How to estimate the resolution of an LES of recirculating flow [1]

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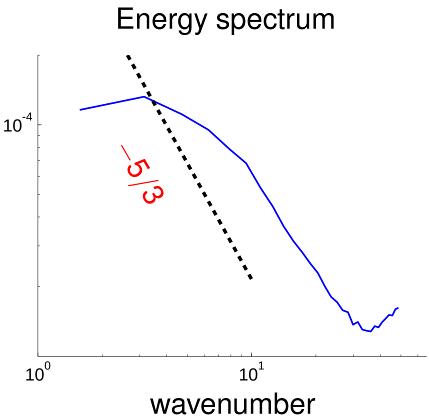
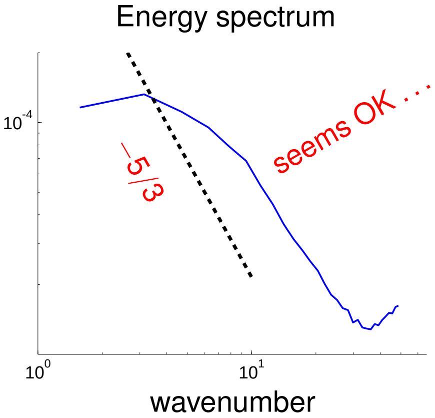
How To Estimate Resolution of an LES?

- In boundary layers there are guidelines *à priori*. The cells size in the streamwise and spanwise direction should be approximately 100 and 30 respectively. First wall-adjacent node at $y^+ \simeq 1$.
- No guidelines in free-flow region (shear layers, re-circulation region . . .)
- Worse: even after having carried out an LES, it is difficult to know if the resolution is good!
- I have recently made a similar study for channel flow [2]

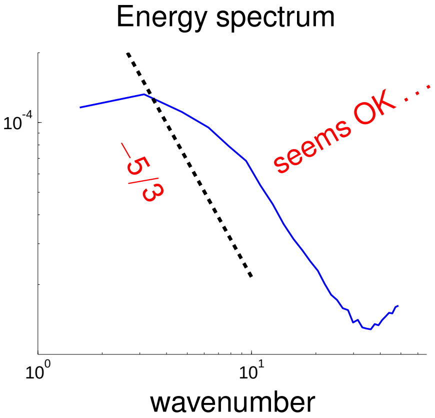
ENERGY SPECTRUM

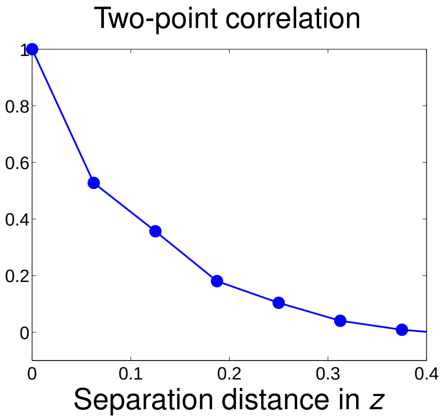


ENERGY SPECTRUM AND TWO-POINT CORRELATION

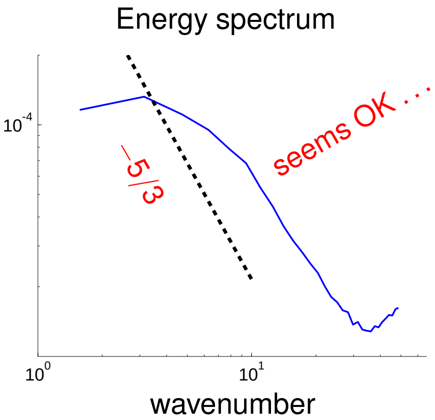


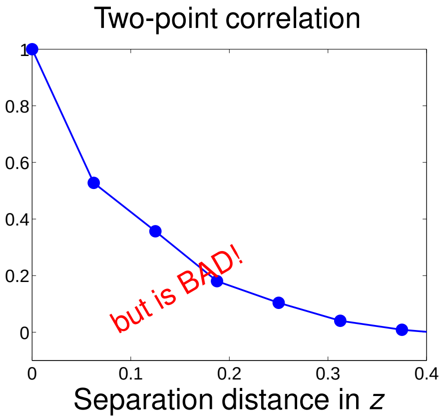
Energy Spectrum and Two-point Correlation



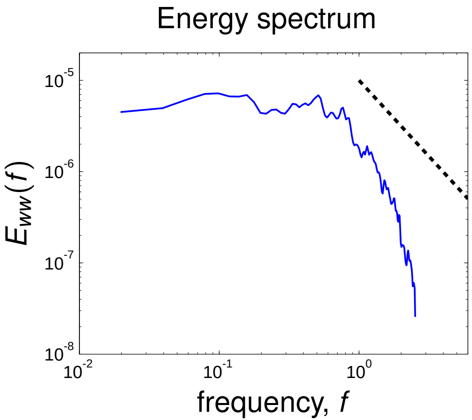


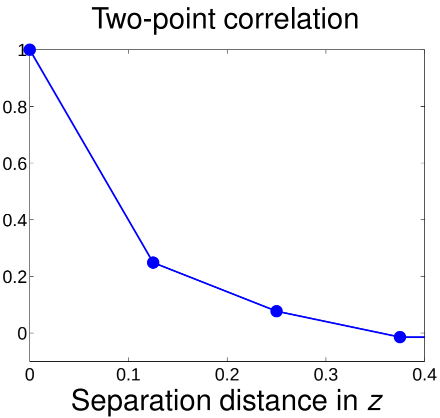
Energy Spectrum and Two-point Correlation





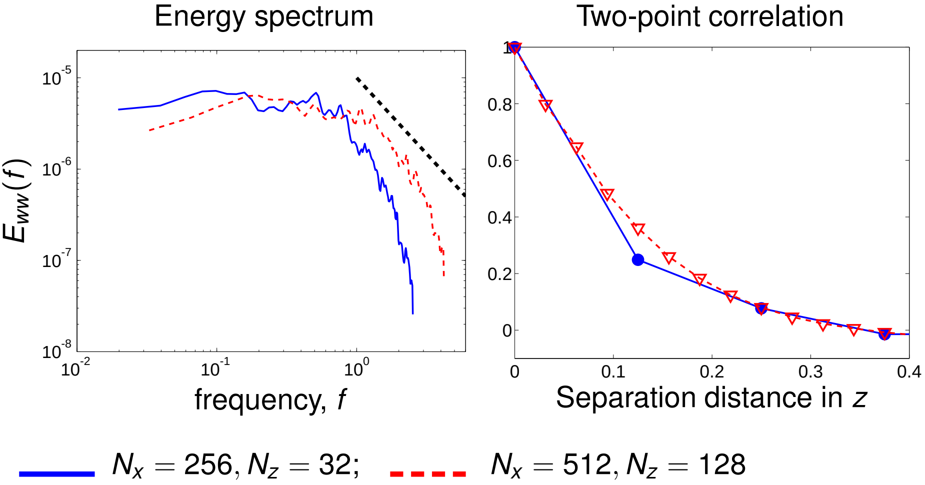
Energy Spectrum vs time and Two-point Correlation



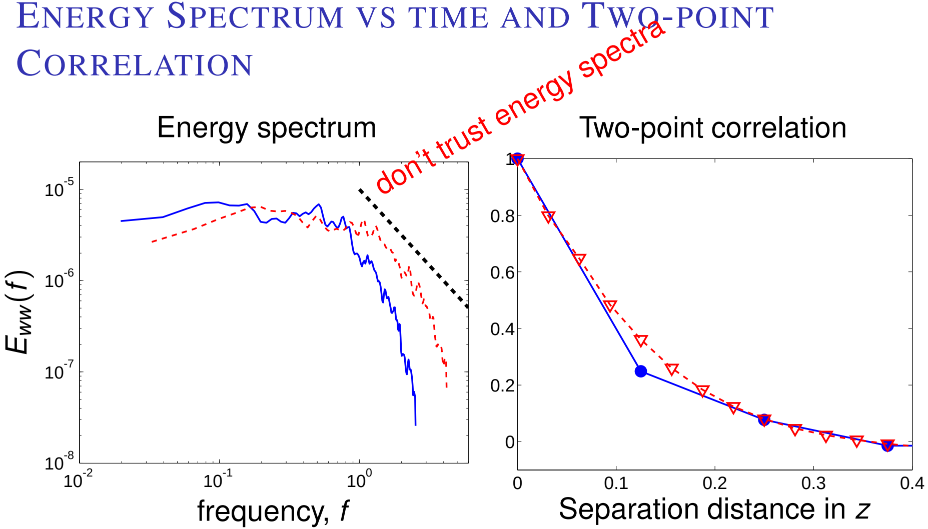


$N_x = 256, N_z = 32$

ENERGY SPECTRUM VS TIME AND TWO-POINT CORRELATION

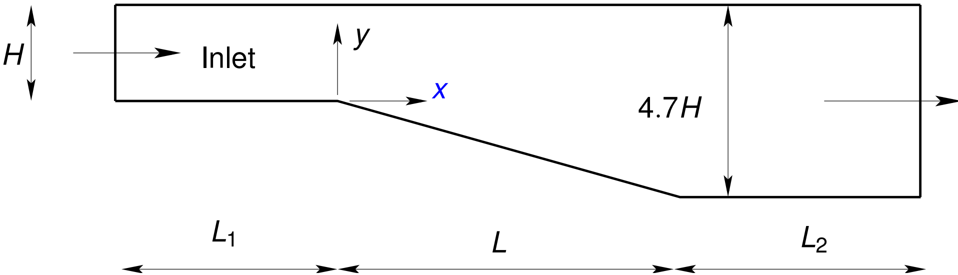


Energy Spectrum vs time and Two-point Correlation



$N_x = 256, N_z = 32;$ $N_x = 512, N_z = 128$

Plane Asymmetric Diffuser (Not to Scale)



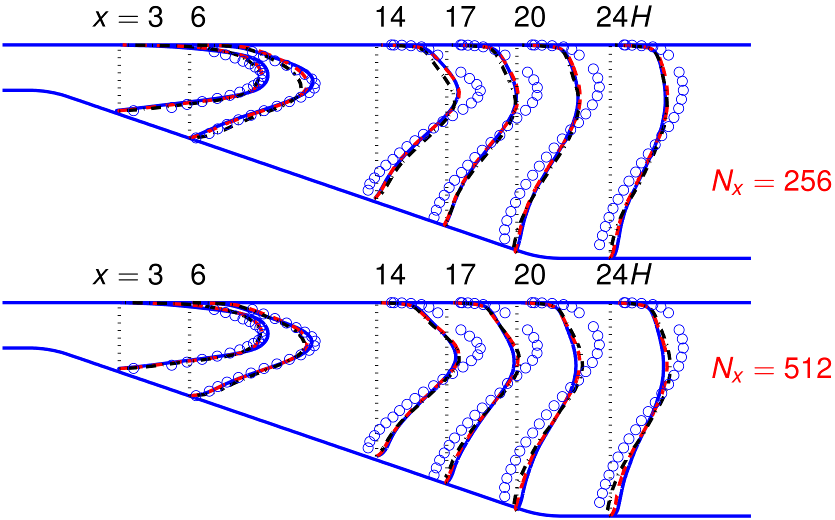
$L_1 = 7.9H$, $L = 21H$, $L_2 = 28H$. The spanwise width is $z_{max} = 4H$.

- Mesh ($x \times y \times z$)
  - $258 \times 64 \times 32$, $258 \times 64 \times 64$, $258 \times 64 \times 128$
  - .
  - $512 \times 64 \times 32$, $512 \times 64 \times 64$, $512 \times 64 \times 128$

COMPUTATIONAL METHOD

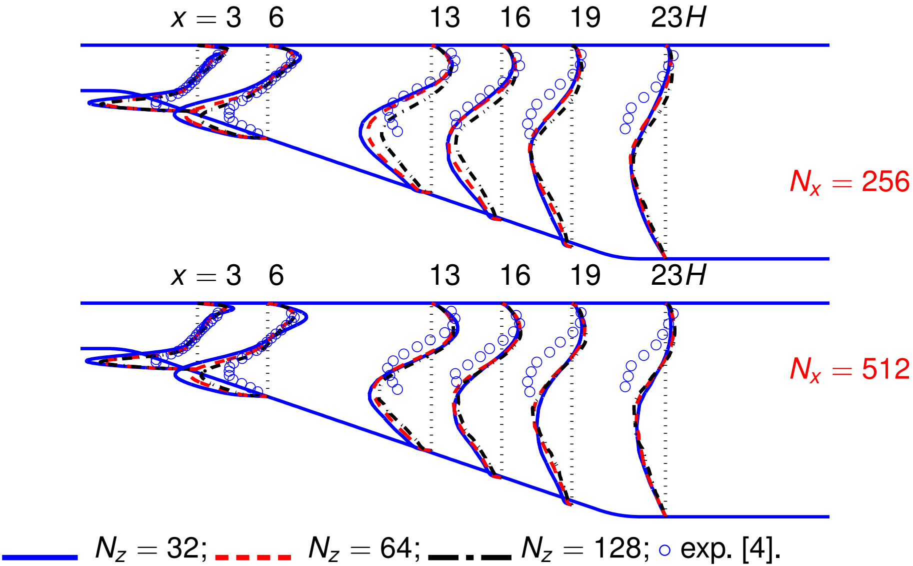
- Finite volume with central differencing in space and time (Crank-Nicolson)
- Fractional step
- Dynamic Smagorinsky model
- Inlet fluctuating boundary conditions: synthetic isotropic turbulence [3]
- All simulations run on a single CPU. Averaging during one week (the finest mesh: two weeks)

$\langle \bar{u} \rangle / U_{b,in}$ PROFILES

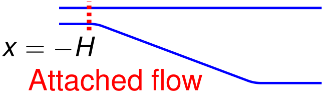


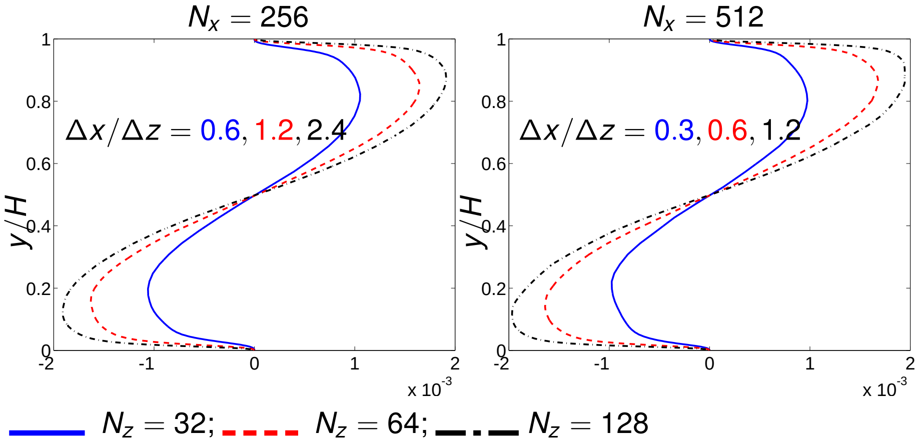
$N_z = 32$; $N_z = 64$; $N_z = 128$; ○ exp. [4].

$\langle u'v' \rangle / U_{b,in}^2$ PROFILES

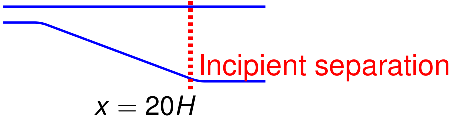


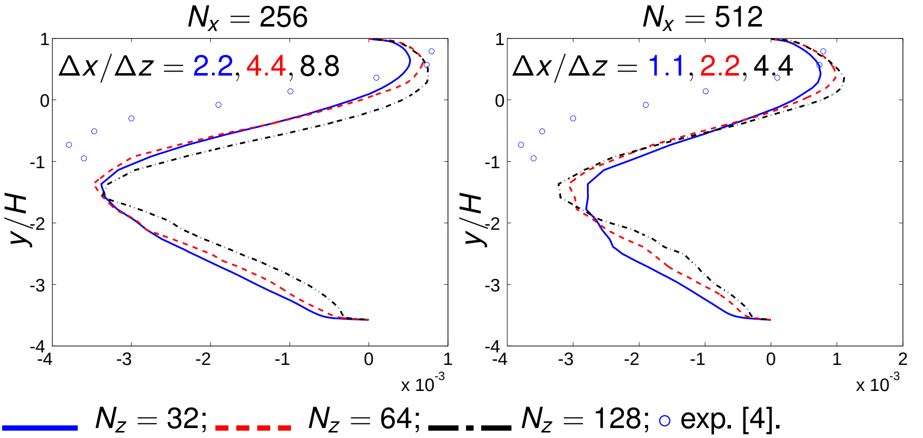
$\langle u'v' \rangle / U_{b,in}^2$ PROFILES AT $x = -H$





$\langle u'v' \rangle / U_{b,in}^2$ PROFILES AT $x = 20H$



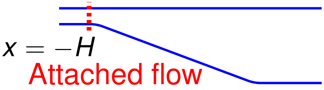


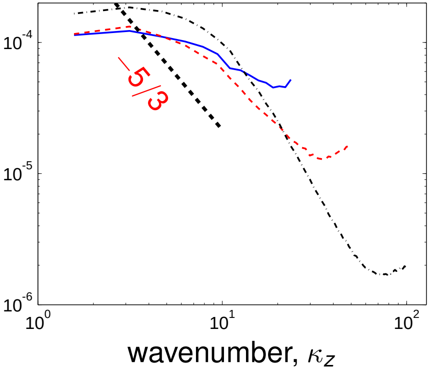
Different Ways to Estimate Resolution

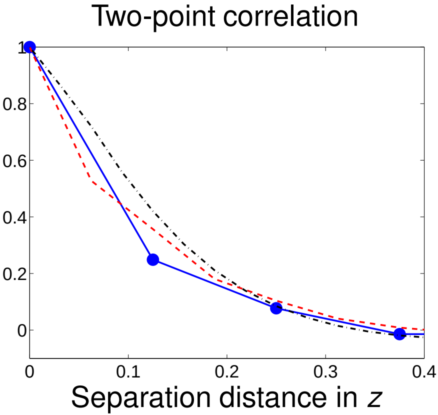
- Energy spectra (both in spanwise direction and time)
- Two-point correlations
- Ratio of SGS shear stress $\langle \tau_{sgs,12} \rangle$ to resolved $\langle u'v' \rangle$
- Ratio of SGS viscosity, $\langle \nu_{sgs} \rangle$ to molecular, ν
- Energy spectra of SGS dissipation
- Comparison of SGS dissipation due to $\partial u'_i / \partial x_j$ and $\partial \langle \bar{u}_i \rangle / \partial x_j$

- Below we will only analyze results from the $N_x = 256$ meshes

ENERGY SPECTRA, TWO-POINT CORR. AT $x = -H$

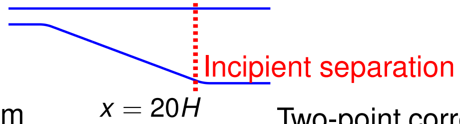


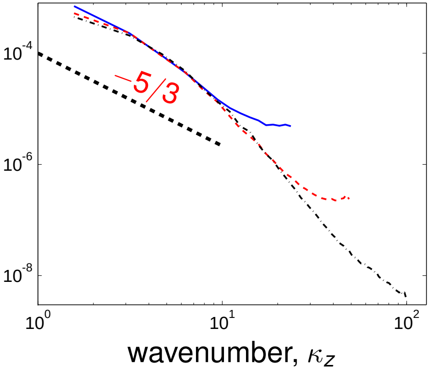


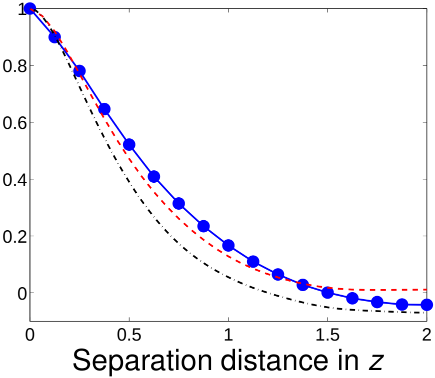


$N_z = 32$; $N_z = 64$; $N_z = 128$.

Energy Spectra, Two-Point Corr. at $x = 20H$

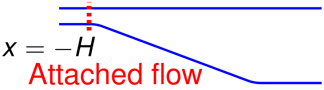


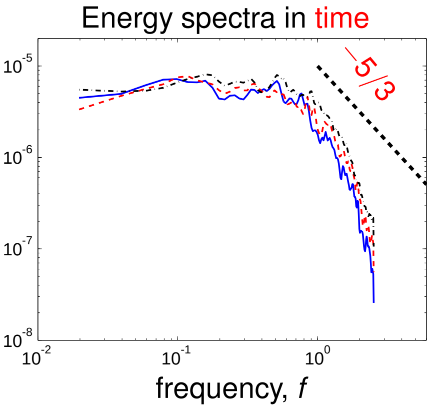


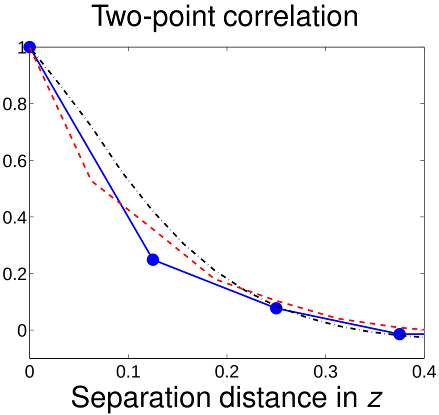


$N_z = 32$; $N_z = 64$; $N_z = 128$.

ENERGY SPECTRA IN TIME. $x = -1$.

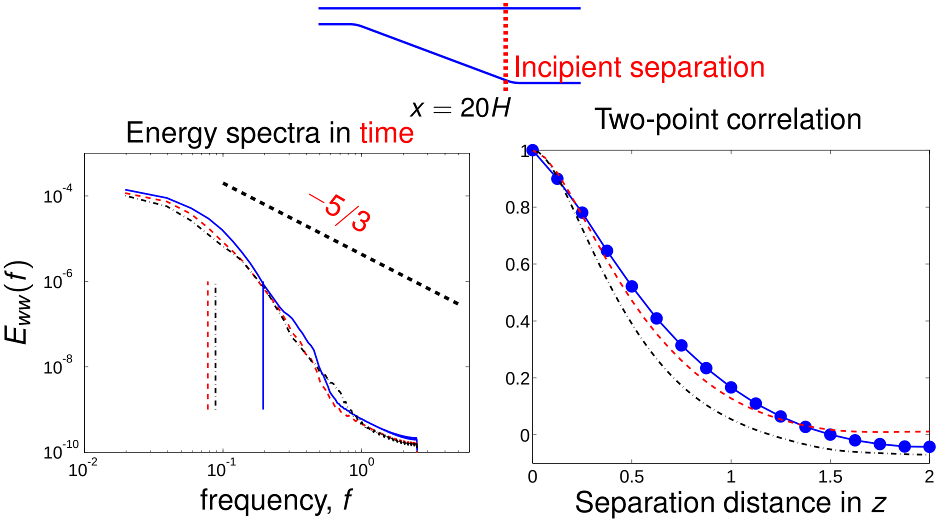






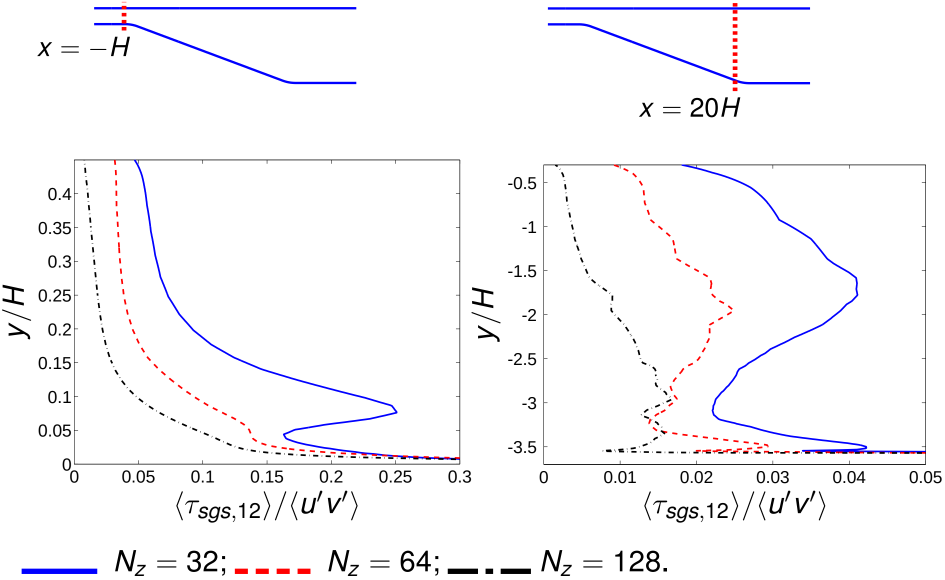
$N_z = 32$; $N_z = 64$; $N_z = 128$.

Energy Spectra in time. $x = 20$.

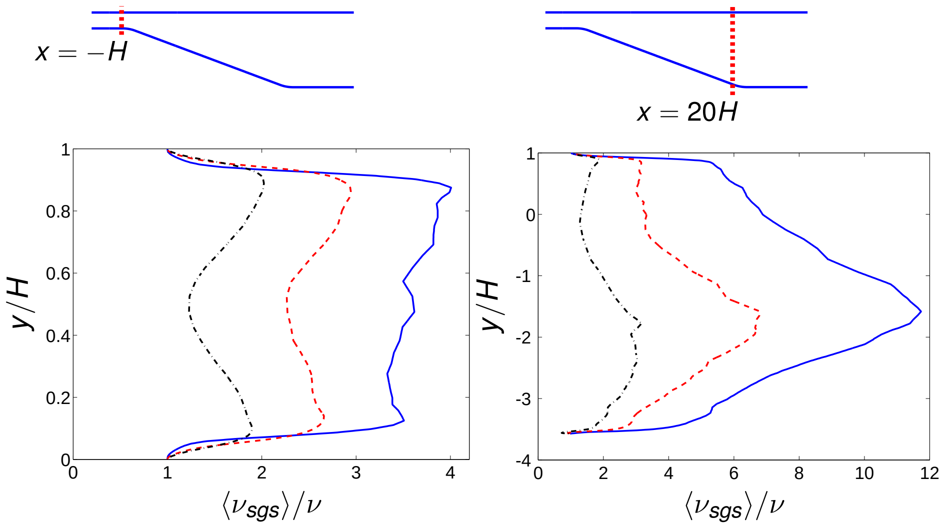


$N_z = 32$; $N_z = 64$; $N_z = 128$.

SGS vs. Resolved Shear Stresses

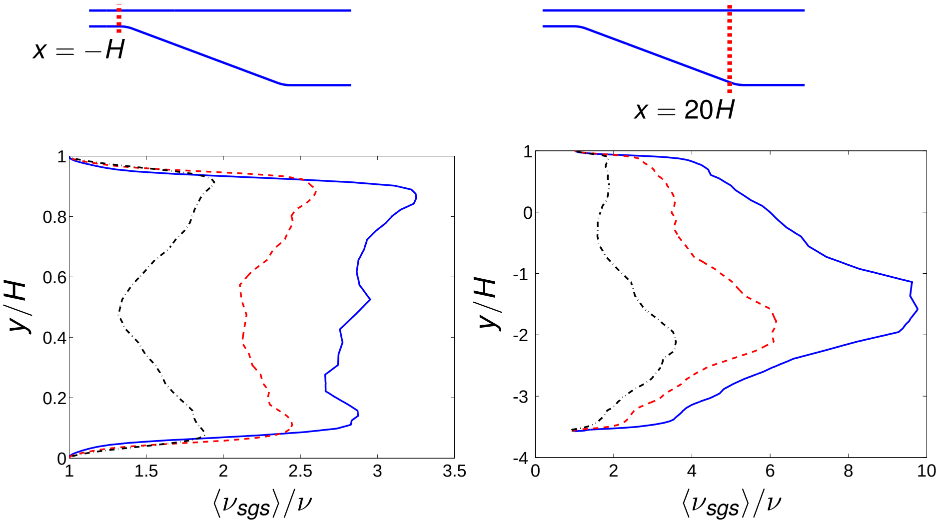


SGS vs. Molecular Viscosity



$N_z = 32$; $N_z = 64$; $N_z = 128$.

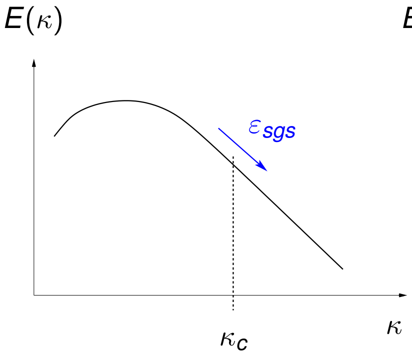
SGS vs. Molecular Viscosity, $N_x = 512$



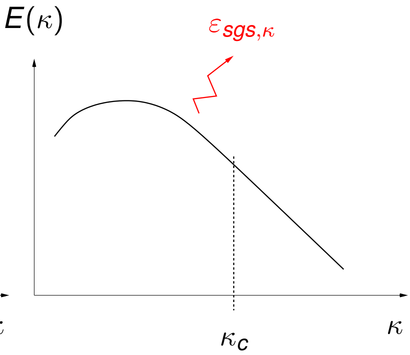
$N_z = 32$; $N_z = 64$; $N_z = 128$.

Dissipation Energy Spectra: Theory vs. Reality

Theory



Reality



$$\varepsilon_{sgs} = \int_0^{\kappa_c} \varepsilon_{sgs,\kappa}(\kappa) d\kappa$$

Approximated Dissipation Energy Spectra

- At which wavenumber is the SGS dissipation largest?
- In the homogeneous direction, z, the SGS dissipation can be analyzed in the wavenumber space
- ε_{wz}, can — in theory — be obtained from the two-point correlation [5] as

$$\varepsilon_{wz} = 2\nu \left\langle \left(\frac{\partial w'}{\partial z} \right)^2 \right\rangle = 2\nu \frac{\partial^2 B_{ww}(\hat{z})}{\partial \hat{z}^2}\bigg|_{\hat{z}=0} = 2\nu \sum_{k_z=1}^{N_z} \kappa_z^2 E_{ww}(k_z)$$

- When the equations are discretized, the left side $\neq$ the right side
- The right side gives $\varepsilon_{wz} \propto \kappa_z^2 E_{ww} = \kappa_z^2 \kappa^{-5/3} = \kappa^{1/3}$

EXACT DISSIPATION ENERGY SPECTRA

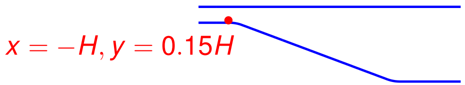
A discrete Fourier transform of $\partial w'/\partial z$ is formed as

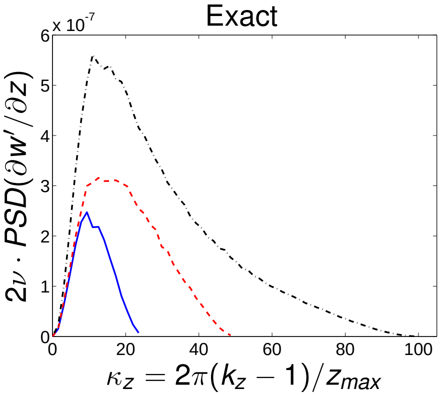
$$\hat{D}_z(k_z) = \frac{1}{N_z}\sum_{n=1}^{N_z}\frac{\partial w'(n)}{\partial z} \left[\cos\left(\frac{2\pi(n-1)(k_z-1)}{N_z}\right) - \imath \sin\left(\frac{2\pi(n-1)(k_z-1)}{N_z}\right)\right] \quad (1)$$

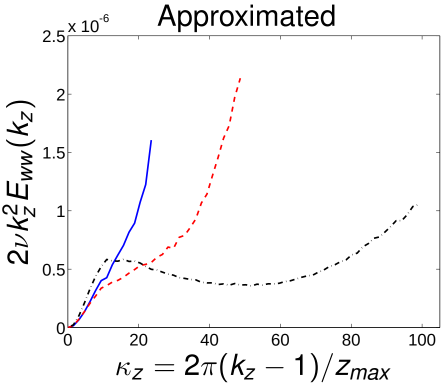
where n is node number in z direction. Power Spectral Density (PSD)

$$\left\langle\left(\frac{\partial w'}{\partial z}\right)^2\right\rangle = \sum_{k_z=1}^{N_z}\langle \hat{D}_z * \hat{D}_z^*\rangle = \sum_{k_z=1}^{N_z} PSD\left(\frac{\partial w'}{\partial z}\right)$$

Predicted Dissipation Energy Spectra



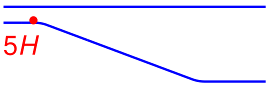


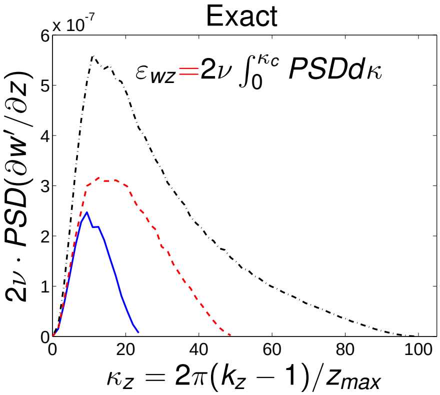


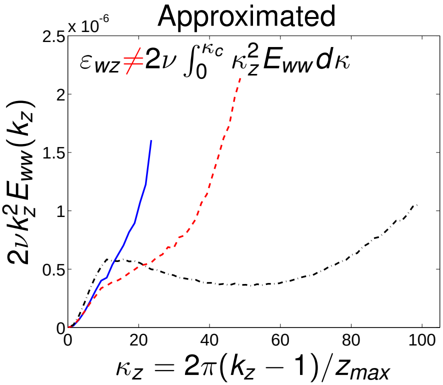
$N_z = 32$; $N_z = 64$; $N_z = 128$.

Predicted Dissipation Energy Spectra

$x = -H, y = 0.15H$

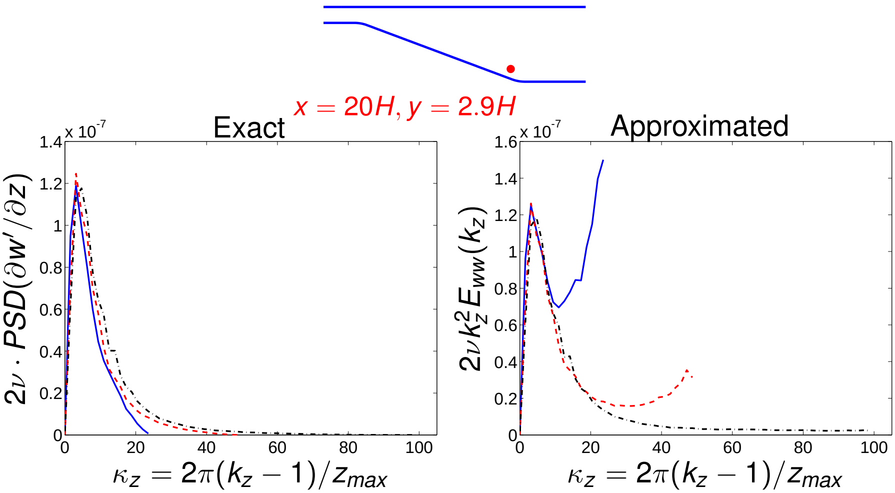






$N_z = 32$; $N_z = 64$; $N_z = 128$.

Predicted Dissipation Energy Spectra



$N_z = 32$; $N_z = 64$; $N_z = 128$.

SGS Dissipation Energy Spectra

- Above, energy spectra for $\partial w'/\partial z$ have been presented which is part of the viscous dissipation
- What about energy spectra for the SGS dissipation

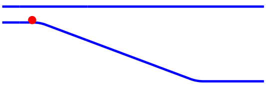
$$\varepsilon_{sgs} = \left\langle \nu_{sgs} \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_i}{\partial x_j} \right\rangle ?$$

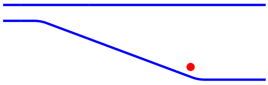
- Form a discrete Fourier transform of $\varepsilon_{sgs}^{1/2}$. Replace $\partial w'/\partial z$ in Eq. 1 on Slide 26 by $\varepsilon_{sgs}^{1/2}$.
- Strange unphysical Fourier coefficients! but the energy spectra

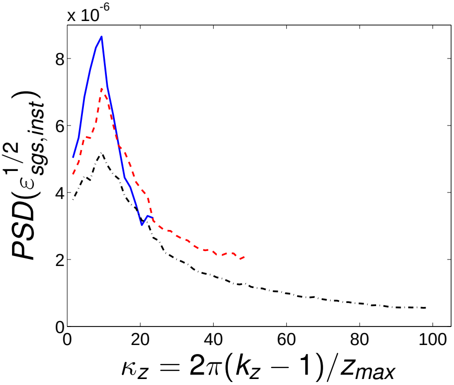
$$\varepsilon_{sgs} = \sum_{k_z=1}^{N_z} \langle \hat{D}_z * \hat{D}_z^* \rangle = \sum_{k_z=1}^{N_z} PSD\left(\varepsilon_{sgs}^{1/2}\right)$$

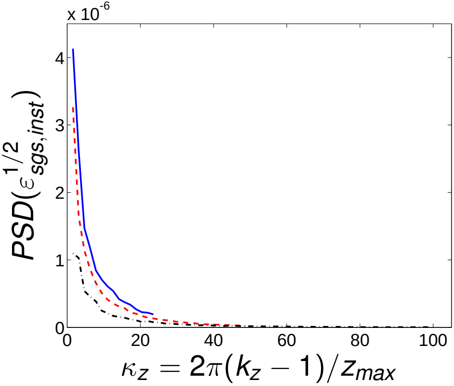
are OK

SGS Dissipation Energy Spectra



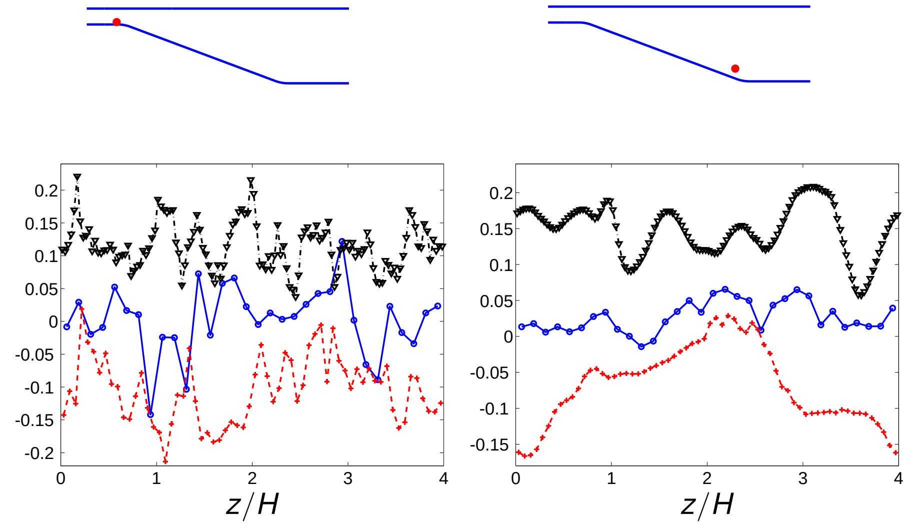






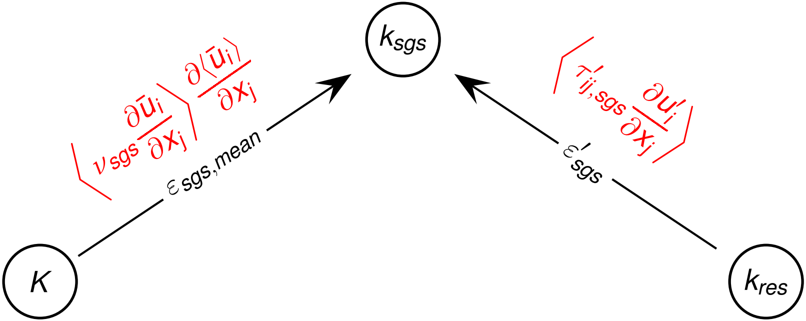
$N_z = 32$; $N_z = 64$; $N_z = 128$.

Snapshots of w' vs. z



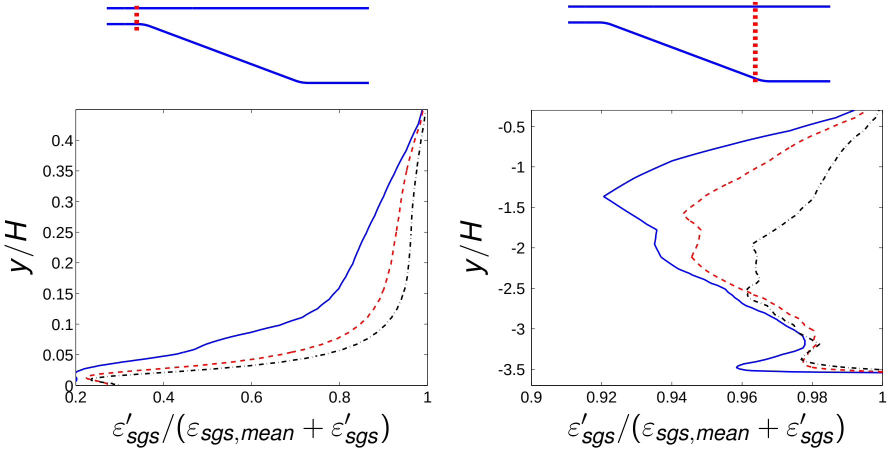
$N_z = 32$; $N_z = 64$, $w' - 0.1$; $N_z = 128$, $w' + 0.14$.

Transfer of Kinetic Turbulent Energy



- time-averaged $K = \frac{1}{2}\langle \bar{u}_i \rangle \langle \bar{u}_i \rangle$ (RANS)
- resolved $k_{res} = \frac{1}{2}\langle u'_i u'_i \rangle$ (RANS and LES)
- SGS kinetic energy, k_{sgs}.

Ratio of SGS Dissipation

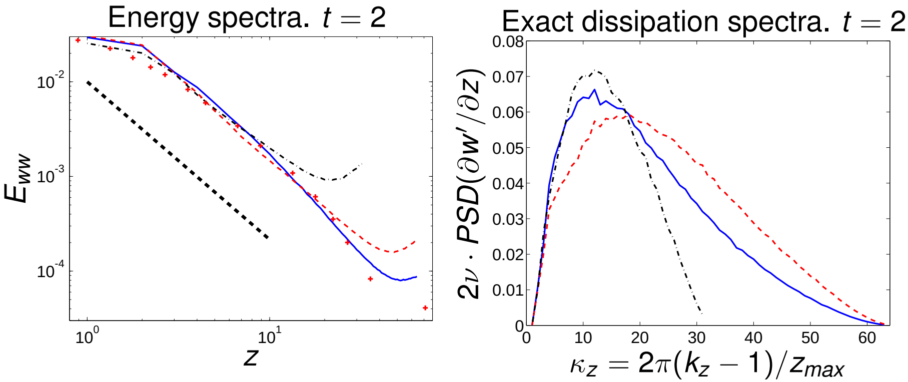


——— $N_z = 32$; - - - - $N_z = 64$; — · — $N_z = 128$.

Decaying Grid Turbulence

- Diffuser flow: peaks in $\partial w'/\partial z$ at surprisingly low wavenumbers.
- The "decaying grid turbulence" is presented below in order to find at which wavenumbers the dissipation attain its peak
- The domain is a cubic box of side 2π. Three computations have been carried out.
  1. Fine LES using a Smagorinsky model ($C_S = 0.1$) on a 128^3 grid.
  2. DNS on a 128^3 grid.
  3. Coarse LES using a Smagorinsky model ($C_S = 0.1$) on a 64^3 grid.

DECAYING GRID TURBULENCE: RESULTS



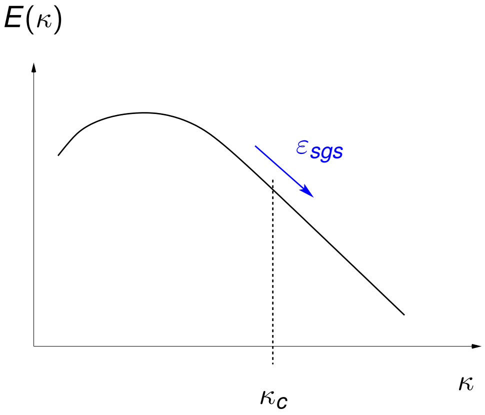
Fine LES; DNS; coarse LES; + exp. [6].

CONCLUSIONS

- Two-point correlation best. They show by how many cells the largest scales are resolved.
- The energy spectra do not give any reliable information on the resolution.
- The $\langle \nu_t/\nu \rangle$ is not a good measure. It compares LES with DNS.
- $\langle \tau_{sgs,12} \rangle / \langle u'v' \rangle$ is a good measure. However, it is difficult to give any quantitative guidelines.
- Ratio of $\varepsilon'_{sgs}/\varepsilon_{sgs,mean}$ useful but difficult to give any quantitative guidelines.

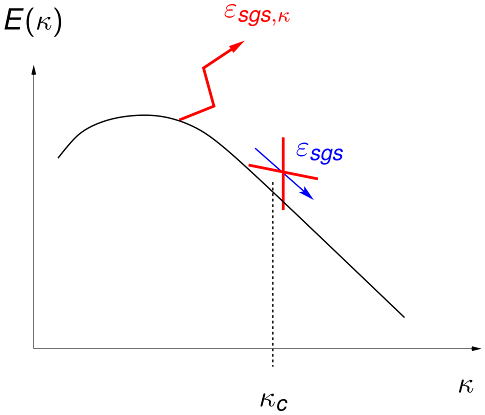
CONCLUSIONS CONT'D

• Energy spectra of the SGS dissipation show that the peak takes place at surprisingly low wavenumber (length scale corresponding to 10 cells or more).



Conclusions cont'd

• Energy spectra of the SGS dissipation show that the peak takes place at surprisingly low wavenumber (length scale corresponding to 10 cells or more).



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