problem

May show that solutions to the shock tube problem have the general form:

$$p = p(x/t)$$
$$\rho = \rho(x/t)$$
$$u = u(x/t)$$
$$T = T(x/t)$$
$$a = a(x/t)$$

where $x = 0$ denotes the position of the initial jump between states 1 and 4

Riemann Problem - Shock Tube

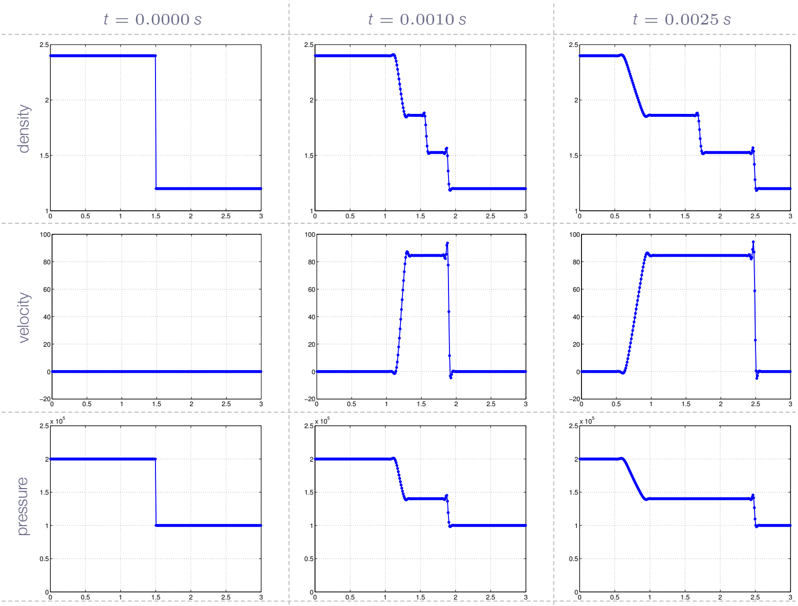
Shock tube simulation:

- left side conditions (state 4):
  - $\rho = 2.4\ kg/m^3$
  - $u = 0.0\ m/s$
  - $p = 2.0\ bar$

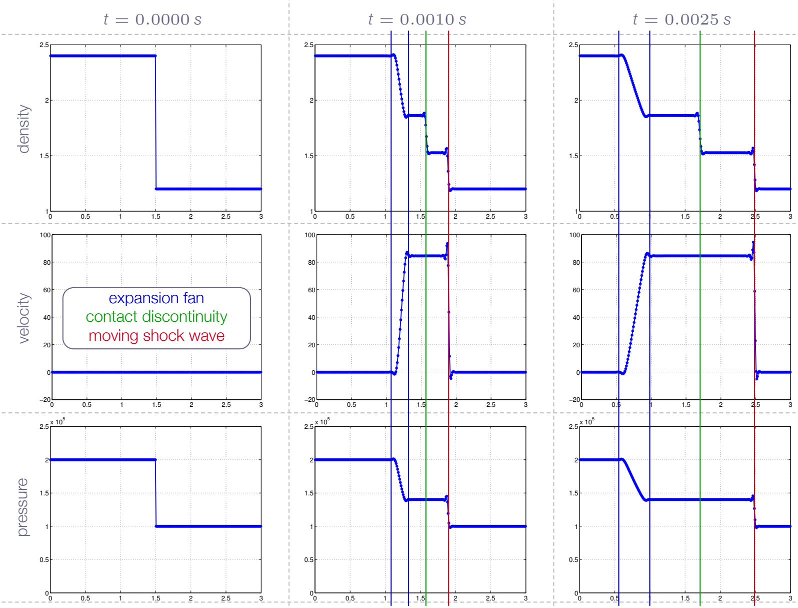
- right side conditions (state 1):
  - $\rho = 1.2\ kg/m^3$
  - $u = 0.0\ m/s$
  - $p = 1.0\ bar$

- Numerical method
  - Finite-Volume Method (FVM) solver
  - three-stage Runge-Kutta time stepping
  - third-order characteristic upwinding scheme
  - local artificial damping

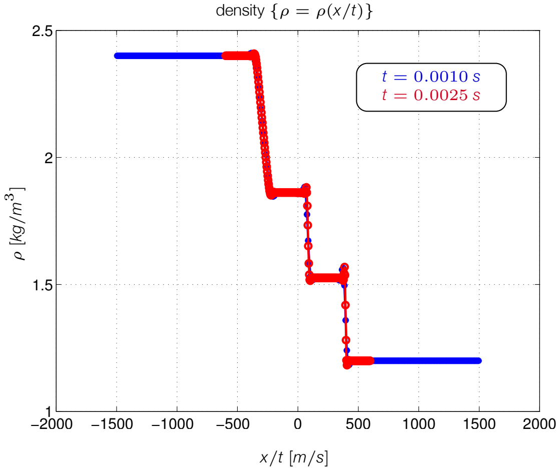
Riemann Problem - Shock Tube



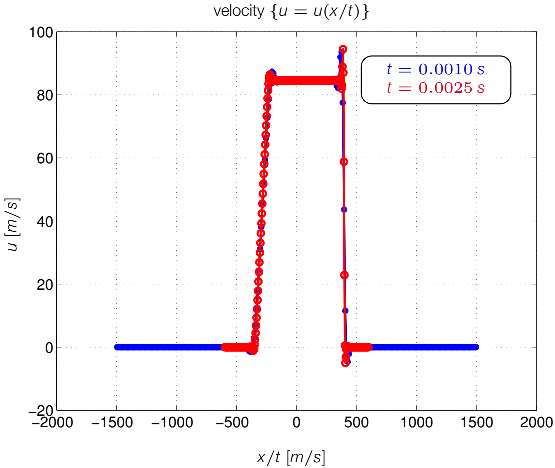
Riemann Problem - Shock Tube



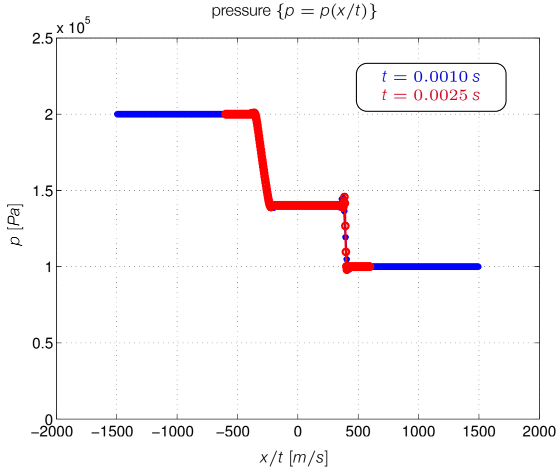
Riemann Problem - Shock Tube



Riemann Problem - Shock Tube



Riemann Problem - Shock Tube



Roadmap - Unsteady Wave Motion