# Compressible Flow - TME085

## Lecture 1

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# Compressible Flow

*"Compressible flow (gas dynamics) is a branch of fluid mechanics that deals with flows having significant changes in fluid density"*

Wikipedia

# Gas Dynamics

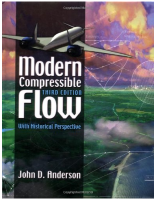
*"... the study of motion of gases and its effects on physical systems ..."*

*"... based on the principles of fluid mechanics and thermodynamics ..."*

*"... gases flowing around or within physical objects at speeds comparable to the speed of sound ..."*

Wikipedia

# Course Details - Literature




Course Literature:

John D. Anderson
Modern Compressible Flow; With Historical Perspective
Third Edition (International Edition 2004)
McGraw-Hill, ISBN 007-124136-1

# Course Details - Literature

Content:

- Chapter 1-7: All
- Chapter 8-11: Excluded
- Chapter 12: Included, supplemented by lecture notes
- Chapter 13-15: Excluded
- Chapter 16-17: Some parts included (see lecture notes)

With the exception of the lecture notes supplementing chapter 12, all lecture notes are based on the book.

# Course Details - Assessment

Written examination (*fail, 3, 4, 5*):

- A theoretical part and a problem solving part.
- The course book by Anderson is not allowed at the examination, nor any additional material that has been handed out during the course.
- There will be a list of formulas which you will have at your disposal during the exam
- Only standard non-programmable electronic calculators (typgodkända) are allowed.
- Historical data, *e.g.* names of researchers, dates, etc are not included.

# Course Details - Assessment

Assignemnts (*fail/pass*):

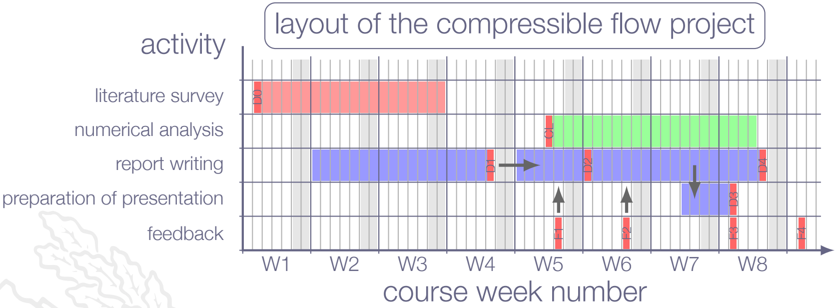
- three computer labs (attendance + results approved by TA)

Compressible Flow Project (*fail/pass*):

- literature survey (report)
- numerical analysis (technical report)
- oral presentation (attendance + presentation)
- bonus points for the written exam awarded for high-quality work (see assessment criteria in project description)

*N.B.* important dates in Course PM

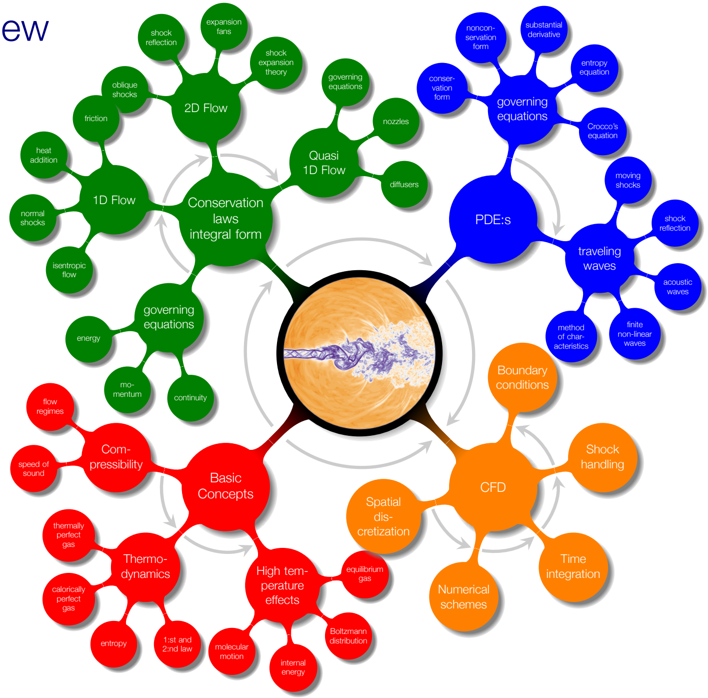
# Course Details - The Compressible Flow Project



# Course Details - Learning Outcomes

1. Define the concept of compressibility for flows
2. Explain how to find out if a given flow is subject to significant compressibility effects
3. Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)
4. Present at least two different formulations of the governing equations for compressible flows and explain what basic conservation principles they are based on
5. Explain how thermodynamic relations enter into the flow equations
6. Define the special cases of calorically perfect gas, thermally perfect gas and real gas and explain the implication of each of these special cases
7. Explain why entropy is important for flow discontinuities
8. Derive (marked) and apply (all) of the presented mathematical formulae for classical gas dynamics
    a. 1D isentropic flow*
    b. normal shocks*
    c. 1D flow with heat addition*
    d. 1D flow with friction*
    e. oblique shocks in 2D*
    f. shock reflection at solid walls*
    g. contact discontinuities
    h. Prandtl-Meyer expansion fans in 2D
    i. detached blunt body shocks, nozzle flows
    j. unsteady waves and discontinuities in 1D
    k. basic acoustics
9. Solve engineering problems involving the above-mentioned phenomena (8a-8k)
10. Explain how the incompressible flow equations are derived as a limiting case of the compressible flow equations
11. Explain how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations
12. Explain the main principles behind a modern Finite Volume CFD code and such concepts as explicit/implicit time stepping, CFL number, conservation, handling of compression shocks, and boundary conditions
13. Apply a given CFD code to a particular compressible flow problem
14. Analyze and verify the quality of the numerical solution
15. Explain the limitations in fluid flow simulation software
16. Report numerical analysis work in form of a technical report
    a. Describe a numerical analysis with details such that it is possible to redo the work based on the provided information
    b. Write a technical report (structure, language)
17. Search for literature relevant for a specific physical problem and summarize the main ideas and concepts found
18. Present engineering work in the form of oral presentations

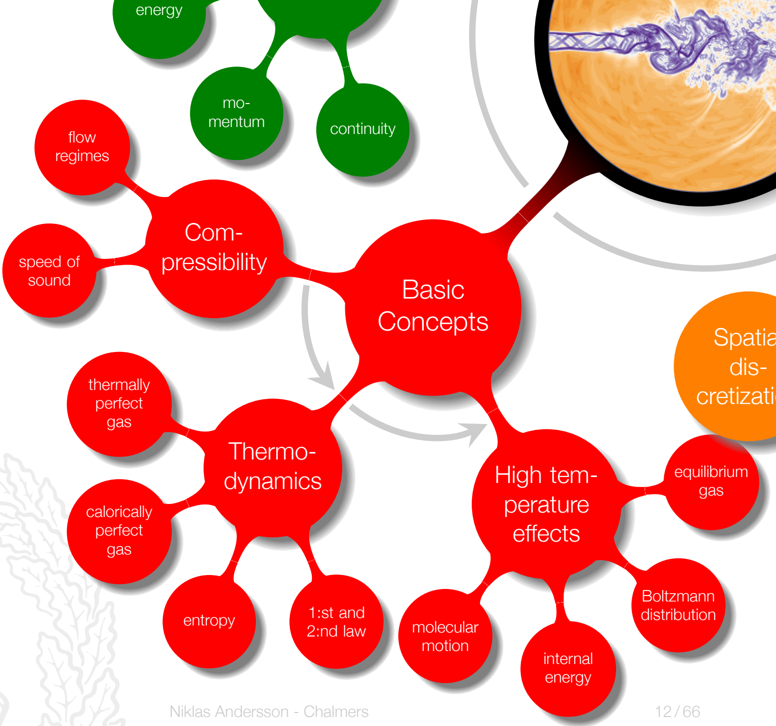
# Overview



Conservation laws integral form
2D Flow
shock reflection
expansion fans
shock expansion theory
oblique shocks
friction
1D Flow
heat addition
normal shocks
isentropic flow
Quasi 1D Flow
governing equations
nozzles
diffusers
governing equations
energy
mo-mentum
continuity
PDE:s
governing equations
noncon-servation form
substantial derivative
entropy equation
conser-vation form
Crocco's equation
traveling waves
moving shocks
shock reflection
acoustic waves
method of char-acteristics
finite non-linear waves
CFD
Boundary conditions
Shock handling
Spatial dis-cretization
Numerical schemes
Time integration
Basic Concepts
Com-pressibility
flow regimes
speed of sound
Thermo-dynamics
thermally perfect gas
calorically perfect gas
entropy
1:st and 2:nd law
High tem-perature effects
equilibrium gas
Boltzmann distribution
molecular motion
internal energy

# Chapter 1
# Compressible Flow

# Overview



energy
mo-
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continuity
flow
regimes
Com-
pressibility
speed of
sound
Basic
Concepts
Spatia
dis-
cretizati
thermally
perfect
gas
Thermo-
dynamics
calorically
perfect
gas
entropy
1:st and
2:nd law
High tem-
perature
effects
equilibrium
gas
Boltzmann
distribution
molecular
motion
internal
energy

# Addressed Learning Outcomes

1 Define the concept of compressibility for flows

2 Explain how to find out if a given flow is subject to significant compressibility effects

3 Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

6 Define the special cases of calorically perfect gas, thermally perfect gas and real gas and explain the implication of each of these special cases

*in this lecture we will find out what compressibility means and do a brief review of thermodynamics*

# Roadmap - Introduction to Compressible Flow

- Introduction
- Compressible Flow: Applications
- Historical milestones
- Compressibility
- Flow regimes

- Review of thermodynamics
- Gas properties
- First and second law of thermodynamics
- Isentropic relations

# Applications - Classical

- Treatment of calorically perfect gas
- Exact solutions of inviscid flow in 1D
- Shock-expansion theory for steady-state 2D flow
- Approximate closed form solutions to linearized equations in 2D and 3D
- Method of Characteristics (MOC) in 2D and axi-symmetric inviscid supersonic flows

# Applications - Modern

- Computational Fluid Dynamics (CFD)
- Complex geometries (including moving boundaries)
- Complex flow features (compression shocks, expansion waves, contact discontinuities)
- Viscous effects
- Turbulence modeling
- High temperature effects (molecular vibration, dissociation, ionization)
- Chemically reacting flow (equilibrium & non-equilibrium reactions)

# Applications - Examples

Turbo-machinery flows:

- Gas turbines, steam turbines, compressors
- Aero engines (turbojets, turbofans, turboprops)

Aeroacoustics:

- Flow induced noise (jets, wakes, moving surfaces)
- Sound propagation in high speed flows

External flows:

- Aircraft (airplanes, helicopters)
- Space launchers (rockets, re-entry vehicles)

Internall flows:

- Nozzle flows
- Inlet flows, diffusers
- Gas pipelines (natural gas, bio gas)

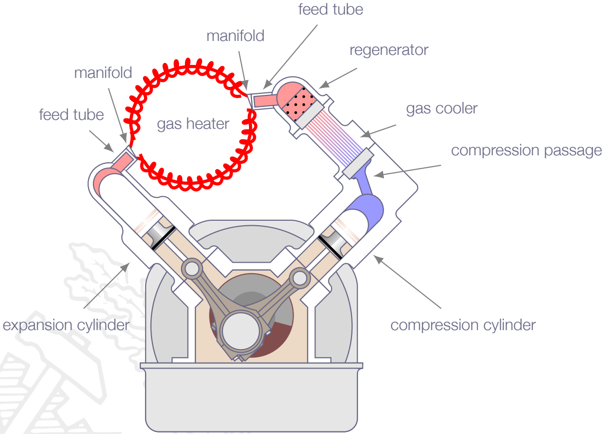
Free-shear flows:

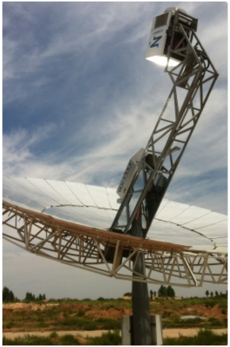
- High speed jets

Combustion:

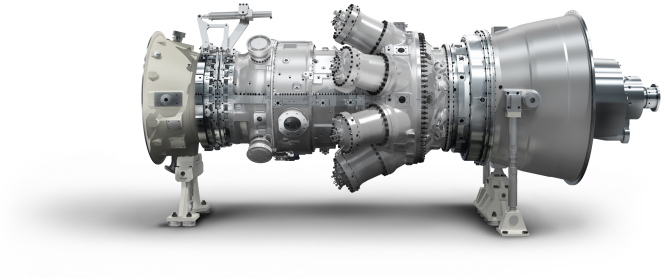
- Internal combustion engines (valve flow, in-cylinder flow, exhaust pipe flow, mufflers)
- Combustion induced noise (turbulent combustion)
- Combustion instabilities

# Applications - Stirling Engine

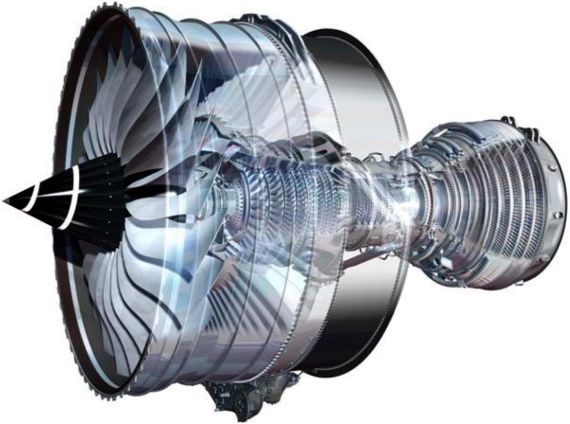


# Applications - Siemens GT750



# Applications - Rolls-Royce Trent XWB



# Applications - Airbus A380

# Applications - Vulcain Nozzle

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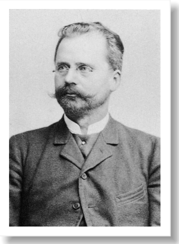
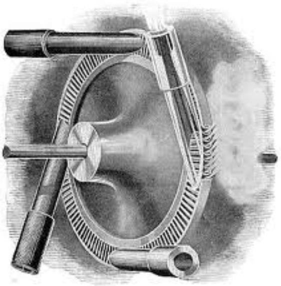
# Historical Milestones

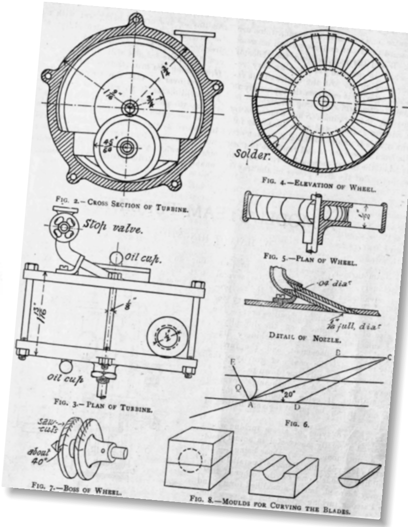


1893 C.G.P. de Laval, first steam turbine with supersonic nozzles (convergent-divergent). At this time, the significance was not fully understood, but it worked!

1947 Charles Yeager, flew first supersonic aircraft (XS-1), M 1.06

# Historical Milestones - C.G.P. de Laval (1893)

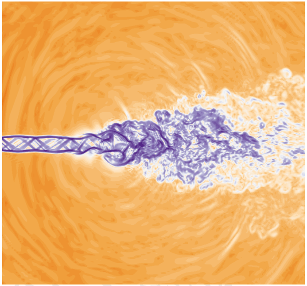


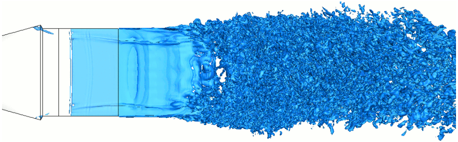


# Historical Milestones - Charles Yeager (1947)

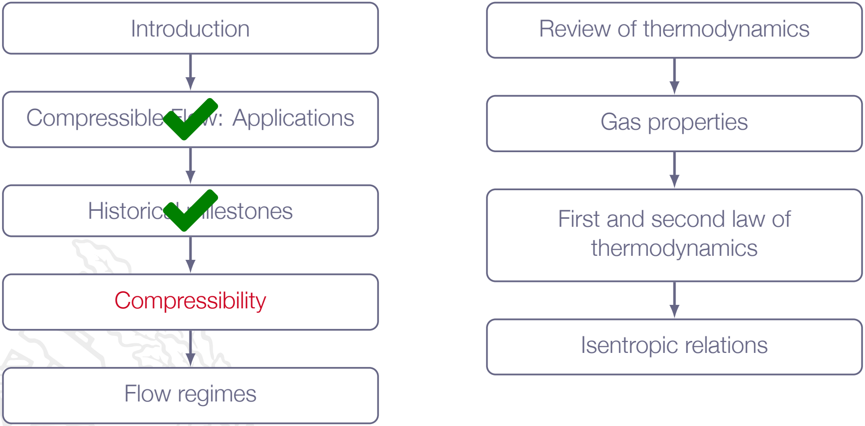

# Modern Compressible Flow

Screeching rectangular supersonic jet





# Roadmap - Introduction to Compressible Flow



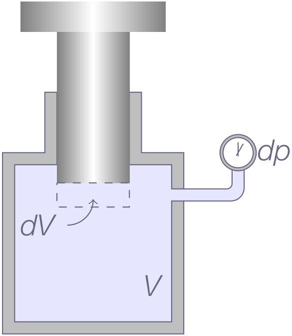
# Chapter 1.2 Compressibility

# Compressibility

$$\tau = -\frac{1}{\nu}\frac{\partial \nu}{\partial p},\ (\nu = \frac{1}{\rho})$$

Not really precise!

Is $T$ held constant during the compression or not?



# Compressibility

Two fundamental cases:

### Constant temperature

- Heat is cooled off to keep $T$ constant inside the cylinder
- The piston is moved slowly

### Adiabatic process

- Thermal insulation prevents heat exchange
- The piston is moved fairly rapidly (*gives negligible flow losses*)

# Compressibility

Isothermal process:

$$\tau_T = -\frac{1}{\nu}\left(\frac{\partial \nu}{\partial p}\right)_T$$

Adiabatic reversible (*isentropic*) process:

$$\tau_S = -\frac{1}{\nu}\left(\frac{\partial \nu}{\partial p}\right)_S$$

| | | |
|---|---|---|
| Air at normal conditions: | $\tau_T \approx 1.0 \times 10^{-5}$ | $[m^2/N]$ |
| Water at normal conditions: | $\tau_T \approx 5.0 \times 10^{-10}$ | $[m^2/N]$ |

# Compressibility

$$\tau = -\frac{1}{\nu}\frac{\partial \nu}{\partial p}$$

but

$$\nu = \frac{1}{\rho}$$

which gives

$$\tau = -\rho\frac{\partial}{\partial p}\left(\frac{1}{\rho}\right) = -\rho\left(-\frac{1}{\rho^2}\right)\frac{\partial \rho}{\partial p} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

Similarly:

$$\tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T, \quad \tau_S = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_S$$

# Compressibility

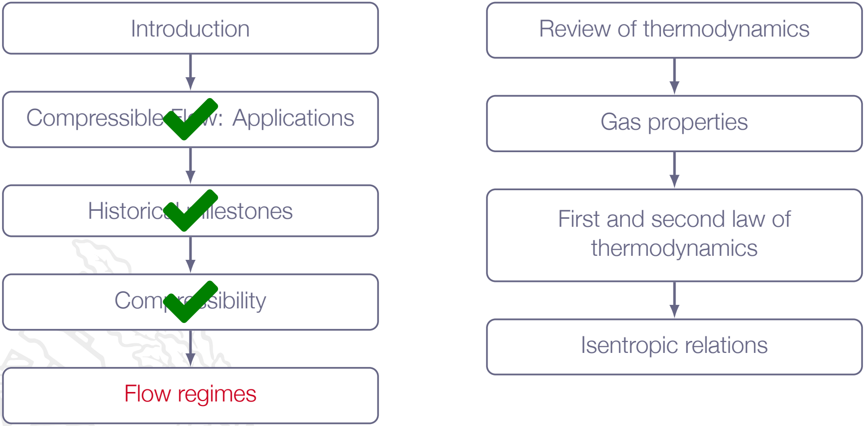
## Definition of compressible flow:

- If $p$ changes with amount $\Delta p$ over a characteristic length scale of the flow, such that the corresponding change in density, given by $\Delta\rho \sim \rho\tau\Delta$ p, is too large to be neglected, the flow is compressible (*typically, if* $\Delta\rho/\rho > 0.05$)

## Important note:

- Bernoulli´s equation is restricted to incompressible flow, *i.e.* it is not valid for compressible flow!

# Roadmap - Introduction to Compressible Flow



# Chapter 1.3
# Flow Regimes

# Flow Regimes

The freestream Mach number is defined as

$$M_\infty = \frac{U_\infty}{a_\infty}$$

where $U_\infty$ is the freestream flow speed and $a_\infty$ is the speed of sound at freestream conditions

# Flow Regimes

Assume first incompressible flow and estimate the max pressure difference using

$$\Delta p \approx \frac{1}{2}\rho_\infty U_\infty^2$$

For air at normal conditions we have

$$\tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T = \frac{1}{\rho R T} = \frac{1}{p}$$

*(ideal gas law for perfect gas $p = \rho R T$)*

# Flow Regimes

Using the relations on previous slide we get

$$\frac{\Delta\rho}{\rho} \approx \tau_T \Delta p \approx \frac{1}{p_\infty}\frac{1}{2}\rho_\infty U_\infty^2 = \frac{\frac{1}{2}\rho_\infty U_\infty^2}{\rho_\infty R T_\infty}$$

for a calorically perfect gas we have

$$a = \sqrt{\gamma R T}$$

which gives us

$$\frac{\Delta\rho}{\rho} \approx \frac{\gamma U_\infty^2}{2a_\infty^2}$$

now, using the definition of Mach number we get

$$\frac{\Delta\rho}{\rho} \approx \frac{\gamma}{2} M_\infty^2$$

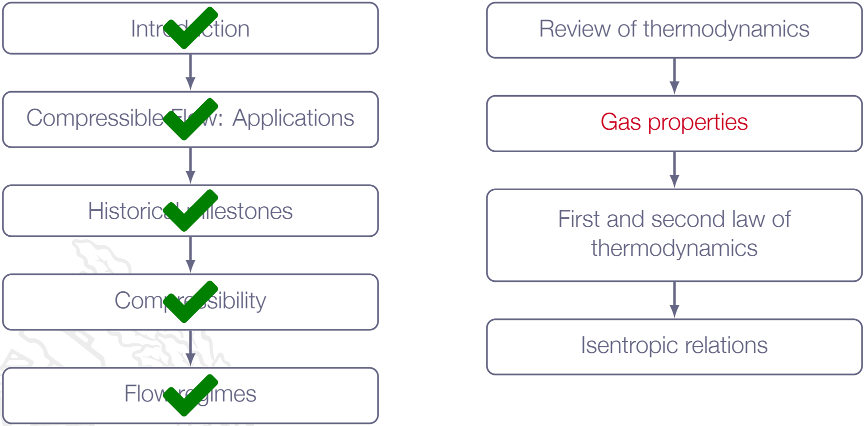
# Flow Regimes

| | |
|---|---|
| Incompressible | $M_\infty < 0.1$ |
| Subsonic | $M_\infty < 1$ and $M < 1$ everywhere |
| Transonic | case 1: $M_\infty < 1$ and $M > 1$ locally<br>case 2: $M_\infty > 1$ and $M < 1$ locally |
| Supersonic | $M_\infty > 1$ and $M > 1$ everywhere |
| Hypersonic | supersonic flow with high-temperature effects |

Compressible

Local Mach number $M$ is based on local flow speed, $U = |\mathbf{U}|$, and local speed of sound, $a$

# Roadmap - Introduction to Compressible Flow



# Chapter 1.4
# Review of Thermodynamics

# Thermodynamic Review

Compressible flow:

*... strong interaction between flow and thermodynamics ...*

# Perfect Gas

All intermolecular forces negligible

Only elastic collitions between molecules

$$p\nu = RT$$

or

$$\frac{p}{\rho} = RT$$

where $R$ is the gas constant $[R] = J/kgK$

Also, $R = R_{univ}/M$ where $M$ is the molecular weight of gas molecules (in $kg/kmol$) and $R_{univ} = 8314\,J/kmol\,K$

# Internal Energy and Enthalpy

Internal energy $e$ ($[e] = J/kg$)
Enthalpy $h$ ($[h] = J/kg$)

$$h = e + p\nu = e + \frac{p}{\rho} \text{ (valid for all gases)}$$

For any gas in thermodynamic equilibrium, e and h are functions of only two thermodynamic variables (*any two variables may be selected*) *e.g.*

$$e = e(T, \rho)$$
$$h = h(T, p)$$

# Internal Energy and Enthalpy

Special cases:

**Thermally perfect gas:**

$e = e(T)$ and $h = h(T)$

OK assumption for air at near atmospheric conditions and $100K < T < 2500K$

**Calorically perfect gas:**

$e = C_v T$ and $h = C_p T$ ($C_v$ and $C_p$ are constants)

OK assumption for air at near atmospheric pressure and $100K < T < 1000K$

# Specific Heat

For thermally perfect (and calorically perfect) gas

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p, \quad C_v = \left(\frac{\partial e}{\partial T}\right)_v$$

since $h = e + p/\rho = e + RT$ we obtain:

$$C_p = C_v + R$$

The ratio of specific heats, $\gamma$, is defined as:

$$\gamma \equiv \frac{C_p}{C_v}$$

# Specific Heat

Important!

calorically perfect gas:

$C_v$, $C_p$, and $\gamma$ are constants

thermally perfect gas:

$C_v$, $C_p$, and $\gamma$ will depend on temperature

# Specific Heat

$$C_p - C_v = R$$

divide by $C_v$

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p - C_v = R$$

divide by $C_p$

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

# Specific Heat

$$C_p - C_v = R$$

divide by $C_v$

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p - C_v = R$$

divide by $C_p$

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

valid for both thermally perfect and calorically perfect gas!

# Roadmap - Introduction to Compressible Flow

- Introduction ✔
- Compressible Flow: Applications ✔
- Historical Milestones ✔
- Compressibility ✔
- Flow regimes ✔

- Review of thermodynamics
- Gas properties ✔
- First and second law of thermodynamics
- Isentropic relations

# First Law of Thermodynamics

A fixed mass of gas, separated from its surroundings by an imaginary flexible boundary, is defined as a ”system”. This system obeys the relation

$$de = \delta q - \delta w$$

where

*de* is a change in internal energy of system

$\delta q$ is heat added to the system

$\delta w$ is work done by the system (on its surroundings)

Note: *de* only depends on starting point and end point of the process while $\delta q$ and $\delta w$ depend on the actual process also

# First Law of Thermodynamics

Examples:

Adiabatic process:

$\delta q = 0.$

Reversible process:

no dissipative phenomena (*no flow losses*)

Isentropic process:

a process which is both adiabatic and reversible

# First Law of Thermodynamics

Reversible process:

$$\delta w = p d\nu = p d(1/\rho)$$

$$de = \delta q - p d(1/\rho)$$

Adiabatic & reversible process:

$$\delta q = 0.$$

$$de = -p d(1/\rho)$$

# Entropy

Entropy $s$ is a property of all gases, uniquely defined by any two thermodynamic variables, *e.g.*

$s = s(p, T)$ or $s = s(\rho, T)$ or $s = s(\rho, p)$ or $s = s(e, h)$ or ...

# Second Law of Thermodynamics

Concept of entropy $s$:

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir}, \quad (ds_{ir} > 0.)$$

or

$$ds \geq \frac{\delta q}{T}$$

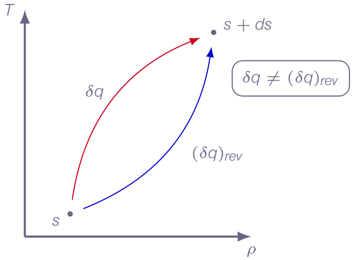
# Second Law of Thermodynamics

Concept of entropy $s$:

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir}, \quad (ds_{ir} > 0.)$$

or

$$ds \geq \frac{\delta q}{T}$$



# Second Law of Thermodynamics

In general:

$$ds \geq \frac{\delta q}{T}$$

For adiabatic processes:

$$ds \geq 0.$$

# Second Law of Thermodynamics

*"In this house, we obey the laws of thermodynamics!"*

Homer Simpson, after Lisa constructs a perpetual motion machine whose energy increases with time

# Calculation of Entropy

For reversible processes ($\delta w = pd(1/\rho)$ and $\delta q = Tds$):

$$de = Tds - pd\left(\frac{1}{\rho}\right)$$

or

$$Tds = de + pd\left(\frac{1}{\rho}\right)$$

from before we have $h = e + p/\rho \Rightarrow$

$$dh = de + pd\left(\frac{1}{\rho}\right) + \left(\frac{1}{\rho}\right)dp \Leftrightarrow de = dh - pd\left(\frac{1}{\rho}\right) - \left(\frac{1}{\rho}\right)dp$$

# Calculation of Entropy

For thermally perfect gases, $p = \rho RT$ and $dh = C_p dT \Rightarrow$

$$ds = C_p \frac{dT}{T} - R\frac{dp}{p}$$

Integration from starting point (1) to end point (2) gives:

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - R \ln\left(\frac{p_2}{p_1}\right)$$

and for calorically perfect gases

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

# Calculation of Entropy

If we instead use $de = C_v dT$ we get

for thermally perfect gases

$$s_2 - s_1 = \int_1^2 C_v \frac{dT}{T} - R \ln \left( \frac{\rho_2}{\rho_1} \right)$$

and for calorically perfect gases

$$s_2 - s_1 = C_v \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{\rho_2}{\rho_1} \right)$$

# Roadmap - Introduction to Compressible Flow

- Introduction
- Compressible Flow: Applications
- Historical milestones
- Compressibility
- Flow regimes

- Review of thermodynamics
- Gas properties
- First and second law of thermodynamics
- Isentropic relations

# Isentropic Relations

For calorically perfect gases, we have

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

For adiabatic reversible processes:

$$ds = 0. \Rightarrow s_1 = s_2 \Rightarrow C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = 0 \Rightarrow$$

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{C_p}{R}\ln\left(\frac{T_2}{T_1}\right)$$

# Isentropic Relations

$$\frac{C_p}{R} = \frac{C_p}{C_p - C_v} = \frac{\gamma}{\gamma - 1} \Rightarrow$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

# Isentropic Relations

Alternatively

$$s_2 - s_1 = 0 = C_v \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{\rho_2}{\rho_1}\right) \Rightarrow$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}}$$

# Isentropic Relations - Summary

For an isentropic process and a calorically perfect gas we have

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

A.K.A. the isentropic relations

# Roadmap - Introduction to Compressible Flow