A NEW APPROACH OF ZONAL HYBRID RANS-LES BASED ON A TWO-EQUATION $k - \varepsilon$ MODEL [2]

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PANS Low Reynolds Number Model [3]

$$\frac{\partial k}{\partial t} + \frac{\partial (kU_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{ku}}\right)\frac{\partial k}{\partial x_j}\right] + (P_k - \varepsilon)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial (\varepsilon U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} P_k \frac{\varepsilon}{k} - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

$$\nu_t = C_\mu f_\mu \frac{k^2}{\varepsilon}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

$C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_k, σ_ε and C_μ same values as [1]. $f_\varepsilon = 1$. f_2 and f_μ read

$$f_2 = \left[1 - \exp\left(-\frac{y^*}{3.1}\right)\right]^2 \left\{1 - 0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}$$

$$f_\mu = \left[1 - \exp\left(-\frac{y^*}{14}\right)\right]^2 \left\{1 + \frac{5}{R_t^{3/4}}\exp\left[-\left(\frac{R_t}{200}\right)^2\right]\right\}$$

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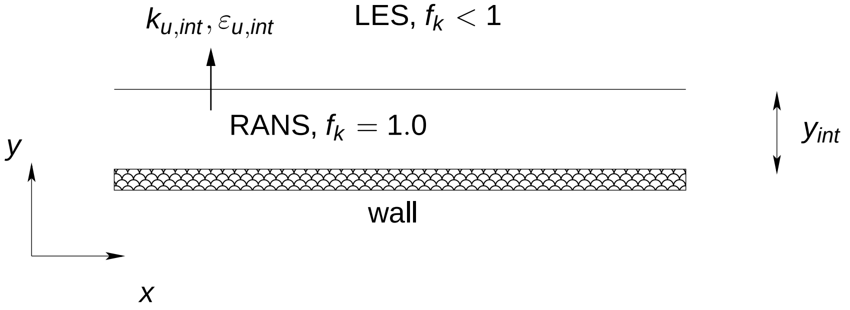
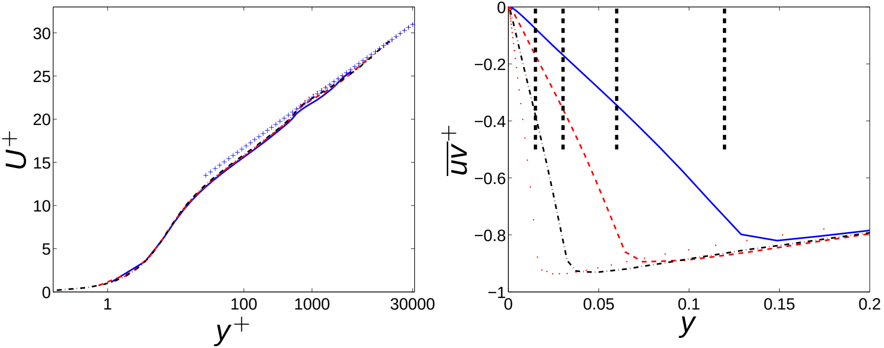
CHANNEL FLOW: ZONAL RANS-LES



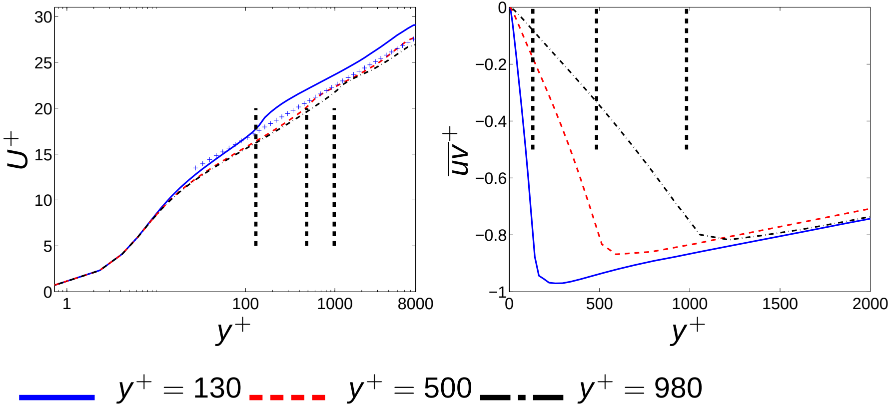
- Interface: how to treat k and ε over the interface? They should be reduced from their RANS values to suitable LES values
- The usual convection and diffusion across the interface is cut off, and new "interface boundary" conditions are prescribed
- $k_{u,int} = f_k k_{RANS}$
- Nothing is done for ε
- $x_{max} = 3.2$ (64 cells), $z_{max} = 1.6$ (64 cells), y dir: $80 - 128$ cells
- CDS in entire region

$(N_x \times N_z) = (64 \times 64)$. $y^+_{int} = 500$

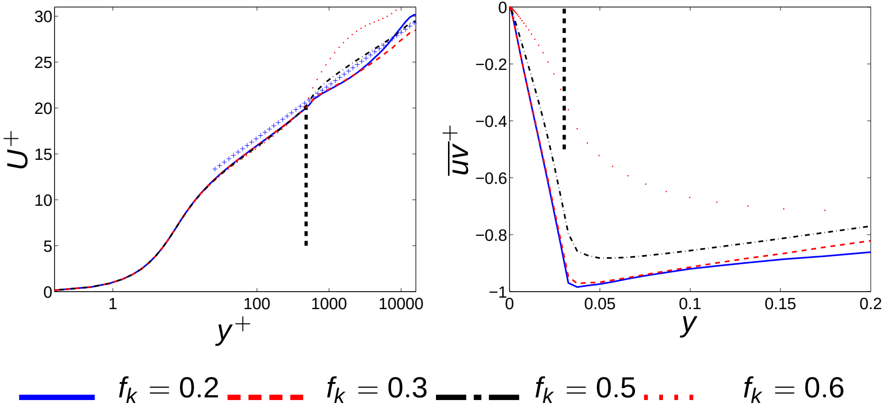


$Re_\tau = 4\,000$ (blue solid), $Re_\tau = 8\,000$ (red dashed), $Re_\tau = 16\,000$ (black dash-dot); $Re_\tau = 32\,000$ (red dotted).

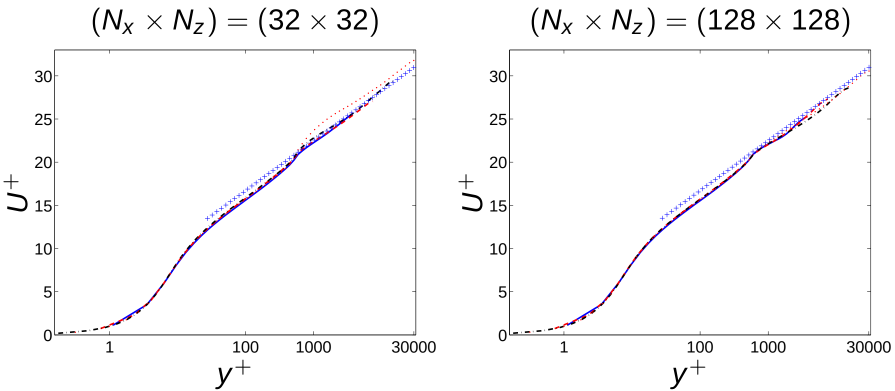
INTERFACE LOCATION. $Re_\tau = 8\,000$.



Effect of f_k. $Re_\tau = 16\,000$. $y^+_{int} = 500$

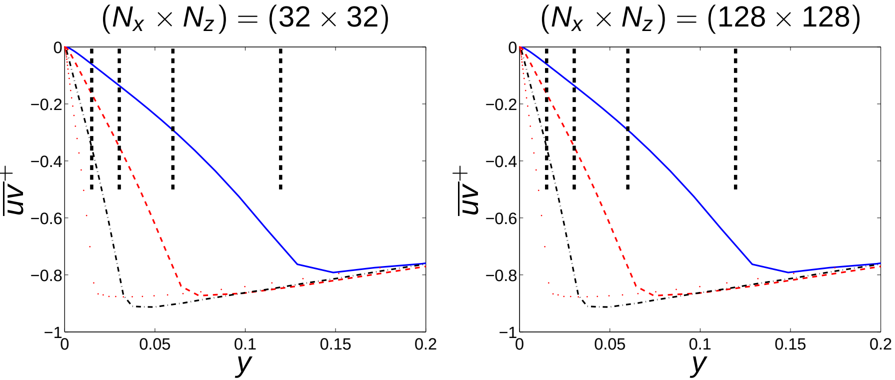


Effect of Resolution: Velocity



$Re_\tau = 4\,000$ (solid blue line), $Re_\tau = 8\,000$ (red dashed line), $Re_\tau = 16\,000$; (black dash-dotted line)
$Re_\tau = 32\,000$. (red dotted line)

Effect of Resolution: Resolved Shear Stress



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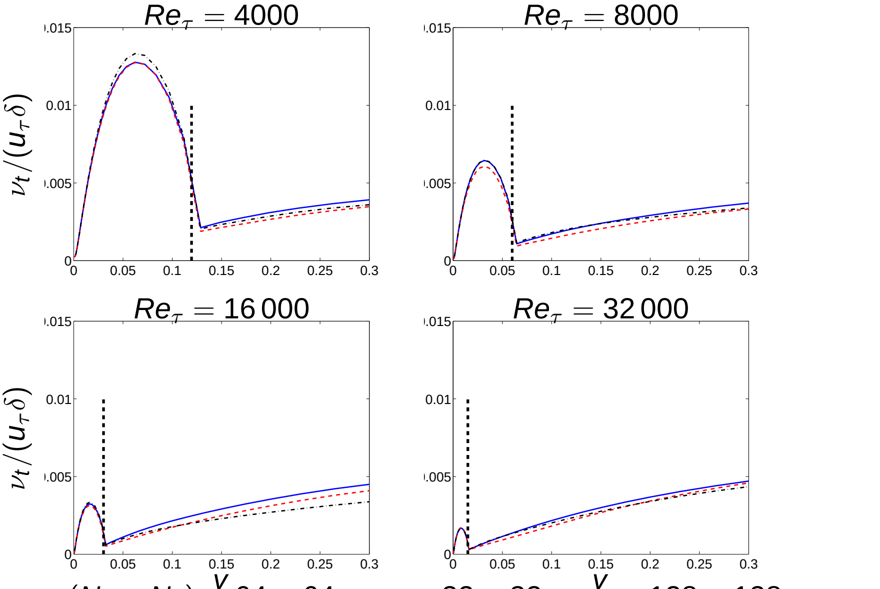
EFFECT OF RESOLUTION: TURBULENT VISCOSITY



$(N_x \times N_z) = 64 \times 64$ (blue solid); 32×32 (red dashed); 128×128 (black dash-dot)

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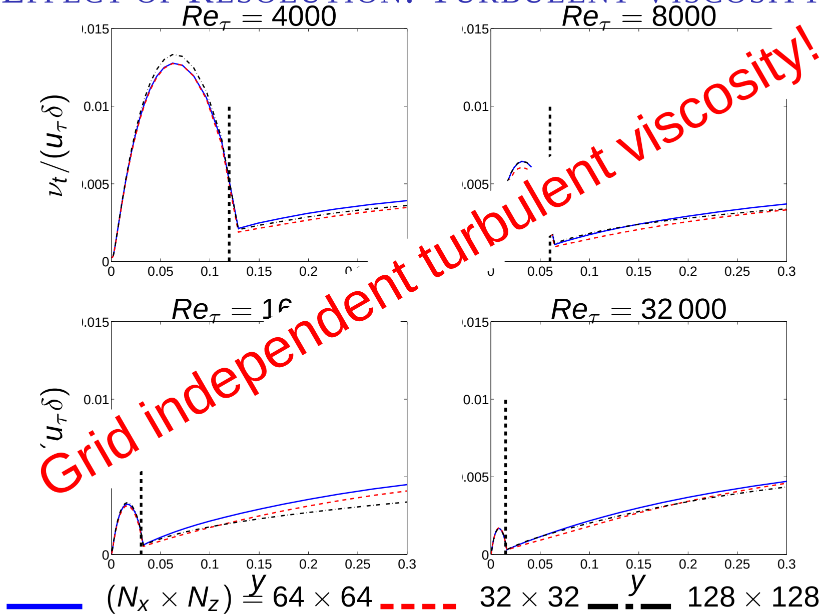

SGS Models Based on Grid Size

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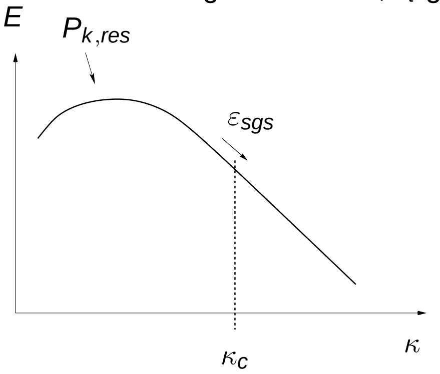

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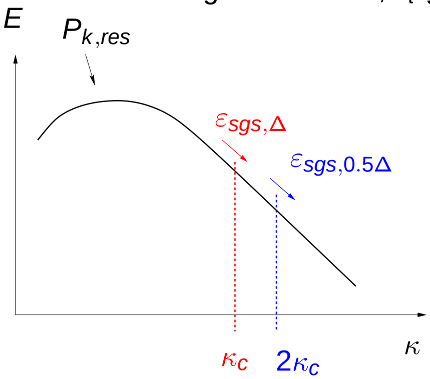

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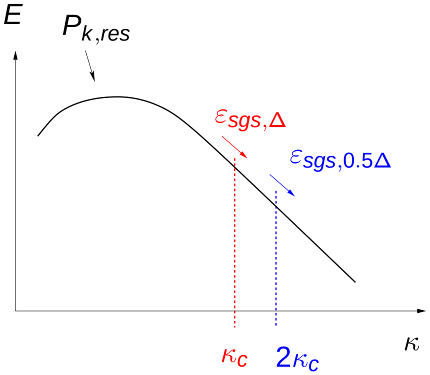
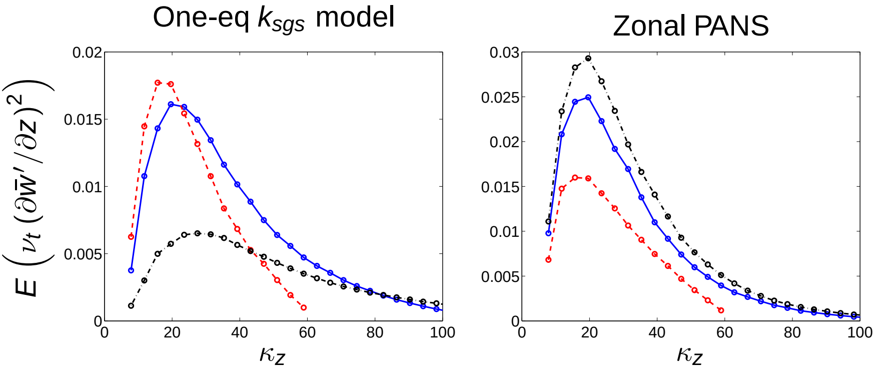


- $\varepsilon_{sgs,\Delta} = \varepsilon_{sgs,0.5\Delta}$
- $\varepsilon_{sgs} = 2\langle \nu_t \bar{s}_{ij}\bar{s}_{ij}\rangle - \langle \tau_{12,t}\rangle \dfrac{\partial \langle \bar{u}\rangle}{\partial y}$
- Grid refinement $\Rightarrow$ must be accompanied with larger $\bar{s}_{ij}\bar{s}_{ij}$
- $\Rightarrow$ $\bar{s}_{ij}\bar{s}_{ij}$ must take place at higher wavenumbers
- if not $\Rightarrow$ grid dependent

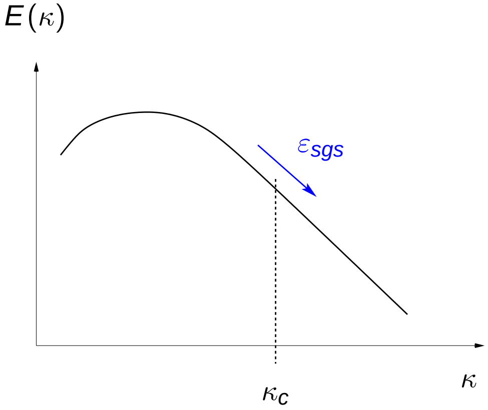
Power Density Spectra of $\nu_t^{0.5}\frac{\partial \bar{w}'}{\partial z}$



$(N_x \times N_z) = 64 \times 64$ ---- 32×32 —·— 128×128

SGS dissipation vs. Wavenumber

- Energy spectra of the SGS dissipation show that the peak takes place at surprisingly low wavenumber (length scale corresponding to 10 cells or more).



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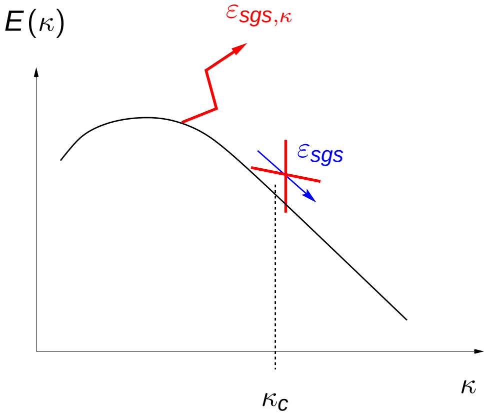
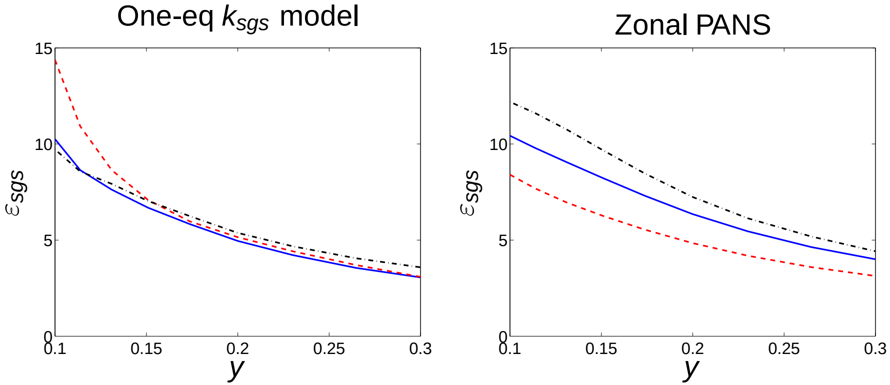


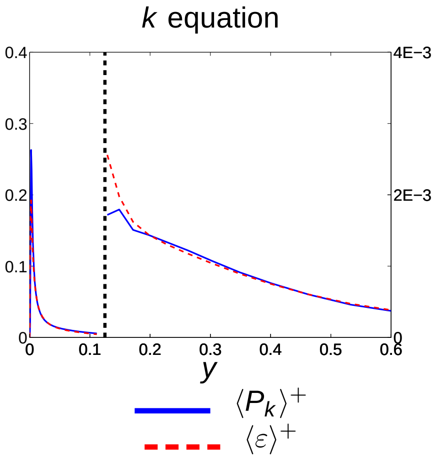
SGS DISSIPATION, $Re_\tau = 8000$

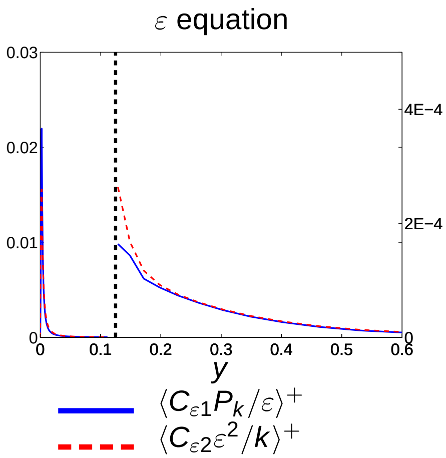
- SGS dissipation in the $\bar{u}'_i\bar{u}'_i/2$ eq, $\varepsilon_{sgs} = 2\langle \nu_t \bar{s}_{ij}\bar{s}_{ij}\rangle - \langle \tau_{12,t}\rangle \dfrac{\partial \langle \bar{u}\rangle}{\partial y}$



$(N_x \times N_z) = 64 \times 64$ (solid blue), 32×32 (dashed red), 128×128 (dash-dotted black)

LOCAL EQUILIBRIUM. $Re_\tau = 4000$, $N_x \times N_z = 64 \times 64$.





Left vertical axes: URANS region; right vertical axes: LES region.

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then

$$C_1 \frac{\langle \varepsilon \rangle}{\langle k \rangle} \langle P_k \rangle \neq C_2^* \frac{\langle \varepsilon \rangle^2}{\langle k \rangle}, \text{because} \quad C_1 \neq C_2^*$$

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However, the figure on previous slide shows

$$C_1 \left\langle \frac{\varepsilon}{k} P_k \right\rangle = C_2^* \left\langle \frac{\varepsilon^2}{k} \right\rangle$$

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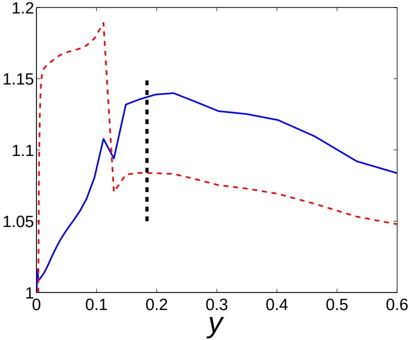
- Answer: when time-averaging $\langle ab \rangle \neq \langle a \rangle \langle b \rangle$

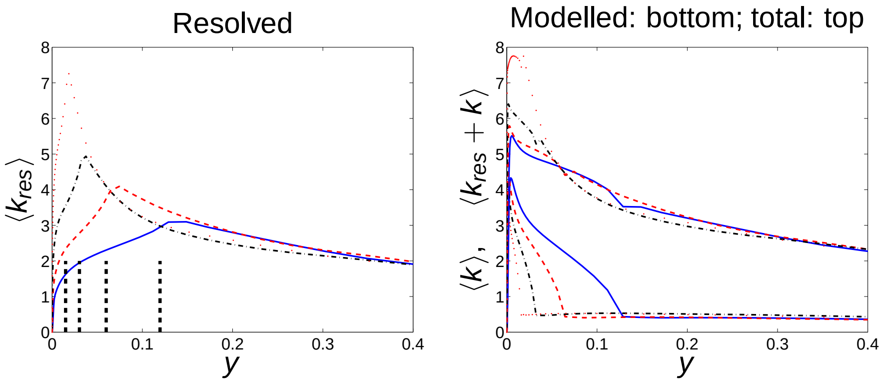
Local Equilibrium in ε Equation.

- The answer is because of time averaging ($\langle ab \rangle < \langle a \rangle \langle b \rangle$, (see below)



$\frac{\langle \varepsilon P_k / k \rangle}{\langle \varepsilon \rangle \langle P_k \rangle / \langle k \rangle}$ (solid blue) $\frac{\langle \varepsilon^2 / k \rangle}{\langle \varepsilon^2 \rangle / \langle k \rangle}$ (dashed red)

Resolved And Modelled Turbulent Kinetic Energy.



——— $Re_\tau = 4\,000$ - - - - $Re_\tau = 8\,000$ —·— $Re_\tau = 16\,000$;

.... $Re_\tau = 32\,000$.

Concluding Remarks

- LRN PANS works well as zonal LES-RANS model for very high Re_τ ($> 32\,000$)
- The model gives grid independent results
- The location of the interface is not important (it should not be too close to the wall)
- Values of $0.2 < f_k < 0.5$ have little impact on the results

[1] ABE, K., KONDOH, T., AND NAGANO, Y.
A new turbulence model for predicting fluid flow and heat transfer in separating and reattaching flows - 1. Flow field calculations.
Int. J. Heat Mass Transfer 37 (1994), 139–151.

[2] DAVIDSON, L.
A new approach of zonal hybrid RANS-LES based on a two-equation $k - \varepsilon$ model.
In *ETMM9: International ERCOFTAC Symposium on Turbulence Modelling and Measurements* (Thessaloniki, Greece, 2012).

[3] MA, J., PENG, S.-H., DAVIDSON, L., AND WANG, F.
A low Reynolds number variant of Partially-Averaged Navier-Stokes model for turbulence.
International Journal of Heat and Fluid Flow 32 (2011), 652–669.