

# EMBEDDED LES USING PANS [2]

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## PANS LOW REYNOLDS NUMBER MODEL [3]

$$\frac{\partial k_u}{\partial t} + \frac{\partial(k_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_u}{\sigma_{ku}} \right) \frac{\partial k_u}{\partial x_j} \right] + (P_u - \varepsilon_u)$$

$$\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial(\varepsilon_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_u}{\sigma_{\varepsilon u}} \right) \frac{\partial \varepsilon_u}{\partial x_j} \right] + C_{\varepsilon 1} P_u \frac{\varepsilon_u}{k_u} - C_{\varepsilon 2}^* \frac{\varepsilon_u^2}{k_u}$$

$$\nu_u = C_\mu f_\mu \frac{k_u^2}{\varepsilon_u}, C_{\varepsilon 2}^* = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon} (C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

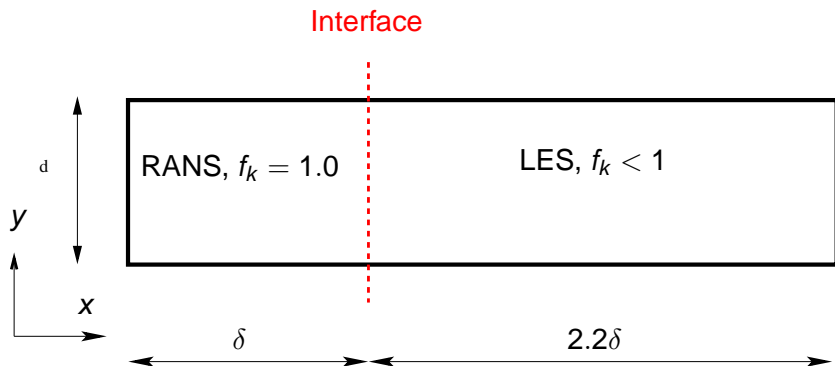
$C_{\varepsilon 1}$ ,  $C_{\varepsilon 2}$ ,  $\sigma_k$ ,  $\sigma_\varepsilon$  and  $C_\mu$  same values as [1].  $f_\varepsilon = 1$ .  $f_2$  and  $f_\mu$  read

$$f_2 = \left[ 1 - \exp\left(-\frac{y^*}{3.1}\right) \right]^2 \left\{ 1 - 0.3 \exp\left[-\left(\frac{R_t}{6.5}\right)^2\right] \right\}$$

$$f_\mu = \left[ 1 - \exp\left(-\frac{y^*}{14}\right) \right]^2 \left\{ 1 + \frac{5}{R_t^{3/4}} \exp\left[-\left(\frac{R_t}{200}\right)^2\right] \right\}$$

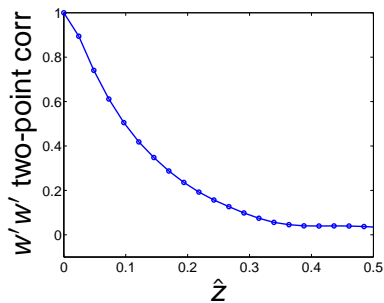
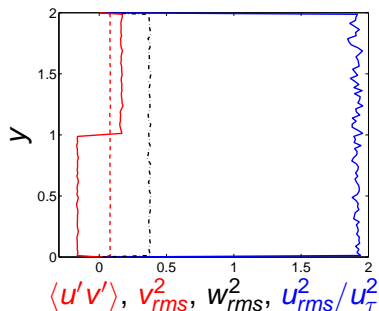
- Baseline model:  $f_k = 0.4$ . Range of  $0.2 < f_k < 0.6$  is evaluated

# CHANNEL FLOW: DOMAIN



- **Interface:** Synthetic turbulent fluctuations are introduced as additional **convective** fluxes in the **momentum** equations and the **continuity** equation
- $f_k = 0.4$  is the baseline value for LES [3]

# INLET FLUCTUATIONS



- Anisotropic synthetic fluctuations,  $u'$ ,  $v'$ ,  $w'$ ,
- Integral length scale  $\mathcal{L} \simeq 0.13$  (see 2-p point correlation)
- Asymmetric time filter  $(u')^m = a(u')^{m-1} + b(u')^m$  with  $a = 0.954$ ,  $b = (1 - a^2)^{1/2}$  gives a time integral scale  $\mathcal{T} = 0.015$  ( $\Delta t = 0.00063$ )

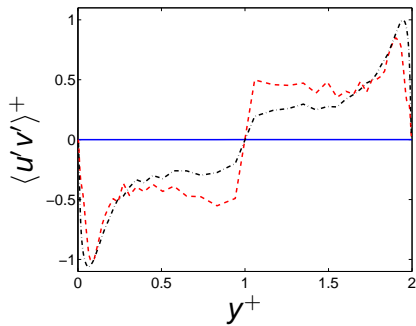
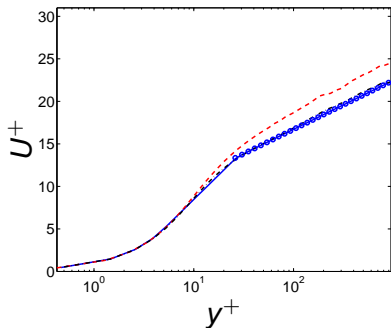
# INTERFACE CONDITIONS FOR $k_u$ AND $\varepsilon_u$

- For  $k_u$  &  $\varepsilon_u$  we prescribe “inlet” boundary conditions at the interface.
- First, the usual convective and diffusive fluxes at the interface are set to zero
- Next, new convective fluxes are added. Which “inlet” values should be used at the interface?
  - ▶  $k_{u,int} = f_k k_{RANS}(x = 0.5\delta)$ ,  $\varepsilon_{u,int} = C_\mu^{3/4} k_{u,int}^{3/2} / \ell_{sgs}$ ,  $\ell_{sgs} = C_s \Delta$ ,  
 $\Delta = V^{1/3}$
  - ▶

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 $\Delta = V^{1/3}$
  - ▶ Baseline  $C_s = 0.07$ ; different  $C_s$  values are tested

# CHANNEL FLOW: VELOCITY AND SHEAR STRESSES

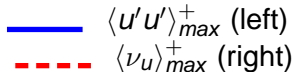
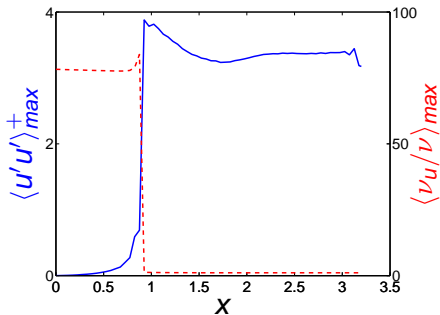
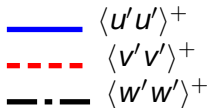
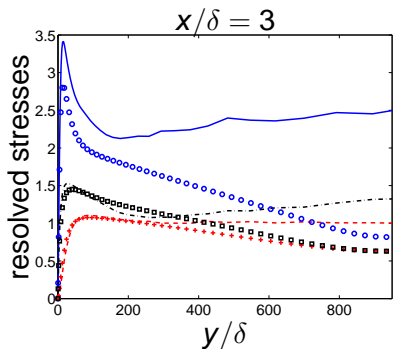


—  $x/\delta = 0.19$

- - -  $x/\delta = 1.25$

- . -  $x/\delta = 3$

# CHANNEL FLOW: STRESSES AND PEAK VALUES VS. $x$

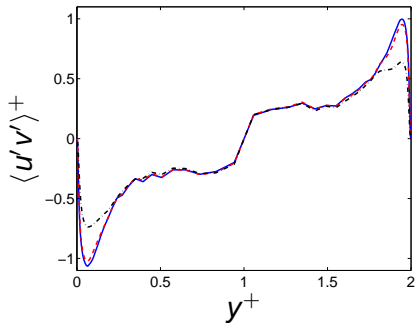
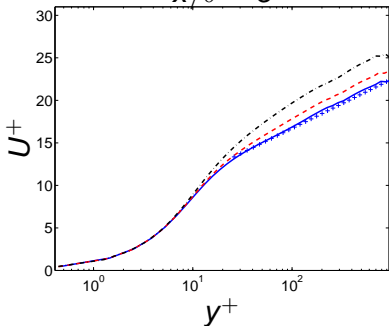




# CHANNEL FLOW: DIFFERENT $C_S$ VALUE FOR $\varepsilon_{interface}$

- $k_{u,int} = f_k k_{RANS}$
- $\varepsilon_{u,int} = C_\mu^{3/4} k_{u,int}^{3/2} / l_{sgs}$ ,  $l_{sgs} = C_S \Delta$

$x/\delta = 3$

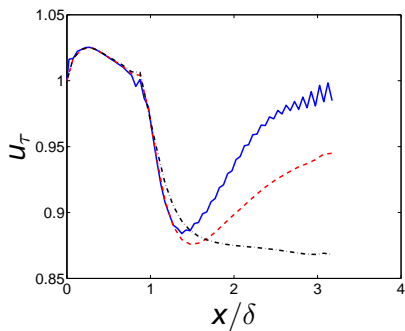
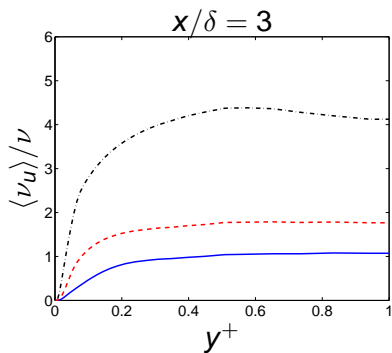


—  $C_S = 0.07$

- - -  $C_S = 0.1$

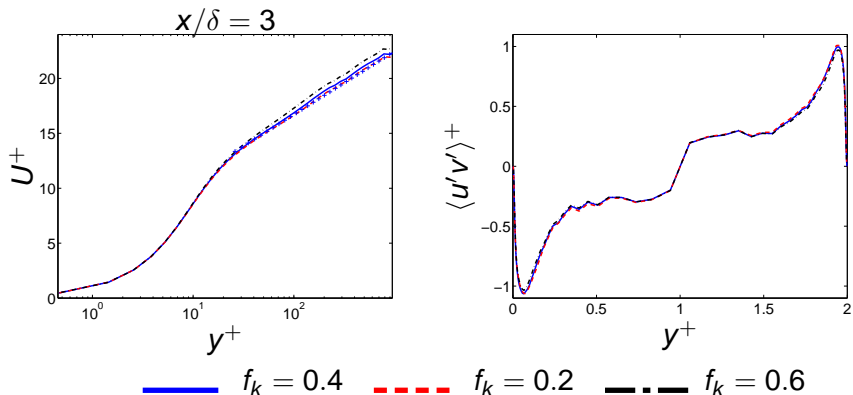
- . -  $C_S = 0.2$

# CHANNEL FLOW: DIFFERENT $C_S$ VALUE FOR $\varepsilon_{interface}$

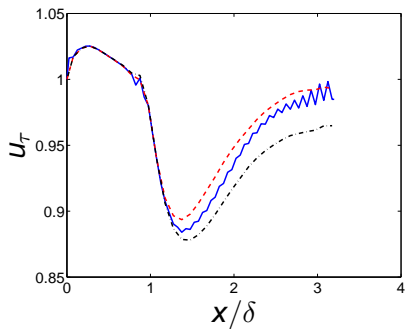
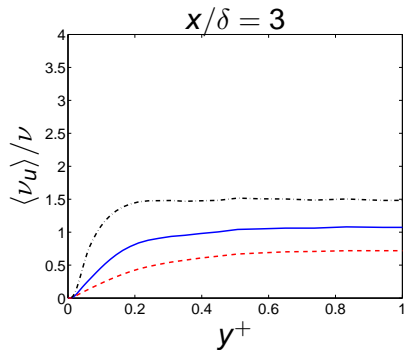


—  $C_S = 0.07$     - - -  $C_S = 0.1$     - . -  $C_S = 0.2$

# CHANNEL FLOW: DIFFERENT $f_k$ VALUES

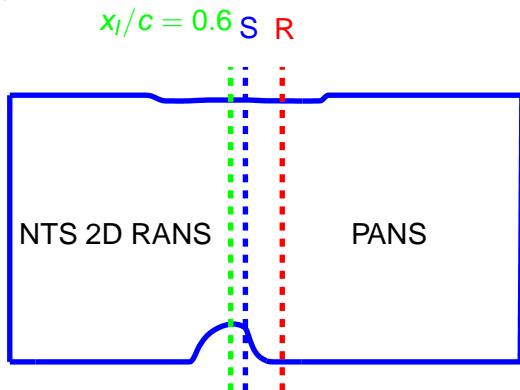


# CHANNEL FLOW: DIFFERENT $f_k$ VALUES



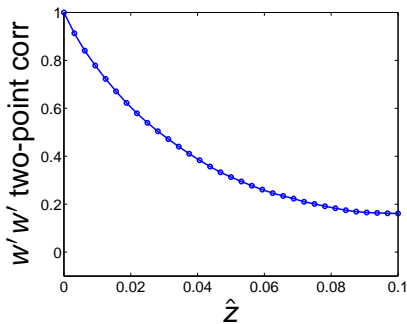
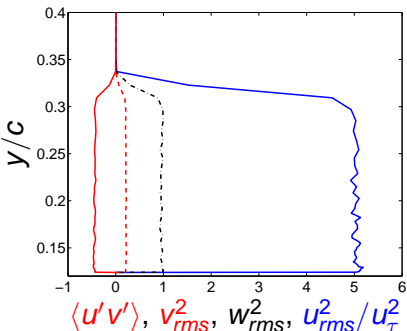
—  $f_k = 0.4$     - - -  $f_k = 0.2$     - . -  $f_k = 0.6$

# HUMP FLOW



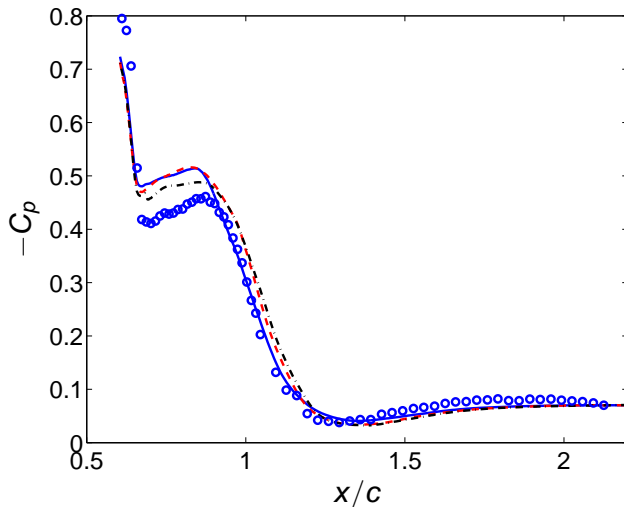
- **Inlet**, **Separation**  $x_S/c = 0.65$ ; **reattachment**  $x_R/c = 1.1$
- $Re_c = 936\,000 \frac{U_{ij}c}{\nu}$  ( $U_{in} = c = \rho = 1, \nu = 1/Re_c$ )
- $H/c = 0.91, h/c = 0.128, x/c = [0.6, 4.2]$
- Mesh:  $312 \times 120 \times 64, Z_{max} = 0.2c$  (baseline)

# BASELINE INLET FLUCTUATIONS



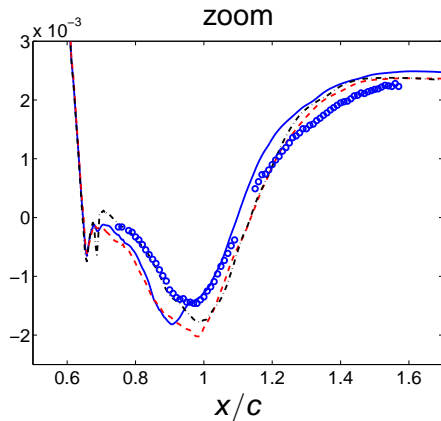
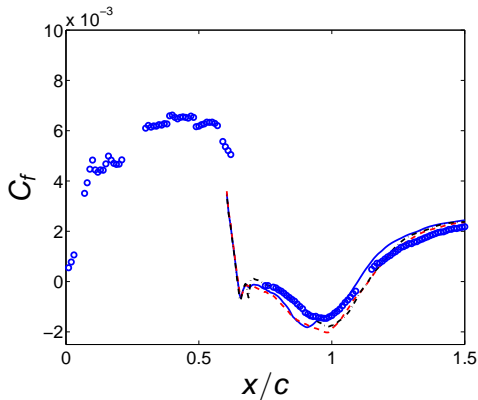
- Integral length scale  $\mathcal{L} \simeq 0.04$  (see 2-p point correlation)
- Asymmetric time filter  $(U')^m = a(U')^{m-1} + b(u')^m$  with  $a = 0.954, b = (1 - a^2)^{1/2}$  gives a time integral scale  $\mathcal{T} = 0.038$
- $\Delta t = 0.002$ . 7500 + 7500 time steps (100 hours one core)
- Fluctuations multiplied by  $f_{bl} = \max\{0.5 [1 - \tanh(y - y_{bl} - y_{wall})/b], 0.02\}$ ,  $y_{bl} = 0.2$ ,  $b = 0.01$ .

# PRESSURE: AMPLITUDES OF INLET FLUCT



— baseline inlet fluct    - - - 1.5× (baseline inlet fluct)  
- . - . 0.5× (baseline inlet fluct)

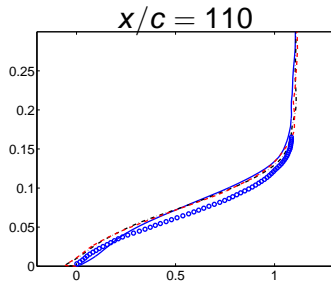
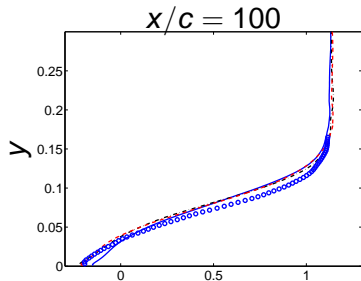
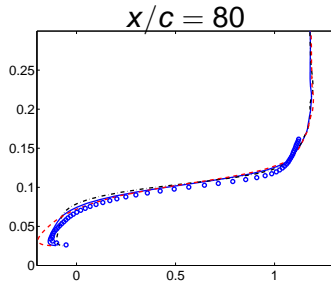
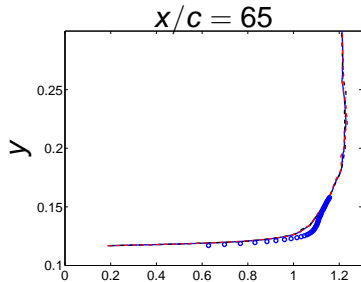
# SKIN FRICTION: AMPLITUDES OF INLET FLUCT



— baseline inlet fluct    - - - 1.5× (baseline inlet fluct)  
- - - 0.5× (baseline inlet fluct)



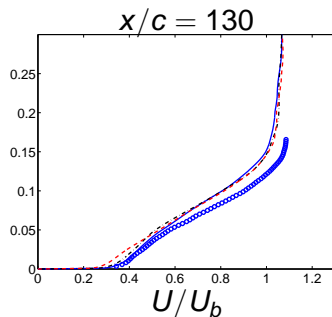
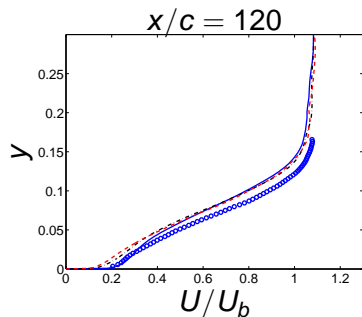
# VELOCITIES: AMPLITUDES OF INLET FLUCT



**—** baseline    **- - -**  $1.5 \times$  (baseline)    **- . -**  $0.5 \times$  (baseline)

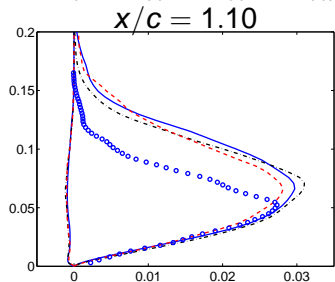
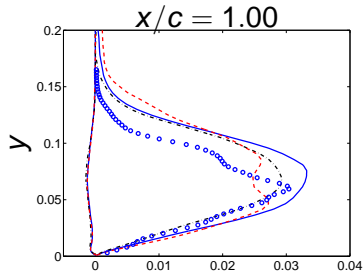
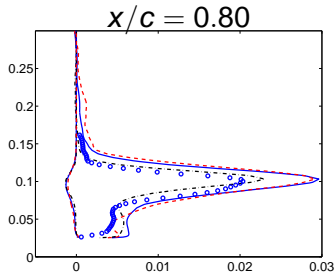
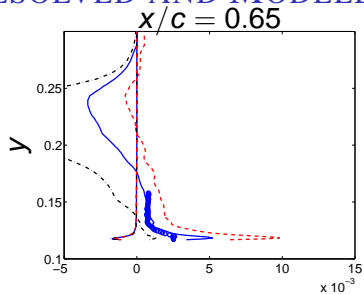
$U/U_b$      $U/U_b$

# VELOCITIES: AMPLITUDES OF INLET FLUCT



— baseline    - - - 1.5× (baseline)    - . - 0.5× (baseline)

# RESOLVED AND MODELLED ( $< 0$ ) SHEAR STRESSES



$\langle \tau_{12,u} \rangle, \langle -u'v' \rangle / U_b^2$

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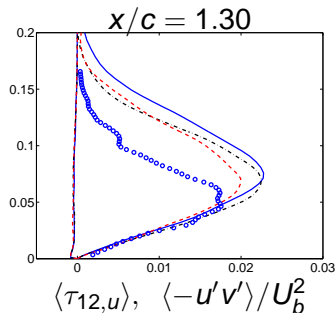
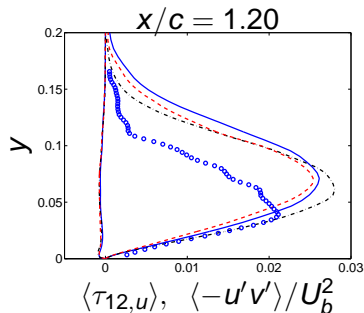
— baseline

- - - 1.5× (baseline)

- - - 0.5× (baseline)

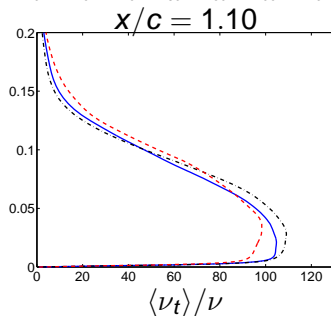
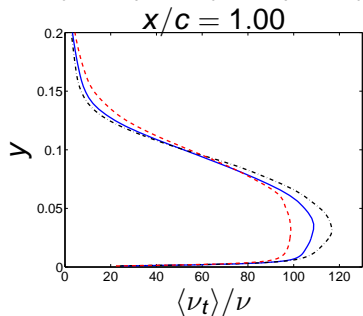
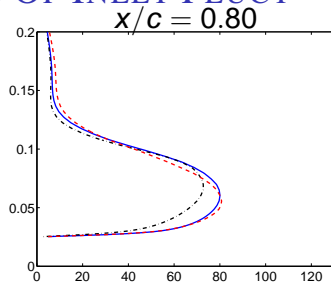
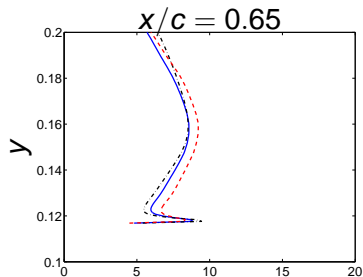
# SHEAR STRESSES: AMPLITUDES OF INLET FLUCT

- Resolved and Modelled ( $< 0$ ) Shear stresses



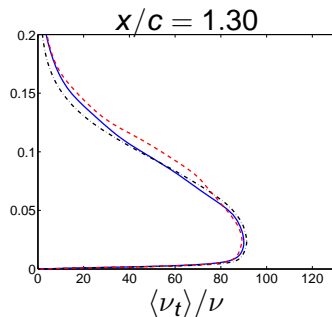
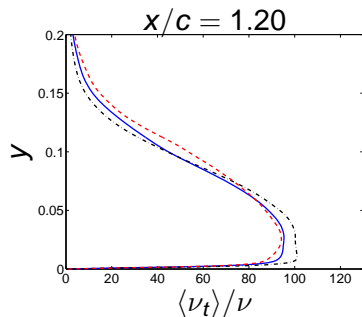
— baseline inlet fluct    - - - 1.5× (baseline inlet fluct)  
- . - 0.5× (baseline inlet fluct)

# TURB VISCOSITY: AMPLITUDES OF INLET FLUCT



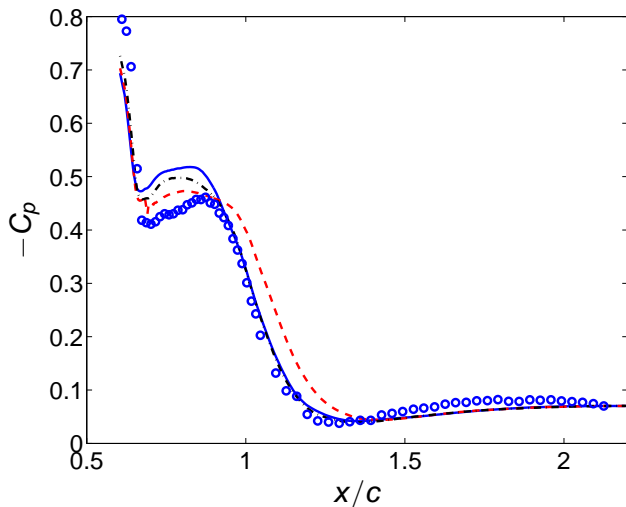
— baseline    - - - 1.5× (baseline)    - · - 0.5× (baseline)

# TURB VISCOSITY: AMPLITUDES OF INLET FLUCT



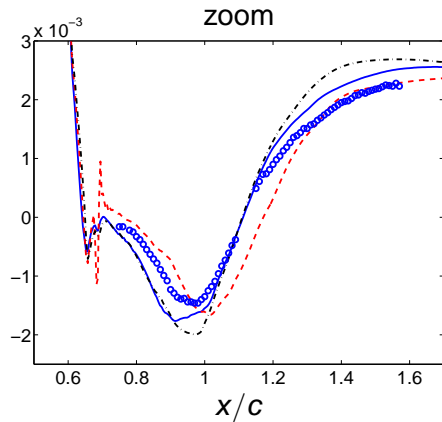
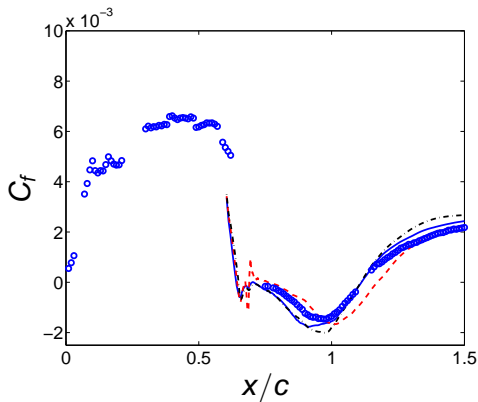
— baseline    - - - 1.5× (baseline)    - . - 0.5× (baseline)

PRESSURE:  $f_k = 0.5$ ; NO INLET FLUCT;  $N_k = 128$



—  $N_k = 128$     - - - no inlet fluct    - . -  $f_k = 0.5$

# SKIN FRICTION: $f_k = 0.5$ ; NO INLET FLUCT; $N_k = 128$

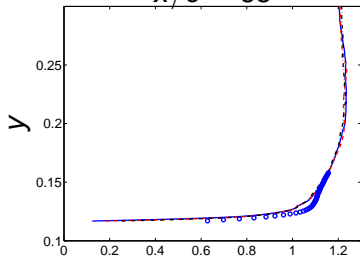


—  $N_k = 128$     - - - no inlet fluct    - . -  $f_k = 0.5$

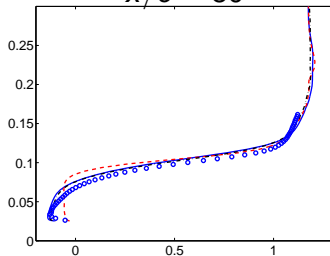


# VELOCITIES: $f_k = 0.5$ ; NO INLET FLUCT; $N_k = 128$

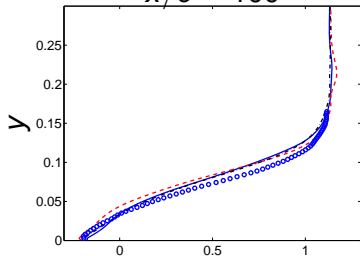
$x/c = 65$



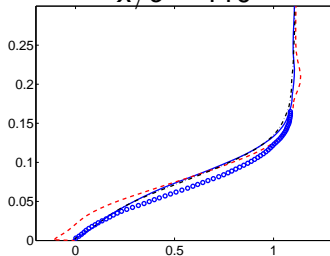
$x/c = 80$



$x/c = 100$



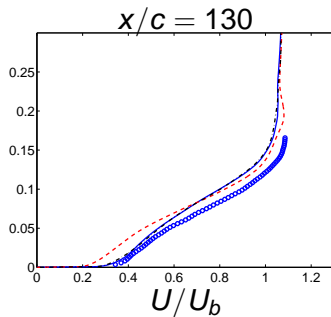
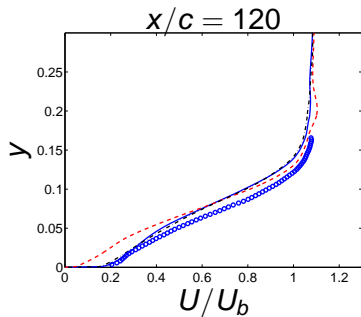
$x/c = 110$



—  $N_k = 128$ 
- - - no inlet fluct

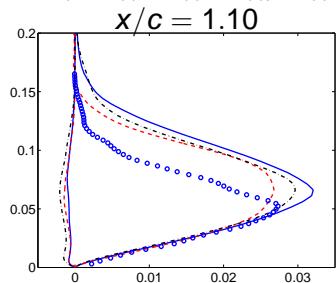
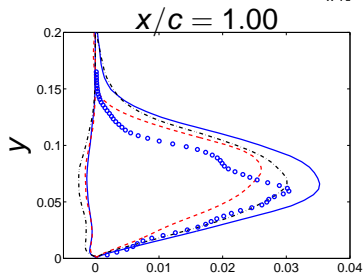
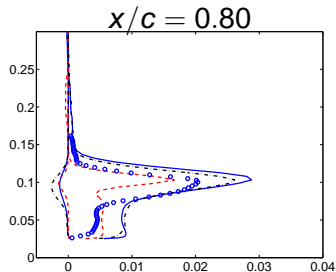
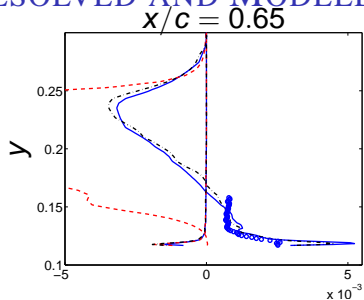
- · - ·  $f_k = 0.5$

VELOCITIES:  $f_k = 0.5$ ; NO INLET FLUCT;  $N_k = 128$



—  $N_k = 128$     - - - no inlet fluct    - . -  $f_k = 0.5$

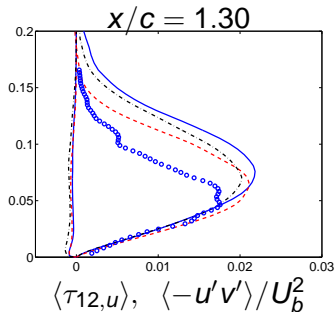
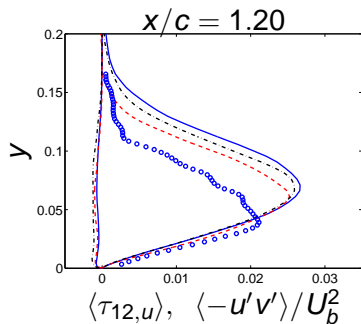
# RESOLVED AND MODELLED ( $< 0$ ) SHEAR STRESSES



$\langle \tau_{12,u} \rangle$ ,  $\langle -u'v' \rangle / U_b^2$   
—●—  $N_k = 128$     - - - no inlet fluct    - · - · -  $f_k = 0.5$

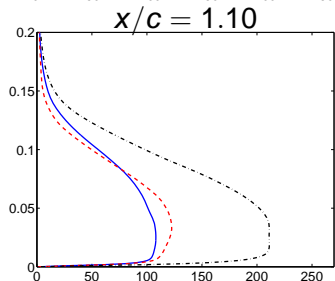
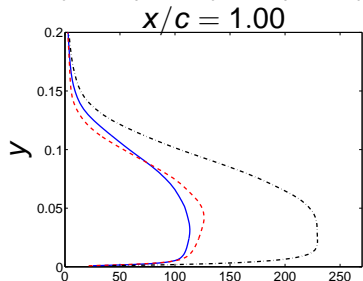
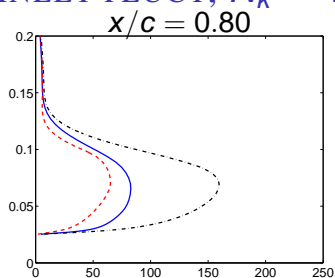
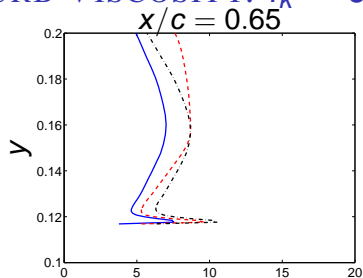
# SHEAR STRESSES: $f_k = 0.5$ ; NO INLET FLUCT; $N_k = 128$

- Resolved and Modelled ( $< 0$ ) Shear stresses



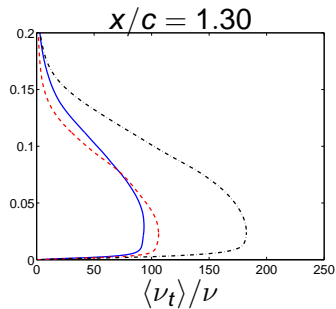
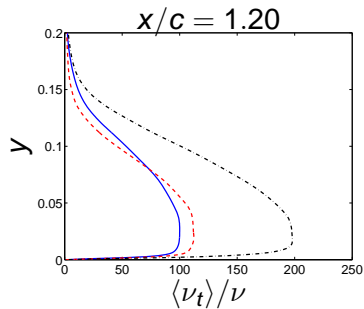
—  $N_k = 128$     - - - no inlet fluct    - . -  $f_k = 0.5$

# TURB VISCOSITY: $f_k = 0.5$ ; NO INLET FLUCT; $N_k = 128$



—  $N_k = 128$    
 - - - no inlet fluct   
 - · - ·  $f_k = 0.5$

# TURB VISCOSITY: $f_k = 0.5$ ; NO INLET FLUCT; $N_k = 128$



—  $N_k = 128$     - - - no inlet fluct    - . -  $f_k = 0.5$

## CONCLUDING REMARKS

- LRN PANS has been shown to **work well** as an embedded LES method
- Channel flow: At **two  $\delta$**  downstream the interface, the **resolved turbulence** in good agreement with DNS data and the wall friction velocity has reached **99%** of its fully developed value.
- Channel flow: The treatment of the modelled  **$k_u$  and  $\varepsilon_u$**  across the interface is important.
- LRN PANS predicts the hump flow **well** but the **recover rate** slightly too slow
- Hump flow: large **(small)** inlet fluctuations gives a smaller **(larger)** recirculation

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