Compressible Flow - TME085

Lecture 5

Niklas Andersson

Chalmers University of Technology
Department of Mechanics and Maritime Sciences
Division of Fluid Mechanics
Gothenburg, Sweden

niklas.andersson@chalmers.se

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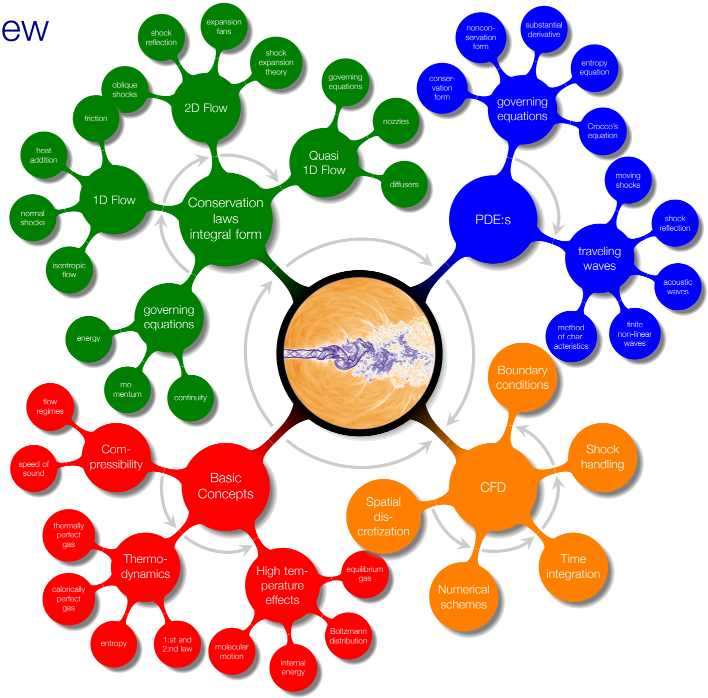
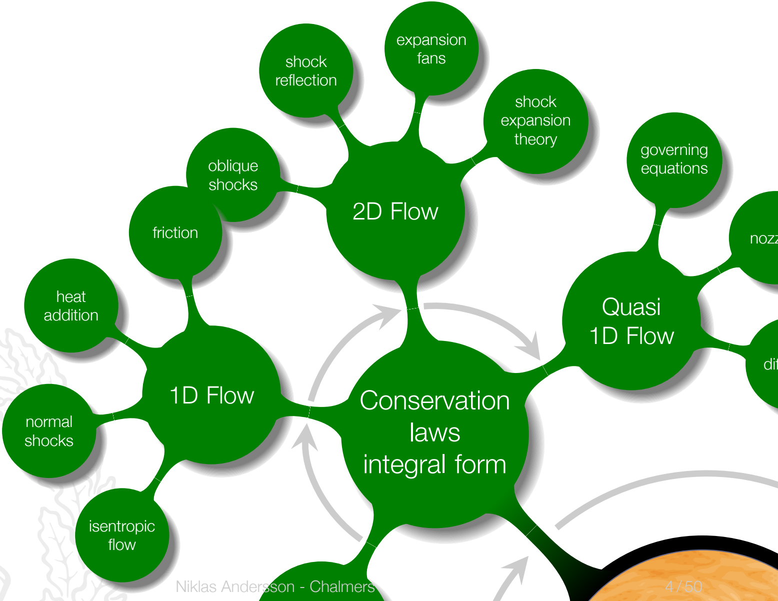
Overview



Conservation laws integral form
1D Flow
heat addition
normal shocks
isentropic flow
friction
2D Flow
oblique shocks
shock reflection
expansion fans
shock expansion theory
Quasi 1D Flow
governing equations
nozzles
diffusers
governing equations
energy
mo-mentum
continuity
PDE:s
governing equations
noncon-servation form
substantial derivative
entropy equation
conser-vation form
Crocco's equation
traveling waves
moving shocks
shock reflection
acoustic waves
finite non-linear waves
method of char-acteristics
Basic Concepts
Com-pressibility
flow regimes
speed of sound
Thermo-dynamics
thermally perfect gas
calorically perfect gas
entropy
1:st and 2:nd law
High tem-perature effects
equilibrium gas
Boltzmann distribution
internal energy
molecular motion
CFD
Boundary conditions
Shock handling
Time integration
Numerical schemes
Spatial dis-cretization

Chapter 4 Oblique Shocks and Expansion Waves

Overview



shock reflection
expansion fans
shock expansion theory
oblique shocks
friction
2D Flow
governing equations
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1D Flow
Quasi 1D Flow
normal shocks
Conservation laws integral form
isentropic flow

Addressed Learning Outcomes

4 Present at least two different formulations of the governing equations for compressible flows and explain what basic conservation principles they are based on

7 Explain why entropy is important for flow discontinuities

8 Derive (marked) and apply (all) of the presented mathematical formulae for classical gas dynamics

  b normal shocks*

  e oblique shocks in 2D*

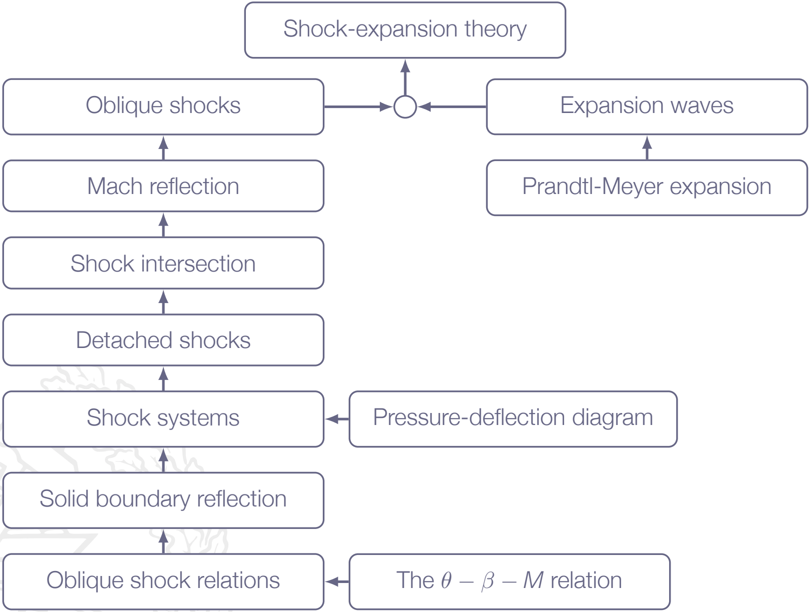
  f shock reflection at solid walls*

  g contact discontinuities

  i detached blunt body shocks, nozzle flows

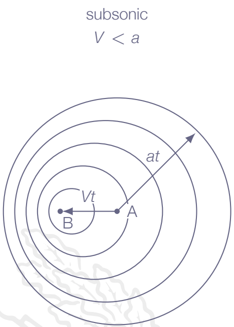
why do we get normal shocks in some cases and oblique shocks in other?

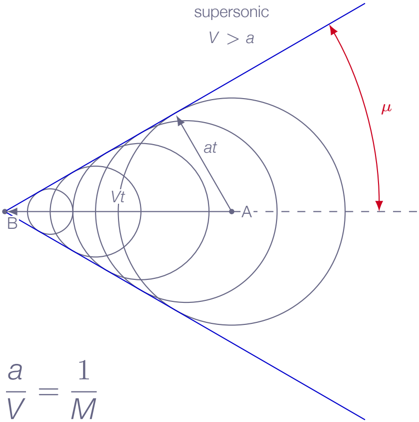
Roadmap - Oblique Shocks and Expansion Waves



Mach Waves

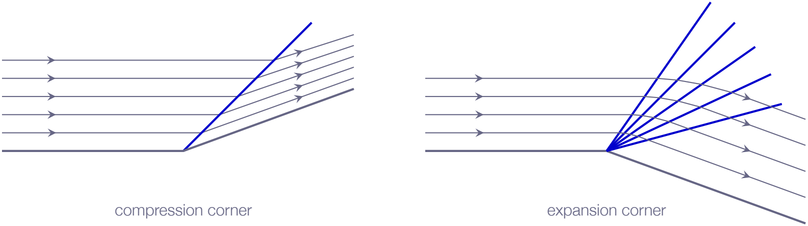
A Mach wave is an infinitely weak oblique shock





$$\sin\mu = \frac{at}{Vt} = \frac{a}{V} = \frac{1}{M}$$

Oblique Shocks and Expansion Waves



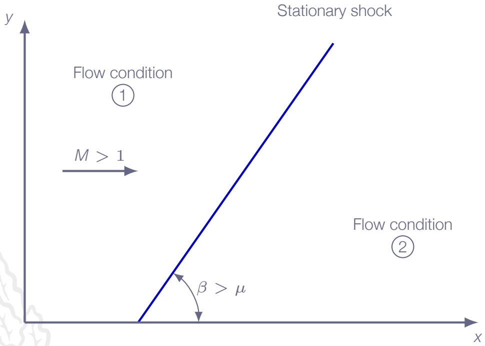
Supersonic two-dimensional steady-state inviscid flow
(no wall friction)

In real flow, viscosity is non-zero $\Rightarrow$ boundary layers

For high-Reynolds-number flows, boundary layers are thin $\Rightarrow$
inviscid theory still relevant!

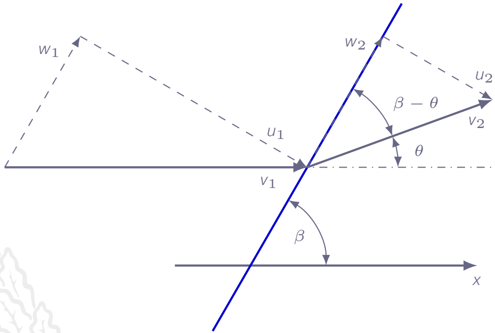
Oblique Shocks

Two-dimensional steady-state flow

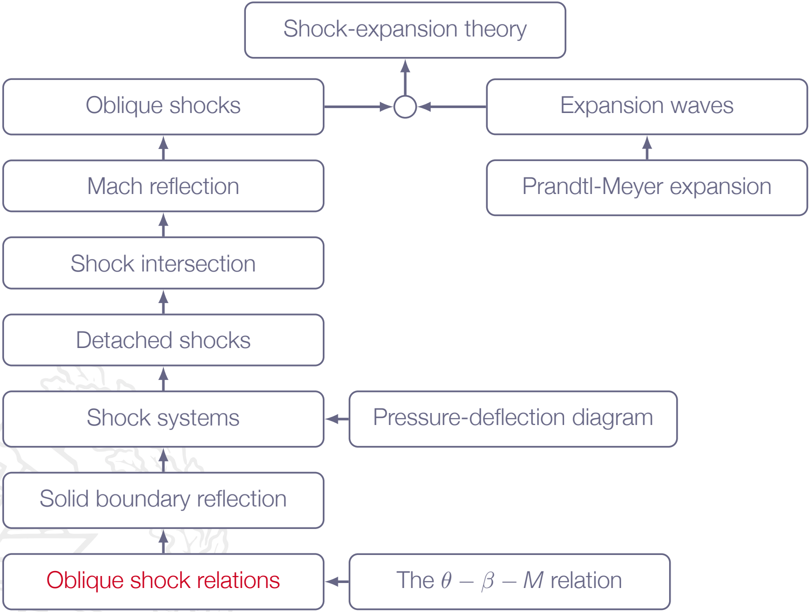


Oblique Shocks

Stationary oblique shock

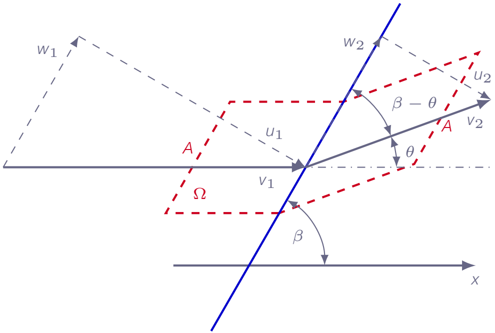


Roadmap - Oblique Shocks and Expansion Waves



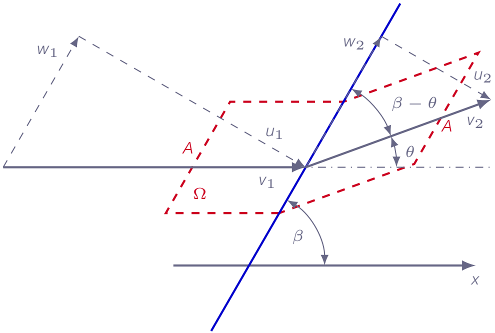
Chapter 4.3
Oblique Shock Relations

Oblique Shock Relations



- Two-dimensional steady-state flow
- Control volume aligned with flow stream lines

Oblique Shock Relations



Velocity notations:

$$M_{n_1} = \frac{u_1}{a_1} = M_1 \sin(\beta) \qquad M_1 = \frac{v_1}{a_1}$$

$$M_{n_2} = \frac{u_2}{a_2} = M_2 \sin(\beta - \theta) \qquad M_2 = \frac{v_2}{a_2}$$

Oblique Shock Relations

Conservation of mass:

$$\frac{d}{dt}\iiint\limits_{\Omega}\rho d\mathscr{V} + \oiint\limits_{\partial\Omega}\rho\mathbf{v}\cdot\mathbf{n}dS = 0$$

Mass conservation for control volume Ω:

$$0 - \rho_1 u_1 A + \rho_2 u_2 A = 0 \Rightarrow$$

$$\boxed{\rho_1 u_1 = \rho_2 u_2}$$

Oblique Shock Relations

Conservation of momentum:

$$\frac{d}{dt}\iiint_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oiint_{\partial\Omega} \left[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}\right] dS = \iiint_{\Omega} \rho \mathbf{f} d\mathscr{V}$$

Momentum in shock-normal direction:

$$0 - (\rho_1 u_1^2 + p_1)A + (\rho_2 u_2^2 + p_2)A = 0 \Rightarrow$$

$$\boxed{\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2}$$

Oblique Shock Relations

Momentum in shock-tangential direction:

$$0 - \rho_1 u_1 w_1 A + \rho_2 u_2 w_2 A = 0 \Rightarrow$$

$$\boxed{w_1 = w_2}$$

Oblique Shock Relations

Conservation of energy:

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho e_o d\mathscr{V} + \oiint\limits_{\partial\Omega} [\rho h_o \mathbf{v}\cdot\mathbf{n}]\, dS = \iiint\limits_{\Omega} \rho \mathbf{f}\cdot\mathbf{v} d\mathscr{V}$$

Energy equation applied to the control volume Ω:

$$0 - \rho_1 u_1 [h_1 + \frac{1}{2}(u_1^2 + w_1^2)]A + \rho_2 u_2 [h_2 + \frac{1}{2}(u_2^2 + w_2^2)]A = 0 \Rightarrow$$

$$\boxed{h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2}$$

Oblique Shock Relations

We can use the equations as for normal shocks if we replace M_1 with M_{n_1} and M_2 with M_{n_2}

$$M_{n_2}^2 = \frac{M_{n_1}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)]M_{n_1}^2 - 1}$$

Ratios such as ρ_2/ρ_1, p_2/p_1, and T_2/T_1 can be calculated using the relations for normal shocks with M_1 replaced by M_{n_1}

Oblique Shock Relations

What about ratios involving stagnation flow properties, can we use the ones previously derived for normal shocks?

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P_{o_1}, T_{o_1}, etc are calculated using M_1 not M_{n_1} (the tangential velocity is included)

Oblique Shock Relations

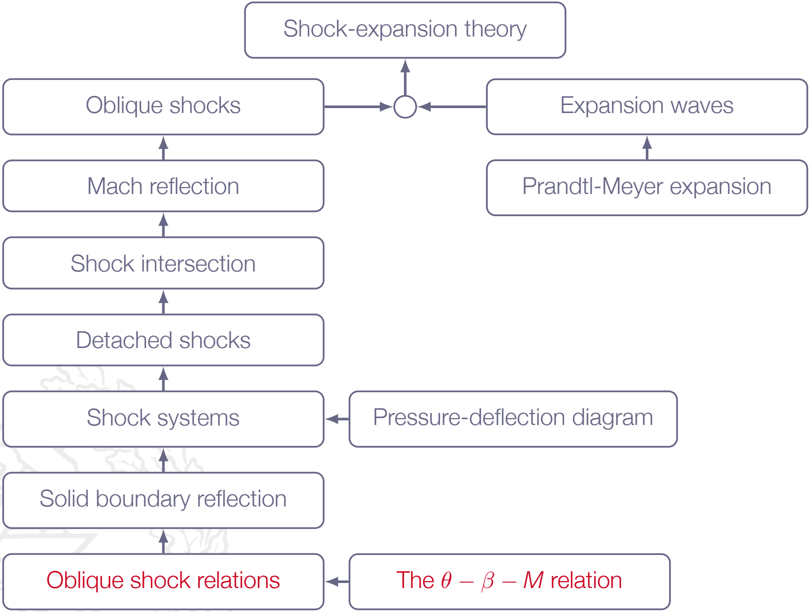
What about ratios involving stagnation flow properties, can we use the ones previously derived for normal shocks?

The answer is no, but why?

P_{o_1}, T_{o_1}, etc are calculated using M_1 not M_{n_1} (the tangential velocity is included)

OBS! Do not not use ratios involving total quantities, *e.g.* p_{o_2}/p_{o_1}, T_{o_2}/T_{o_1}, obtained from formulas or tables for normal shock

Roadmap - Oblique Shocks and Expansion Waves



The θ-β-M Relation

It can be shown that

$$\tan\theta = 2\cot\beta\left(\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right)$$

which is the θ-β-M relation

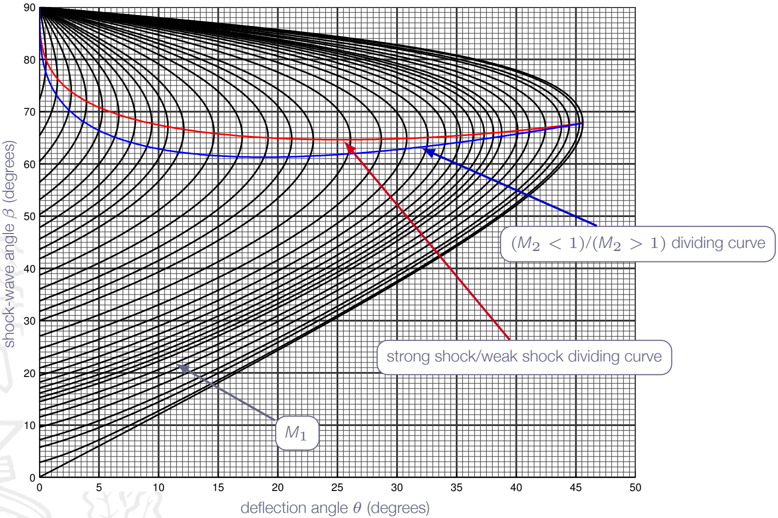
Does this give a complete specification of flow state 2 as function of flow state 1?

The θ-β-M Relation

$$\tan\theta = 2\cot\beta\left(\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right)$$

In general there are two solutions for a given M_1 (or none)

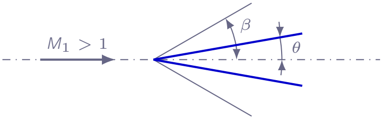
Oblique shock properties (the θ-β-M relation for $\gamma = 1.4$)



The θ-β-M Relation

$$\tan\theta = 2\cot\beta\left(\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right)$$

Example: Wedge flow



Two solution case:

Weak solution:

- smaller β, $M_2 > 1$ (except in some cases)

Strong solution:

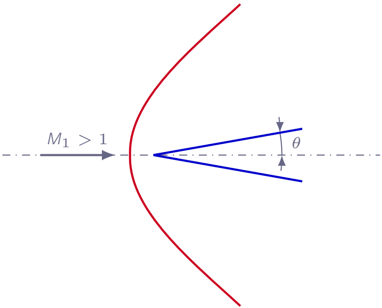
- larger β, $M_2 < 1$

Note: In Chapter 3 we learned that the mach number always reduces to subsonic values behind a shock. This is true for normal shocks (shocks that are normal to the flow direction). It is also true for oblique shocks if looking in the shock-normal direction.

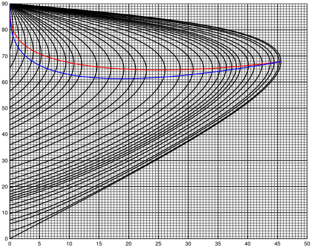
The θ-β-M Relation

$$\tan\theta = 2\cot\beta\left(\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right)$$

No solution case: Detached curved shock



The θ-β-M Relation - Shock Strength



- There is a small region where we may find weak shock solutions for which $M_2 < 1$
- In most cases weak shock solutions have $M_2 > 1$
- Strong shock solutions always have $M_2 < 1$
- In practical situations, weak shock solutions are most common
- Strong shock solution may appear in special situations due to high back pressure, which forces $M_2 < 1$

The θ-β-M Relation - Wedge Flow

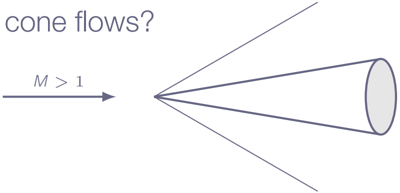
Wedge flow oblique shock analysis:

1. θ-β-M relation $\Rightarrow$ β for given M_1 and θ
2. β gives M_{n_1} according to: $M_{n_1} = M_1 \sin(\beta)$
3. normal shock formula with M_{n_1} instead of M_1 $\Rightarrow$ M_{n_2} (instead of M_2)
4. M_2 given by $M_2 = M_{n_2} / \sin(\beta - \theta)$
5. normal shock formula with M_{n_1} instead of M_1 $\Rightarrow$ ρ_2/ρ_1, p_2/p_1, etc
6. upstream conditions + ρ_2/ρ_1, p_2/p_1, etc $\Rightarrow$ downstream conditions

Chapter 4.4 Supersonic Flow over Wedges and Cones

Supersonic Flow over Wedges and Cones

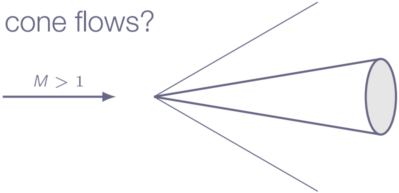
What about cone flows?



- Similar to wedge flow, we do get a constant-strength shock wave, attached at the cone tip (or else a detached curved shock)
- The attached shock is also cone-shaped

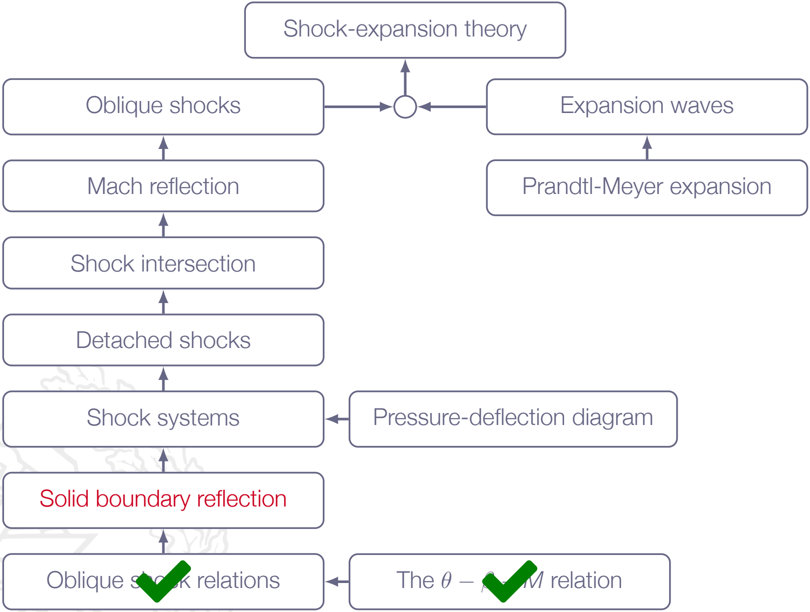
Supersonic Flow over Wedges and Cones

What about cone flows?



- The flow condition immediately downstream of the shock is uniform
- However, downstream of the shock the streamlines are curved and the flow varies in a more complex manner (3D relieving effect - as R increases there is more and more space around cone for the flow)
- β for cone shock is always smaller than that for wedge shock, if M_1 is the same

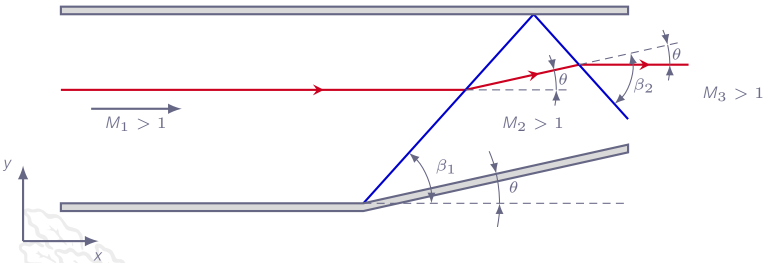
Roadmap - Oblique Shocks and Expansion Waves



Chapter 4.6
Regular Reflection from a Solid Boundary

Shock Reflection

Regular reflection of oblique shock at solid wall
(see example 4.10)



Assumptions:

- steady-state inviscid flow
- weak shocks

Shock Reflection

first shock:

- upstream condition:

  $M_1 > 1$, flow in x-direction
- downstream condition:

  weak shock $\Rightarrow M_2 > 1$
  deflection angle θ
  shock angle β_1

second shock:

- upstream condition:

  same as downstream condition of first shock
- downstream condition:

  weak shock $\Rightarrow M_3 > 1$
  deflection angle θ
  shock angle β_2

Shock Reflection

Solution:

first shock:

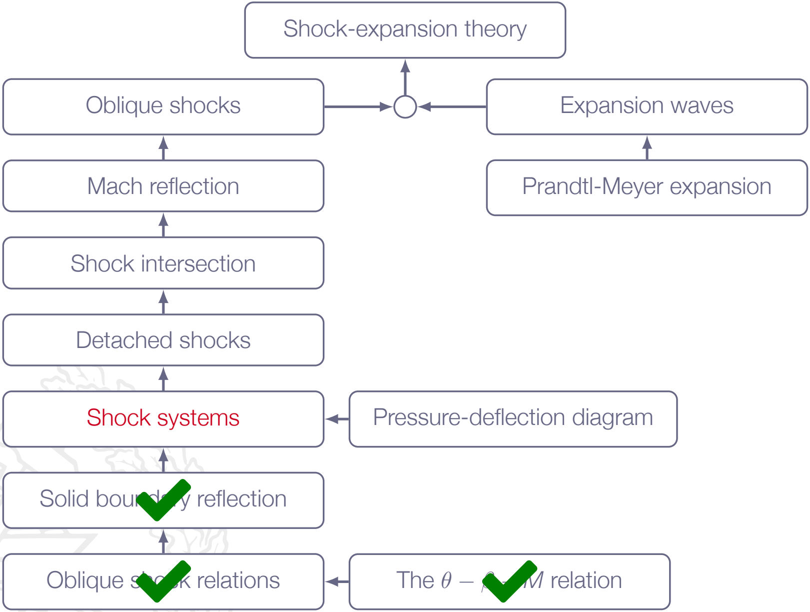
- β_1 calculated from θ-β-M relation for specified θ and M_1 (*weak solution*)
- flow condition 2 according to formulas for normal shocks ($M_{n_1} = M_1 \sin(\beta_1)$ and $M_{n_2} = M_2 \sin(\beta_1 - \theta)$)

second shock:

- β_2 calculated from θ-β-M relation for specified θ and M_2 (*weak solution*)
- flow condition 3 according to formulas for normal shocks ($M_{n_2} = M_2 \sin(\beta_2)$ and $M_{n_3} = M_3 \sin(\beta_2 - \theta)$)

$\Rightarrow$ complete description of flow and shock waves
(angle between upper wall and second shock: $\Phi = \beta_2 - \theta$)

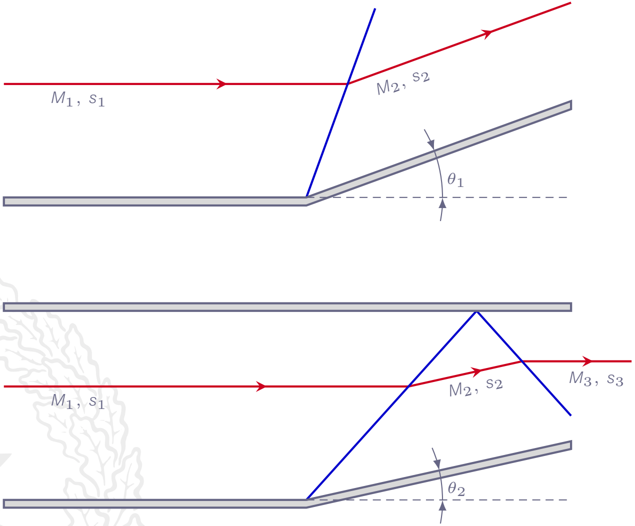
Roadmap - Oblique Shocks and Expansion Waves



Chapter 4.7 Comments on Flow Through Multiple Shock Systems

Flow Through Multiple Shock Systems

Single-shock compression versus multiple-shock compression:



Flow Through Multiple Shock Systems

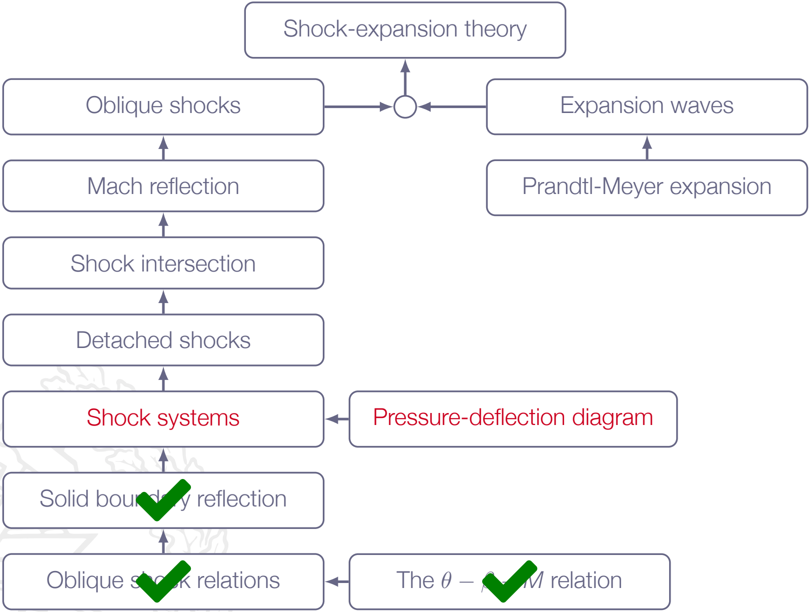
We may find θ_1 and θ_2 (for same M_1) which gives the same final Mach number

In such cases, the multiple shock flow has smaller losses

Explanation: entropy generation at a shock is a very non-linear function of shock strength

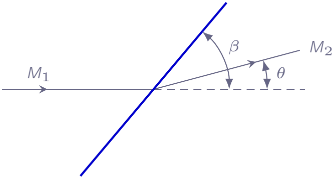
Note: θ_1 might very well be less than $2\theta_2$

Roadmap - Oblique Shocks and Expansion Waves

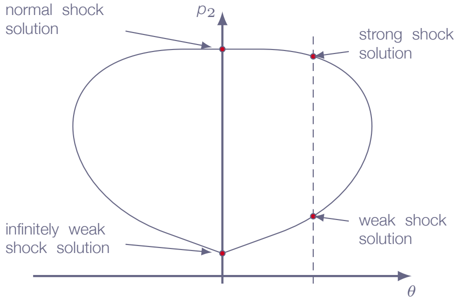


Chapter 4.8 Pressure Deflection Diagrams

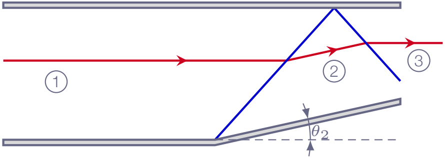
Pressure Deflection Diagrams

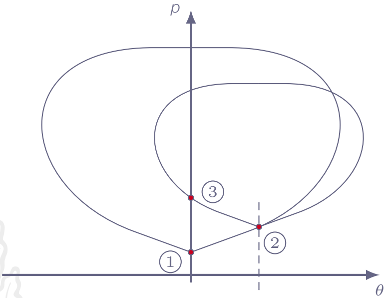


$\Rightarrow$ relation between p_2 and θ

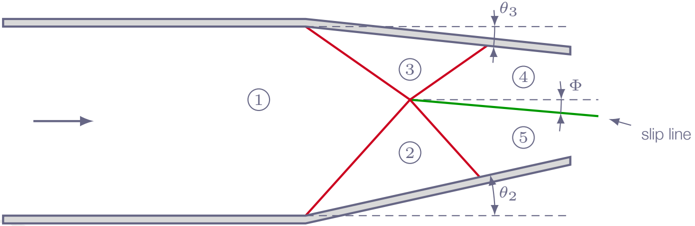


Pressure Deflection Diagrams - Shock Reflection





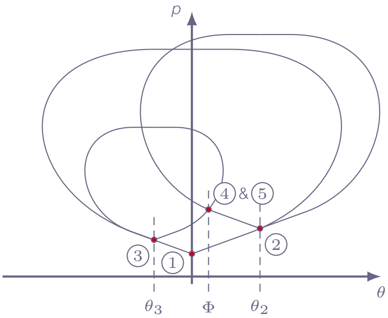
Pressure Deflection Diagrams - Shock Intersection



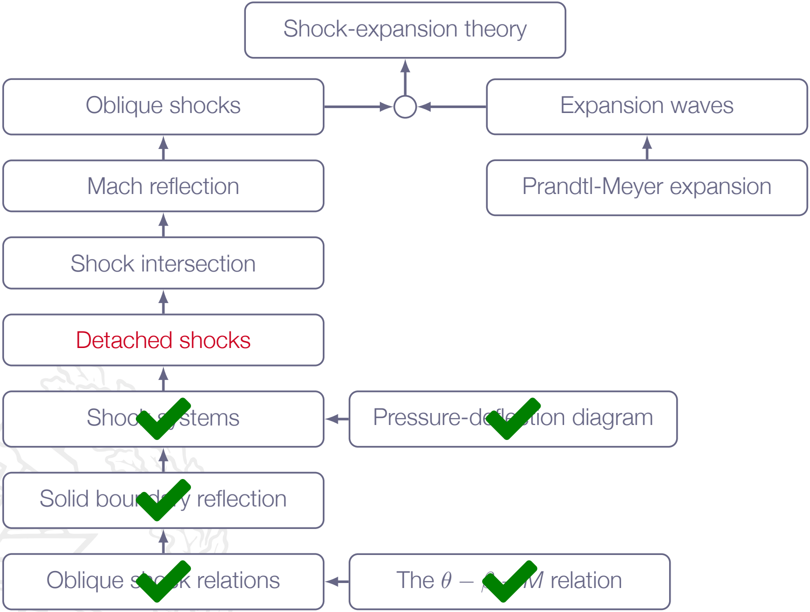
A slip line is a contact discontinuity

- discontinuity in ρ, T, s, v, and M
- continuous in p and flow angle

Pressure Deflection Diagrams - Shock Intersection

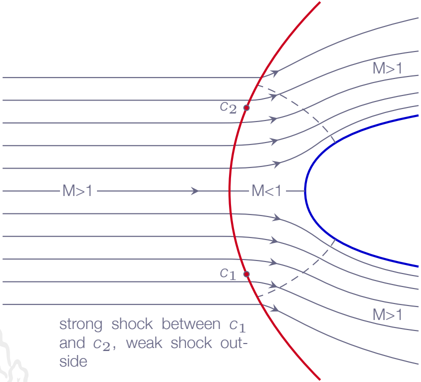


Roadmap - Oblique Shocks and Expansion Waves



Chapter 4.12
Detached Shock Wave in Front of a Blunt Body

Detached Shocks



Detached Shocks

As we move along the detached shock form the centerline, the shock will change in nature as

- right in front of the body we will have a normal shock
- strong oblique shock
- weak oblique shock
- far away from the body it will approach a Mach wave, *i.e.* an infinitely weak oblique shock

Roadmap - Oblique Shocks and Expansion Waves