Compressible Flow - TME085

Lecture 2

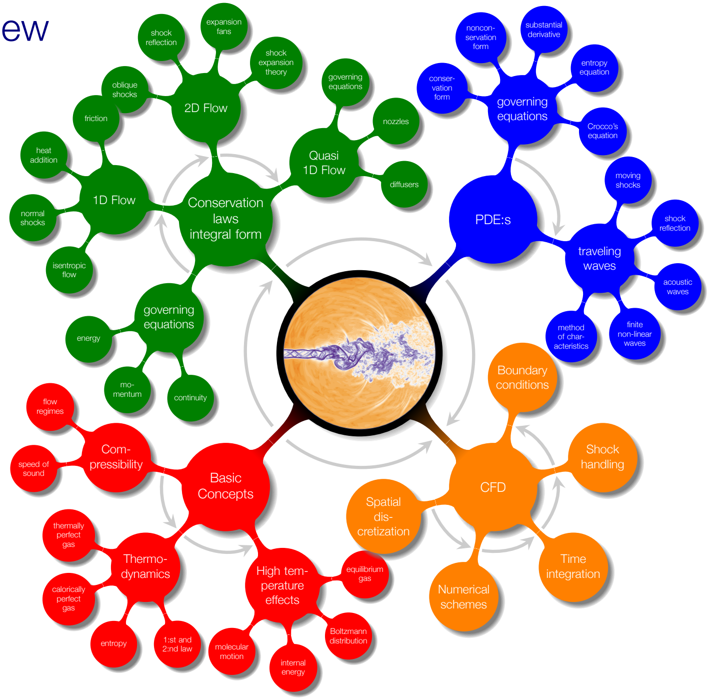
Niklas Andersson

Chalmers University of Technology
Department of Mechanics and Maritime Sciences
Division of Fluid Mechanics
Gothenburg, Sweden

niklas.andersson@chalmers.se

CHALMERS
UNIVERSITY OF TECHNOLOGY

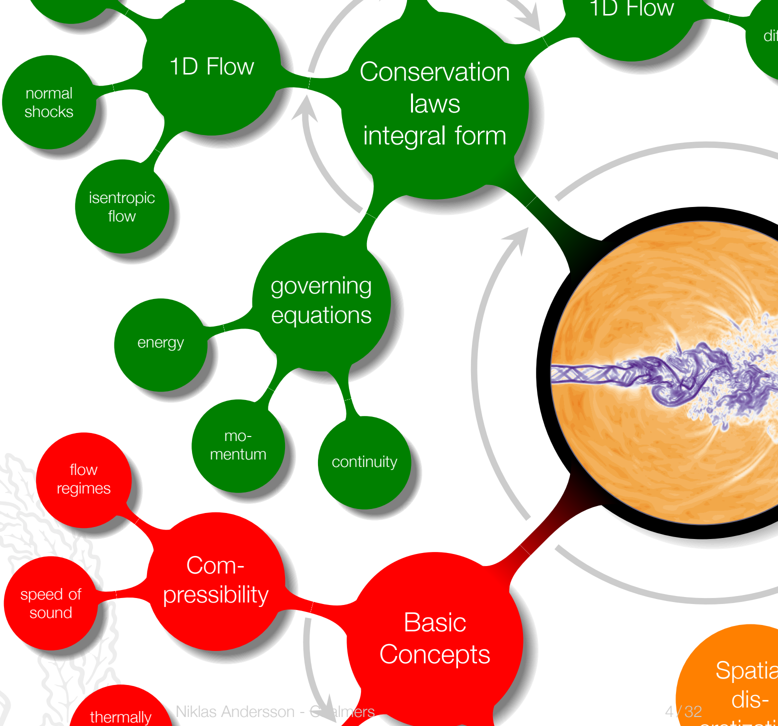
Overview



Conservation laws integral form
2D Flow
shock reflection
expansion fans
shock expansion theory
oblique shocks
friction
1D Flow
heat addition
normal shocks
isentropic flow
Quasi 1D Flow
governing equations
nozzles
diffusers
governing equations
energy
mo-mentum
continuity
PDE:s
governing equations
noncon-servation form
substantial derivative
entropy equation
conser-vation form
Crocco's equation
traveling waves
moving shocks
shock reflection
acoustic waves
method of char-acteristics
finite non-linear waves
CFD
Boundary conditions
Shock handling
Spatial dis-cretization
Time integration
Numerical schemes
Basic Concepts
Com-pressibility
flow regimes
speed of sound
Thermo-dynamics
thermally perfect gas
calorically perfect gas
entropy
1:st and 2:nd law
High tem-perature effects
equilibrium gas
Boltzmann distribution
molecular motion
internal energy

Chapter 2 Integral Forms of the Conservation Equations for Inviscid Flows

Overview


1D Flow
normal shocks
isentropic flow
Conservation laws integral form
1D Flow
governing equations
energy
mo-mentum
continuity
flow regimes
Com-pressibility
speed of sound
Basic Concepts
thermally

Addressed Learning Outcomes

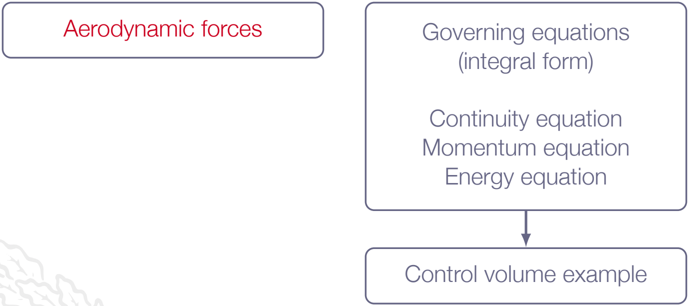
4 Present at least two different formulations of the governing equations for compressible flows and explain what basic conservation principles they are based on

5 Explain how thermodynamic relations enter into the flow equations

7 Explain why entropy is important for flow discontinuities

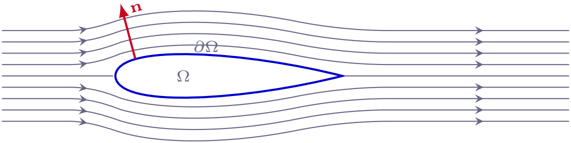
equations, equations and more equations

Roadmap - Integral Relations



Chapter 1.5
Aerodynamic Forces

Aerodynamic Forces



Ω	region occupied by body
$\partial\Omega$	surface of body
$\mathbf{n}$	outward facing unit normal vector

Aerodynamic Forces

Overall forces on the body du to the flow

$$\mathbf{F} = \oiint (-p\mathbf{n} + \tau \cdot \mathbf{n}) dS$$

where p is static pressure and τ is a stress tensor

Aerodynamic Forces

Drag is the component of $\mathbf{F}$ which is parallel with the freestream direction:

$$D = D_p + D_f$$

where D_p is drag due to pressure and D_f is drag due to friction

Lift is the component of $\mathbf{F}$ which is normal to the free stream direction:

$$L = L_p + L_f$$

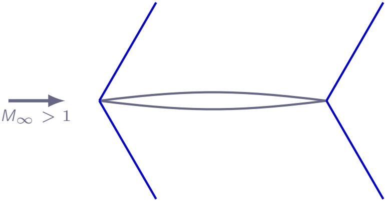
(L_f is usually negligible)

Aerodynamic Forces

Inviscid flow around slender body (*attached flow*)

- subsonic flow: $D = 0$
- transonic or supersonic flow: $D > 0$

Explanation: Wave drag



Aerodynamic Forces

- Wave drag is an inviscid phenomena, connected to the formation of compression shocks and entropy increase
- Viscous effects are present in all Mach regimes
- At transonic and supersonic conditions a particular phenomena named "shock/boundary-layer interaction" may appear
  - shocks trigger flow separation
  - usually leads to unsteady flow

Roadmap - Integral Relations

Aerodynamic forces ✔

Governing equations
(integral form)

Continuity equation
Momentum equation
Energy equation

↓

Control volume example

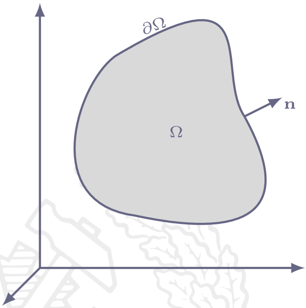
Integral Forms of the Conservation Equations

Conservation principles:

- conservation of mass
- conservation of momentum (*Newton's second law*)
- conservation of energy (*first law of thermodynamics*)

Integral Forms of the Conservation Equations

The control volume approach:



Notation:

Ω: fixed control volume

$\partial\Omega$: boundary of Ω

$\mathbf{n}$: outward facing unit normal vector

$\mathbf{v}$: fluid velocity

$v = |\mathbf{v}|$

Chapter 2.3 Continuity Equation

Continuity Equation

Conservation of mass:

$$\underbrace{\frac{d}{dt}\iiint\limits_{\Omega} \rho d\mathscr{V}}_{\text{rate of change of total mass in } \Omega} + \underbrace{\oiint\limits_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dS}_{\text{net mass flow out from } \Omega} = 0$$

Note: notation in the text book $\mathbf{n} \cdot dS = d\mathbf{S}$

Chapter 2.4
Momentum Equation

Momentum Equation

Conservation of momentum:

$$\underbrace{\frac{d}{dt}\iiint\limits_{\Omega} \rho \mathbf{v} d\mathscr{V}}_{\substack{\text{rate of change of total} \\ \text{momentum in } \Omega}} + \underbrace{\oint\!\!\oint\limits_{\partial\Omega} \left[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}\right] dS}_{\substack{\text{net momentum flow out from} \\ \Omega \text{ plus surface force on } \partial\Omega \\ \text{due to pressure}}} = \underbrace{\iiint\limits_{\Omega} \rho \mathbf{f} d\mathscr{V}}_{\substack{\text{rate of momentum} \\ \text{generation due to} \\ \text{forces inside } \Omega}}$$

Note: friction forces due to viscosity are not included here. To account for these forces, the term $-(\tau \cdot \mathbf{n})$ must be added to the surface integral term.

Note: the body force, f, is force per unit mass.

Chapter 2.5
Energy Equation

Energy Equation

Conservation of energy:

$$\underbrace{\frac{d}{dt}\iiint\limits_{\Omega}\rho e_o d\mathscr{V}}_{\substack{\text{rate of change of total}\\ \text{internal energy in }\Omega}} + \underbrace{\oiint\limits_{\partial\Omega}\left[\rho e_o(\mathbf{v}\cdot\mathbf{n}) + p\mathbf{v}\cdot\mathbf{n}\right]dS}_{\substack{\text{net flow of total internal energy}\\ \text{out from }\Omega\text{ plus work due to}\\ \text{surface pressure on }\partial\Omega}} = \underbrace{\iiint\limits_{\Omega}\rho\mathbf{f}\cdot\mathbf{v}d\mathscr{V}}_{\substack{\text{work due to forces}\\ \text{inside }\Omega}}$$

where

$$\rho e_o = \rho\left(e + \frac{1}{2}\mathbf{v}\cdot\mathbf{v}\right) = \rho\left(e + \frac{1}{2}v^2\right)$$

is the total internal energy

Energy Equation

The surface integral term may be rewritten as follows:

$$\oiint_{\partial\Omega} \left[\rho\left(e+\frac{1}{2}v^2\right)(\mathbf{v}\cdot\mathbf{n})+p\mathbf{v}\cdot\mathbf{n}\right]dS$$

$$\Leftrightarrow$$

$$\oiint_{\partial\Omega} \left[\rho\left(e+\frac{p}{\rho}+\frac{1}{2}v^2\right)(\mathbf{v}\cdot\mathbf{n})\right]dS$$

$$\Leftrightarrow$$

$$\oiint_{\partial\Omega} \left[\rho\left(h+\frac{1}{2}v^2\right)(\mathbf{v}\cdot\mathbf{n})\right]dS$$

Energy Equation

Introducing total enthalpy

$$h_o = h + \frac{1}{2}v^2$$

we get

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho e_o d\mathscr{V} + \oiint\limits_{\partial\Omega} [\rho h_o \mathbf{v} \cdot \mathbf{n}] \, dS = \iiint\limits_{\Omega} \rho \mathbf{f} \cdot \mathbf{v} d\mathscr{V}$$

Energy Equation

Note 1: to include friction work on $\partial\Omega$, the energy equation is extended as

$$\frac{d}{dt}\iiint\limits_{\Omega}\rho e_o d\mathscr{V} + \oiint\limits_{\partial\Omega}\left[\rho h_o \mathbf{v}\cdot\mathbf{n} - (\tau\cdot\mathbf{n})\cdot\mathbf{v}\right] dS = \iiint\limits_{\Omega}\rho\mathbf{f}\cdot\mathbf{v} d\mathscr{V}$$

Note 2: to include heat transfer on $\partial\Omega$, the energy equation is further extended

$$\frac{d}{dt}\iiint\limits_{\Omega}\rho e_o d\mathscr{V} + \oiint\limits_{\partial\Omega}\left[\rho h_o \mathbf{v}\cdot\mathbf{n} - (\tau\cdot\mathbf{n})\cdot\mathbf{v} + \mathbf{q}\cdot\mathbf{n}\right] dS = \iiint\limits_{\Omega}\rho\mathbf{f}\cdot\mathbf{v} d\mathscr{V}$$

where $\mathbf{q}$ is the heat flux vector

Energy Equation

Note 3: the force $\mathbf{f}$ inside Ω may be a distributed body force field

Examples:

- gravity
- Coriolis and centrifugal acceleration terms in a rotating frame of reference

Energy Equation

Note 4: there may be objects inside Ω which we choose to represent as sources of momentum and energy.

For example, there may be a solid object inside Ω which acts on the fluid with a force $\mathbf{F}$ and performs work $\dot{W}$ on the fluid

Momentum equation:

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oint\!\!\!\oint\limits_{\partial\Omega} \left[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}\right] dS = \iiint\limits_{\Omega} \rho \mathbf{f} d\mathscr{V} + \mathbf{F}$$

Energy equation:

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho e_o d\mathscr{V} + \oint\!\!\!\oint\limits_{\partial\Omega} \left[\rho h_o \mathbf{v}\cdot\mathbf{n}\right] dS = \iiint\limits_{\Omega} \rho \mathbf{f}\cdot\mathbf{v} d\mathscr{V} + \dot{W}$$

Roadmap - Integral Relations

Aerodynamic forces

Governing equations
(integral form)

Continuity equation
Momentum equation
Energy equation

Control volume example

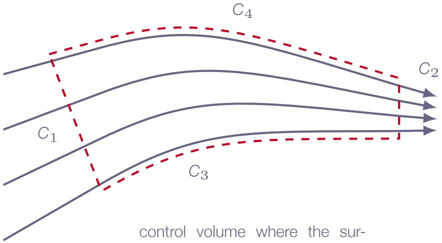
Integral Equations - Applications

How can we use control volume formulations of conservation laws?

- Let $\Omega \to 0$: In the limit of vanishing volume the control volume formulations give the Partial Differential Equations (PDE:s) for mass, momentum and energy conservation (see Chapter 6)
- Apply in a "smart" way $\Rightarrow$ Analysis tool for many practical problems involving compressible flow (see Chapter 2, Section 2.8)

Integral Equations - Applications

Example: Steady-state adiabatic inviscid flow



control volume where the surfaces C_1 and C_2 are normal to the flow and C_3 and C_4 are parallel to the stream lines

Integral Equations - Applications

Conservation of mass:

$$\underbrace{\frac{d}{dt}\iiint\limits_{\Omega} \rho d\mathscr{V}}_{=0} + \underbrace{\oiint\limits_{\partial\Omega} \rho \mathbf{v}\cdot\mathbf{n} dS}_{-\rho_1 v_1 A_1 + \rho_2 v_2 A_2} = 0$$

Conservation of energy:

$$\underbrace{\frac{d}{dt}\iiint\limits_{\Omega} \rho e_o d\mathscr{V}}_{=0} + \underbrace{\oiint\limits_{\partial\Omega} \left[\rho h_o \mathbf{v}\cdot\mathbf{n}\right] dS}_{-\rho_1 h_{o_1} v_1 A_1 + \rho_2 h_{o_2} v_2 A_2} = 0$$

Integral Equations - Applications

Conservation of mass:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

Conservation of energy:

$$\rho_1 h_{o_1} v_1 A_1 = \rho_2 h_{o_2} v_2 A_2$$

$$\Leftrightarrow$$

$$h_{o_1} = h_{o_2}$$

Total enthalpy h_o is conserved along streamlines in steady-state adiabatic inviscid flow

Roadmap - Integral Relations

Aerodynamic forces ✔

Governing equations (integral form) ✔

- Continuity equation
- Momentum equation
- Energy equation

↓

Control volume example ✔