

Compressible Flow - TME085

Lecture 1

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Compressible Flow

*"Compressible flow (**gas dynamics**) is a branch of fluid mechanics that deals with flows having **significant changes in fluid density**"*

Wikipedia



Gas Dynamics

"... the study of *motion of gases* and its effects on physical systems ..."

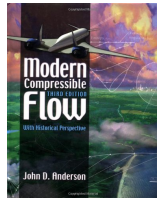
"... based on the principles of *fluid mechanics* and *thermodynamics* ..."

"... gases flowing around or within physical objects at speeds comparable to the *speed of sound* ..."

Wikipedia



Course Details - Literature



Course Literature:

John D. Anderson

Modern Compressible Flow; With Historical Perspective

Third Edition (International Edition 2004)

McGraw-Hill, ISBN 007-124136-1

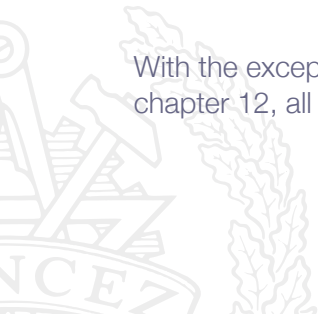


Course Details - Literature

Content:

- ▶ Chapter 1-7: All
- ▶ Chapter 8-11: Excluded
- ▶ Chapter 12: Included, supplemented by lecture notes
- ▶ Chapter 13-15: Excluded
- ▶ Chapter 16-17: Some parts included (see lecture notes)

With the exception of the lecture notes supplementing chapter 12, all lecture notes are based on the book.



Course Details - Assessment

Written examination (*fail, 3, 4, 5*):

- ▶ A theoretical part and a problem solving part.
- ▶ The course book by Anderson is not allowed at the examination, nor any additional material that has been handed out during the course.
- ▶ There will be a list of formulas which you will have at your disposal during the exam
- ▶ Only standard non-programmable electronic calculators (typgodkända) are allowed.
- ▶ Historical data, e.g. names of researchers, dates, etc are not included.



Course Details - Assessment

Assignemnts (*fail/pass*):

- ▶ three computer labs (**attendance + results approved by TA**)

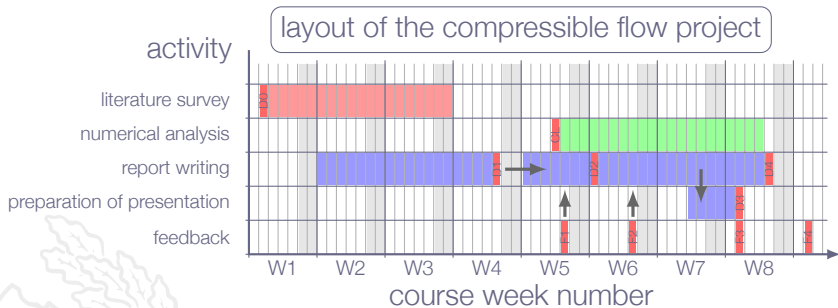
Compressible Flow Project (*fail/pass*):

- ▶ literature survey (**report**)
- ▶ numerical analysis (**technical report**)
- ▶ oral presentation (**attendance + presentation**)
- ▶ bonus points for the written exam awarded for high-quality work (see assessment criteria in project description)

N.B. important dates in Course PM



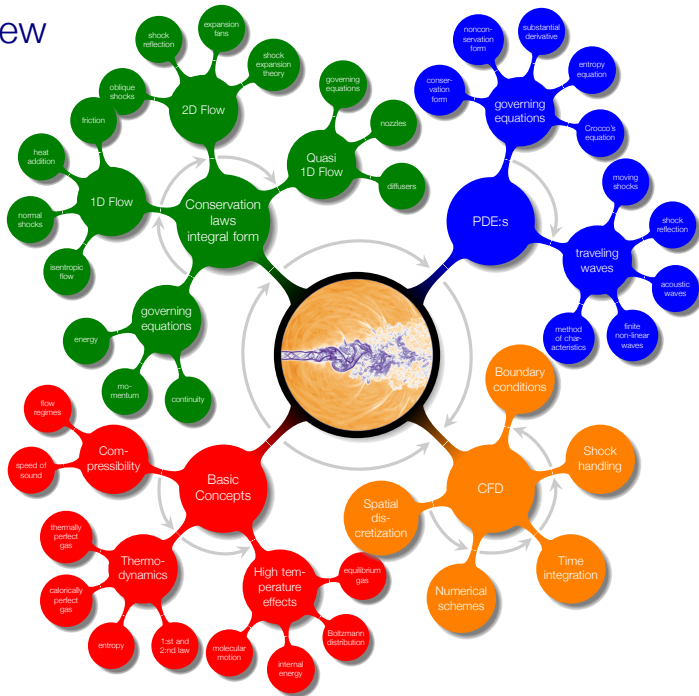
Course Details - The Compressible Flow Project



Course Details - Learning Outcomes

- 1 **Define** the concept of compressibility for flows
- 2 **Explain** how to find out if a given flow is subject to significant compressibility effects
- 3 **Describe** typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)
- 4 **Present** at least two different formulations of the governing equations for compressible flows and **explain** what basic conservation principles they are based on
- 5 **Explain** how thermodynamic relations enter into the flow equations
- 6 **Define** the special cases of calorically perfect gas, thermally perfect gas and real gas and **explain** the implication of each of these special cases
- 7 **Explain** why entropy is important for flow discontinuities
- 8 **Derive** (marked) and **apply** (all) of the presented mathematical formulae for classical gas dynamics
 - a 1D isentropic flow*
 - b normal shocks*
 - c 1D flow with heat addition*
 - d 1D flow with friction*
 - e oblique shocks in 2D*
 - f shock reflection at solid walls*
 - g contact discontinuities
 - h Prandtl-Meyer expansion fans in 2D
 - i detached blunt body shocks, nozzle flows
 - j unsteady waves and discontinuities in 1D
 - k basic acoustics
- 9 **Solve** engineering problems involving the above-mentioned phenomena (8a-8k)
- 10 **Explain** how the incompressible flow equations are derived as a limiting case of the compressible flow equations
- 11 **Explain** how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations
- 12 **Explain** the main principles behind a modern Finite Volume CFD code and such concepts as explicit/implicit time stepping, CFL number, conservation, handling of compression shocks, and boundary conditions
- 13 **Apply** a given CFD code to a particular compressible flow problem
- 14 **Analyze** and **verify** the quality of the numerical solution
- 15 **Explain** the limitations in fluid flow simulation software
- 16 **Report** numerical analysis work in form of a technical report
 - a **Describe** a numerical analysis with details such that it is possible to redo the work based on the provided information
 - b **Write** a technical report (structure, language)
- 17 **Search** for literature relevant for a specific physical problem and **summarize** the main ideas and concepts found
- 18 **Present** engineering work in the form of oral presentations

Overview

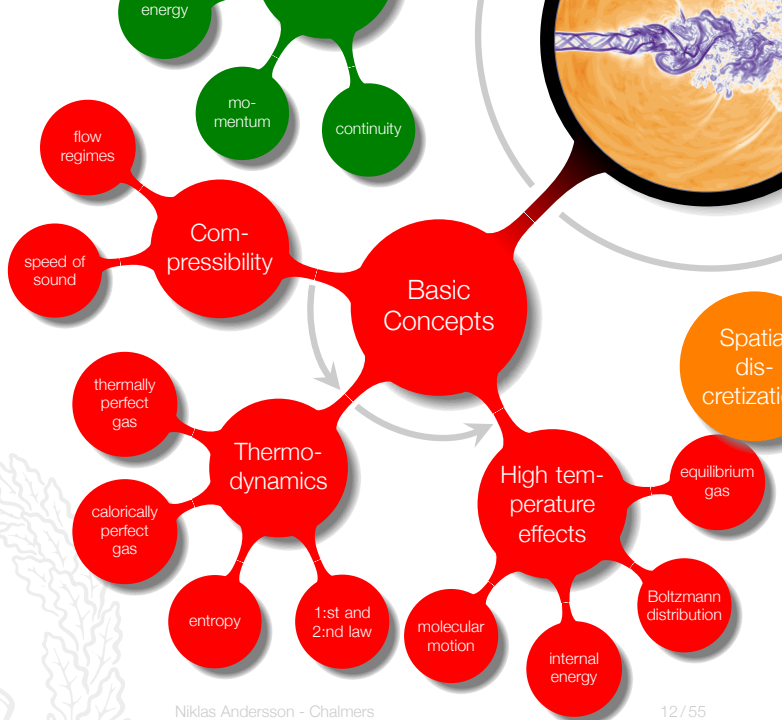


Chapter 1

Compressible Flow



Overview

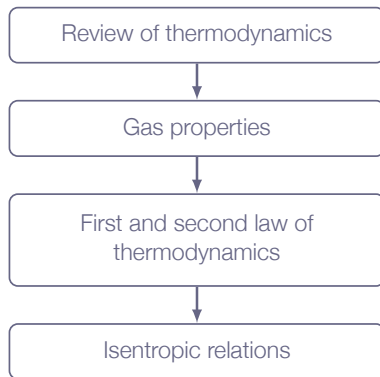
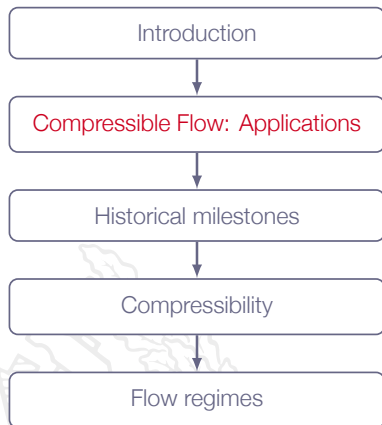


Addressed Learning Outcomes

- 1 **Define** the concept of compressibility for flows
- 2 **Explain** how to find out if a given flow is subject to significant compressibility effects
- 3 **Describe** typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)
- 6 **Define** the special cases of calorically perfect gas, thermally perfect gas and real gas and **explain** the implication of each of these special cases

in this lecture we will find out what compressibility means and do a brief review of thermodynamics

Roadmap - Introduction to Compressible Flow



Applications - Classical

- ▶ Treatment of calorically perfect gas
- ▶ Exact solutions of inviscid flow in 1D
- ▶ Shock-expansion theory for steady-state 2D flow
- ▶ Approximate closed form solutions to linearized equations in 2D and 3D
- ▶ Method of Characteristics (MOC) in 2D and axi-symmetric inviscid supersonic flows



Applications - Modern

- ▶ Computational Fluid Dynamics (CFD)
- ▶ Complex geometries (including moving boundaries)
- ▶ Complex flow features (compression shocks, expansion waves, contact discontinuities)
- ▶ Viscous effects
- ▶ Turbulence modeling
- ▶ High temperature effects (molecular vibration, dissociation, ionization)
- ▶ Chemically reacting flow (equilibrium & non-equilibrium reactions)



Applications - Examples

Turbo-machinery flows:

- ▶ Gas turbines, steam turbines, compressors
- ▶ Aero engines (turbojets, turbofans, turboprops)

Aeroacoustics:

- ▶ Flow induced noise (jets, wakes, moving surfaces)
- ▶ Sound propagation in high speed flows

External flows:

- ▶ Aircraft (airplanes, helicopters)
- ▶ Space launchers (rockets, re-entry vehicles)

Internal flows:

- ▶ Nozzle flows
- ▶ Inlet flows, diffusers
- ▶ Gas pipelines (natural gas, bio gas)

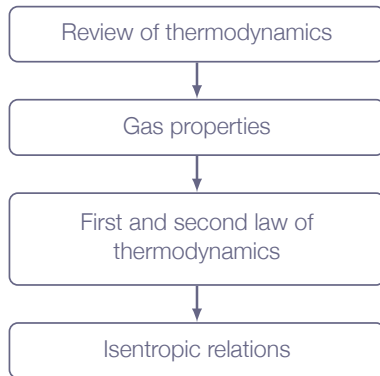
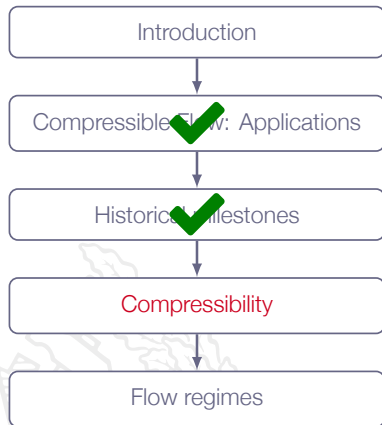
Free-shear flows:

- ▶ High speed jets

Combustion:

- ▶ Internal combustion engines (valve flow, in-cylinder flow, exhaust pipe flow, mufflers)
- ▶ Combustion induced noise (turbulent combustion)
- ▶ Combustion instabilities

Roadmap - Introduction to Compressible Flow



Chapter 1.2

Compressibility

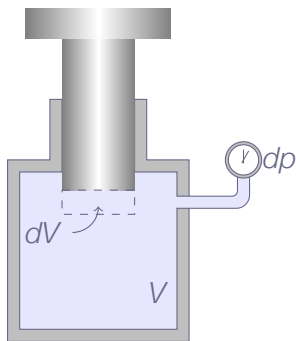


Compressibility

$$\tau = -\frac{1}{\nu} \frac{\partial \nu}{\partial \rho}, \quad (\nu = \frac{1}{\rho})$$

Not really precise!

Is T held constant during the compression or not?



Compressibility

Two fundamental cases:

Constant temperature

- ▶ Heat is cooled off to keep T constant inside the cylinder
- ▶ The piston is moved slowly

Adiabatic process

- ▶ Thermal insulation prevents heat exchange
- ▶ The piston is moved fairly rapidly (*gives negligible flow losses*)



Compressibility

Isothermal process:

$$\tau_T = -\frac{1}{\nu} \left(\frac{\partial \nu}{\partial \rho} \right)_T$$

Adiabatic reversible (*isentropic*) process:

$$\tau_S = -\frac{1}{\nu} \left(\frac{\partial \nu}{\partial \rho} \right)_S$$

Air at normal conditions: $\tau_T \approx 1.0 \times 10^{-5} \quad [m^2/N]$

Water at normal conditions: $\tau_T \approx 5.0 \times 10^{-10} \quad [m^2/N]$

Compressibility

$$\tau = -\frac{1}{\nu} \frac{\partial \nu}{\partial \rho}$$

but

$$\nu = \frac{1}{\rho}$$

which gives

$$\tau = -\rho \frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \right) = -\rho \left(-\frac{1}{\rho^2} \right) \frac{\partial \rho}{\partial \rho} = \frac{1}{\rho} \frac{\partial \rho}{\partial \rho}$$

Similarly:

$$\tau_T = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial \rho} \right)_T, \quad \tau_S = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial \rho} \right)_S$$

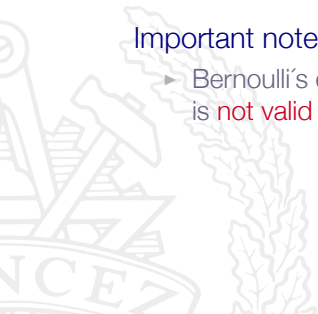
Compressibility

Definition of compressible flow:

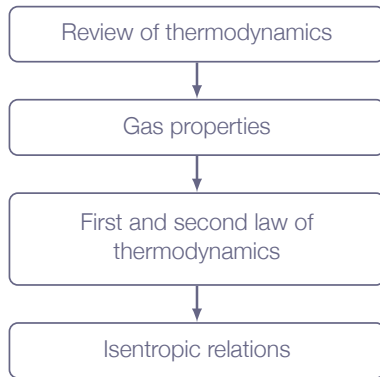
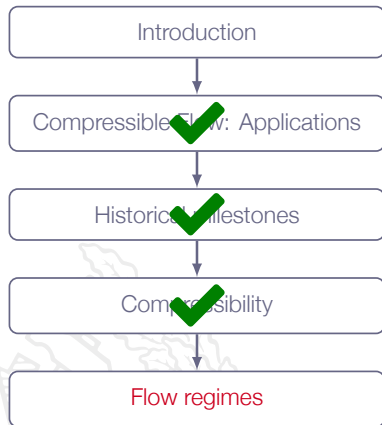
- ▶ If p changes with amount Δp over a characteristic length scale of the flow, such that the corresponding change in density, given by $\Delta\rho \sim \rho\tau\Delta p$, is **too large to be neglected**, the flow is compressible (*typically, if $\Delta\rho/\rho > 0.05$*)

Important note:

- ▶ Bernoulli's equation is restricted to incompressible flow, *i.e.* it is **not valid** for compressible flow!



Roadmap - Introduction to Compressible Flow



Chapter 1.3

Flow Regimes



Flow Regimes

The freestream Mach number is defined as

$$M_{\infty} = \frac{U_{\infty}}{a_{\infty}}$$

where U_{∞} is the freestream flow speed and a_{∞} is the speed of sound at freestream conditions



Flow Regimes

Assume first incompressible flow and estimate the max pressure difference using

$$\Delta p \approx \frac{1}{2} \rho_{\infty} U_{\infty}^2$$

For air at normal conditions we have

$$\tau_T = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial p} \right)_T = \frac{1}{\rho RT} = \frac{1}{p}$$

(ideal gas law for perfect gas $p = \rho RT$)

Flow Regimes

Using the relations on previous slide we get

$$\frac{\Delta\rho}{\rho} \approx \tau_T \Delta\rho \approx \frac{1}{\rho_\infty} \frac{1}{2} \rho_\infty U_\infty^2 = \frac{\frac{1}{2} \rho_\infty U_\infty^2}{\rho_\infty R T_\infty}$$

for a calorically perfect gas we have

$$a = \sqrt{\gamma R T}$$

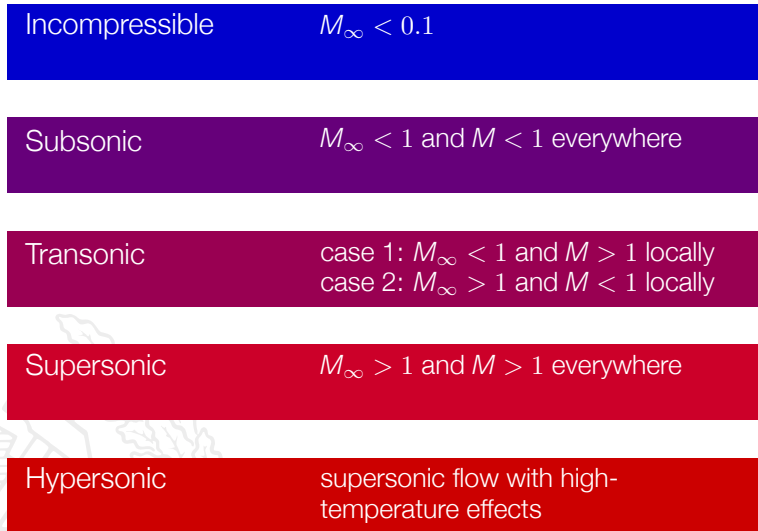
which gives us

$$\frac{\Delta\rho}{\rho} \approx \frac{\gamma U_\infty^2}{2 a_\infty^2}$$

now, using the definition of Mach number we get

$$\frac{\Delta\rho}{\rho} \approx \frac{\gamma}{2} M_\infty^2$$

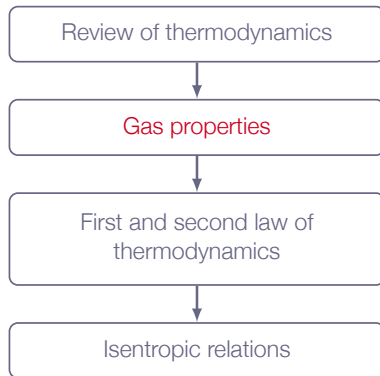
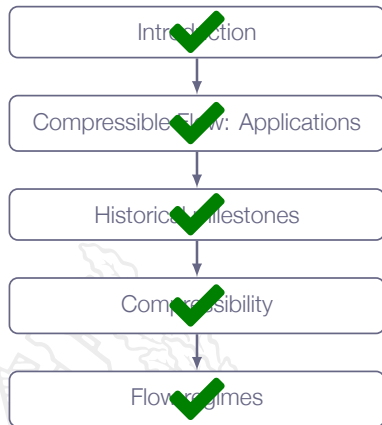
Flow Regimes



Compressible

Local Mach number M is based on local flow speed, $U = |\mathbf{U}|$, and local speed of sound, a

Roadmap - Introduction to Compressible Flow



Chapter 1.4

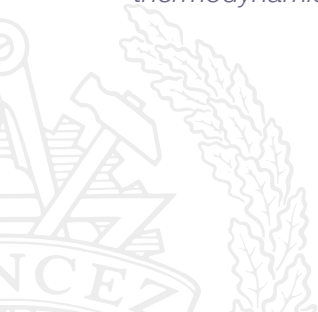
Review of Thermodynamics



Thermodynamic Review

Compressible flow:

... strong interaction between flow and thermodynamics ...



Perfect Gas

All intermolecular forces negligible

Only elastic collisions between molecules

$$p\nu = RT$$

or

$$\frac{p}{\rho} = RT$$

where R is the gas constant $[R] = J/kgK$

Also, $R = R_{univ}/M$ where M is the molecular weight of gas molecules (in $kg/kmol$) and $R_{univ} = 8314 J/kmol K$

Internal Energy and Enthalpy

Internal energy e ($[e] = J/kg$)

Enthalpy h ($[h] = J/kg$)

$$h = e + p\nu = e + \frac{p}{\rho} \text{ (valid for all gases)}$$

For any gas in thermodynamic equilibrium, e and h are functions of only two thermodynamic variables (*any two variables may be selected*) e.g.

$$e = e(T, \rho)$$

$$h = h(T, p)$$



Internal Energy and Enthalpy

Special cases:

Thermally perfect gas:

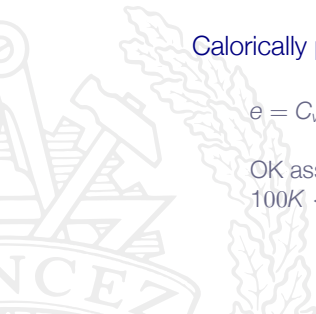
$$e = e(T) \text{ and } h = h(T)$$

OK assumption for air at near atmospheric conditions and $100K < T < 2500K$

Calorically perfect gas:

$$e = C_v T \text{ and } h = C_p T \text{ (} C_v \text{ and } C_p \text{ are constants)}$$

OK assumption for air at near atmospheric pressure and $100K < T < 1000K$



Specific Heat

For thermally perfect (and calorically perfect) gas

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p, \quad C_v = \left(\frac{\partial e}{\partial T} \right)_v$$

since $h = e + p/\rho = e + RT$ we obtain:

$$C_p = C_v + R$$

The ratio of specific heats, γ , is defined as:

$$\gamma \equiv \frac{C_p}{C_v}$$



Specific Heat

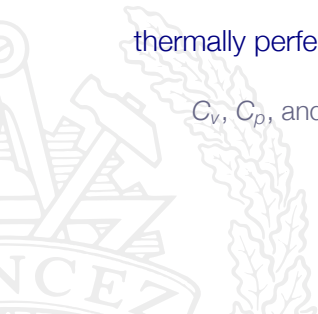
Important!

calorically perfect gas:

C_v , C_p , and γ are constants

thermally perfect gas:

C_v , C_p , and γ will depend on temperature



Specific Heat

$$C_p - C_v = R$$

divide by C_v

$$\gamma - 1 = \frac{R}{C_v}$$

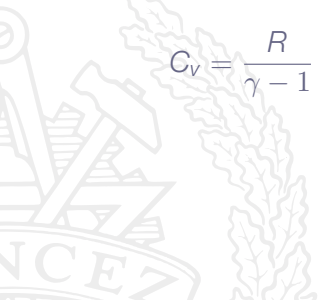
$$C_v = \frac{R}{\gamma - 1}$$

$$C_p - C_v = R$$

divide by C_p

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$



Specific Heat

$$C_p - C_v = R$$

divide by C_v

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p - C_v = R$$

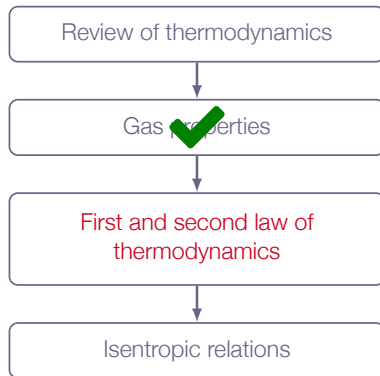
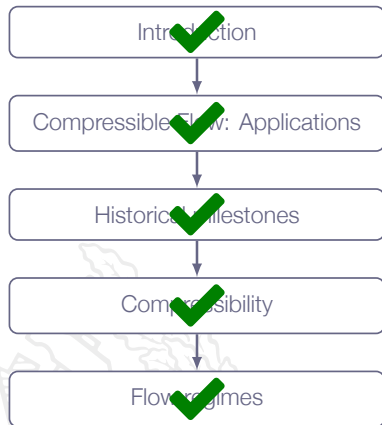
divide by C_p

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

valid for both thermally perfect and calorically perfect gas!

Roadmap - Introduction to Compressible Flow



First Law of Thermodynamics

A fixed mass of gas, separated from its surroundings by an imaginary flexible boundary, is defined as a "system". This system obeys the relation

$$de = \delta q - \delta w$$

where

de is a change in internal energy of system

δq is heat added to the system

δw is work done by the system (on its surroundings)

Note: de only depends on starting point and end point of the process while δq and δw depend on the actual process also

First Law of Thermodynamics

Examples:

Adiabatic process:

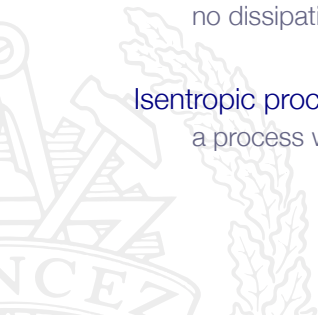
$$\delta q = 0.$$

Reversible process:

no dissipative phenomena (*no flow losses*)

Isentropic process:

a process which is both adiabatic and reversible



First Law of Thermodynamics

Reversible process:

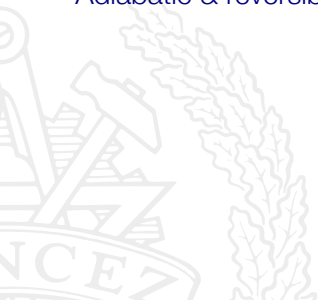
$$\delta w = p d\nu = p d(1/\rho)$$

$$de = \delta q - p d(1/\rho)$$

Adiabatic & reversible process:

$$\delta q = 0.$$

$$de = -p d(1/\rho)$$



Entropy

Entropy s is a property of all gases, uniquely defined by any two thermodynamic variables, e.g.

$s = s(p, T)$ or $s = s(\rho, T)$ or $s = s(\rho, p)$ or $s = s(e, h)$ or ...



Second Law of Thermodynamics

Concept of entropy s :

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir}, \quad (ds_{ir} > 0.)$$

or

$$ds \geq \frac{\delta q}{T}$$



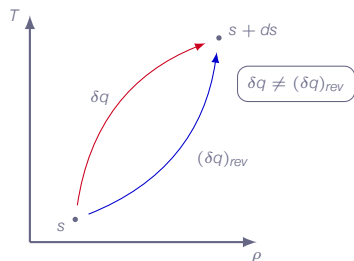
Second Law of Thermodynamics

Concept of entropy s :

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir}, \quad (ds_{ir} > 0.)$$

or

$$ds \geq \frac{\delta q}{T}$$



Second Law of Thermodynamics

In general:

$$ds \geq \frac{\delta q}{T}$$

For adiabatic processes:

$$ds \geq 0.$$



Calculation of Entropy

For reversible processes ($\delta w = pd(1/\rho)$ and $\delta q = Tds$):

$$de = Tds - pd \left(\frac{1}{\rho} \right)$$

or

$$Tds = de + pd \left(\frac{1}{\rho} \right)$$

from before we have $h = e + p/\rho \Rightarrow$

$$dh = de + pd \left(\frac{1}{\rho} \right) + \left(\frac{1}{\rho} \right) dp \Leftrightarrow de = dh - pd \left(\frac{1}{\rho} \right) - \left(\frac{1}{\rho} \right) dp$$

Calculation of Entropy

For thermally perfect gases, $p = \rho RT$ and $dh = C_p dT \Rightarrow$

$$ds = C_p \frac{dT}{T} - R \frac{dp}{p}$$

Integration from starting point (1) to end point (2) gives:

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - R \ln \left(\frac{p_2}{p_1} \right)$$

and for calorically perfect gases

$$s_2 - s_1 = C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right)$$

Calculation of Entropy

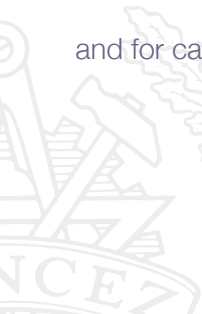
If we instead use $de = C_v dT$ we get

for thermally perfect gases

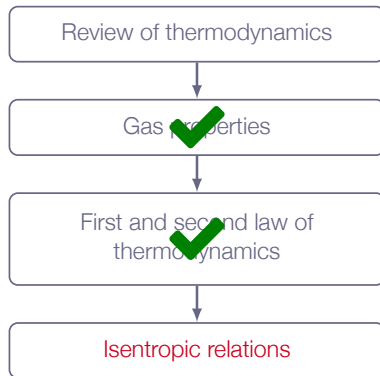
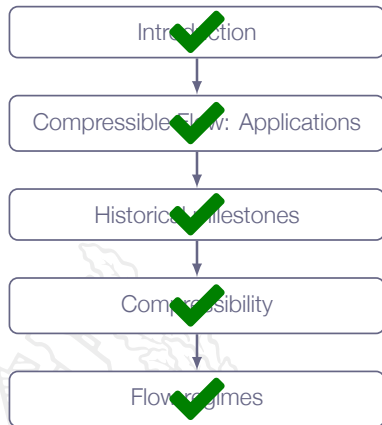
$$s_2 - s_1 = \int_1^2 C_v \frac{dT}{T} - R \ln \left(\frac{\rho_2}{\rho_1} \right)$$

and for calorically perfect gases

$$s_2 - s_1 = C_v \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{\rho_2}{\rho_1} \right)$$



Roadmap - Introduction to Compressible Flow



Isentropic Relations

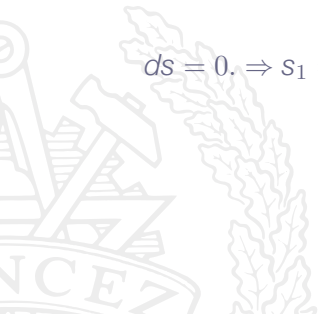
For calorically perfect gases, we have

$$s_2 - s_1 = C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right)$$

For adiabatic reversible processes:

$$ds = 0. \Rightarrow s_1 = s_2 \Rightarrow C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right) = 0 \Rightarrow$$

$$\ln \left(\frac{p_2}{p_1} \right) = \frac{C_p}{R} \ln \left(\frac{T_2}{T_1} \right)$$



Isentropic Relations

$$\frac{C_p}{R} = \frac{C_p}{C_p - C_v} = \frac{\gamma}{\gamma - 1} \Rightarrow$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma - 1}}$$



Isentropic Relations

Alternatively

$$s_2 - s_1 = 0 = C_v \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{\rho_2}{\rho_1} \right) \Rightarrow$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}}$$

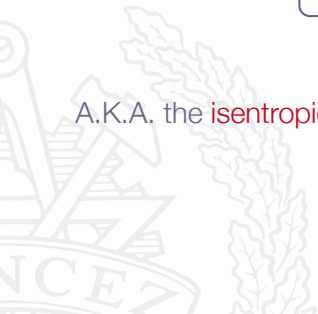


Isentropic Relations - Summary

For an isentropic process and a calorically perfect gas we have

$$\frac{\rho_2}{\rho_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

A.K.A. the **isentropic relations**



Roadmap - Introduction to Compressible Flow

