LARGE EDDY SIMULATION (LES)

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THREE-DAY CFD COURSE AT CHALMERS

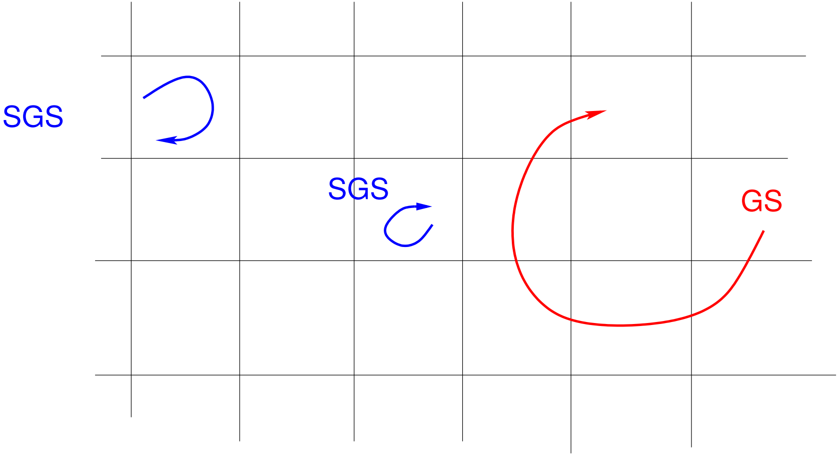
▶ This lecture is a condensed version of the course

- Unsteady Simulations for Industrial Flows: LES, DES, hybrid LES-RANS and URANS
- 5-7 November 2012 at Chalmers, Gothenburg, Sweden
- Max 16 participants
- 50% lectures and 50% workshops in front of a PC
- For info, see http://www.tfd.chalmers.se/˜lada/cfdkurs/cfdkurs.html

LECTURE NOTES

- The slides are partly based on the course material at (click here) `http://www.tfd.chalmers.se/~lada/comp_turb_model/lecture_notes.html`
- This course is part of the MSc programme **Applied Mechanics** at Chalmers. For Fluid courses, click here `http://www.tfd.chalmers.se/~lada/msc/msc-programme.html`
- The MSc programme is presented here `http://www.chalmers.se/en/education/programmes/mast`

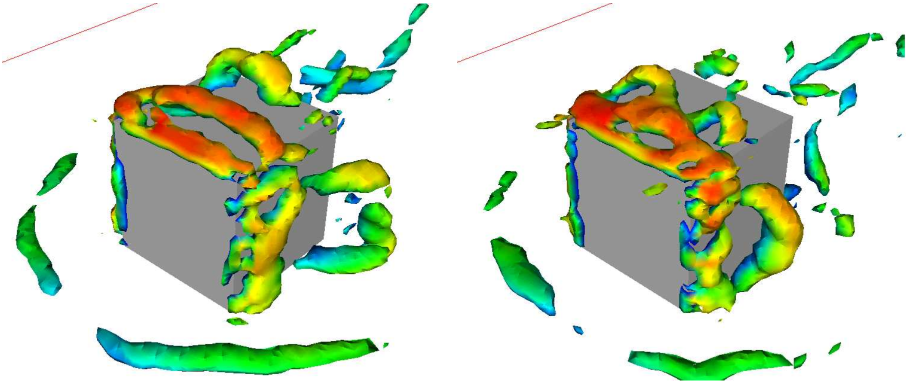
LARGE EDDY SIMULATIONS



- In LES, large (Grid) Scales (GS) are resolved and the small (Sub-Grid) Scales (SGS) are modelled.
- LES is suitable for bluff body flows where the flow is governed by large turbulent scales

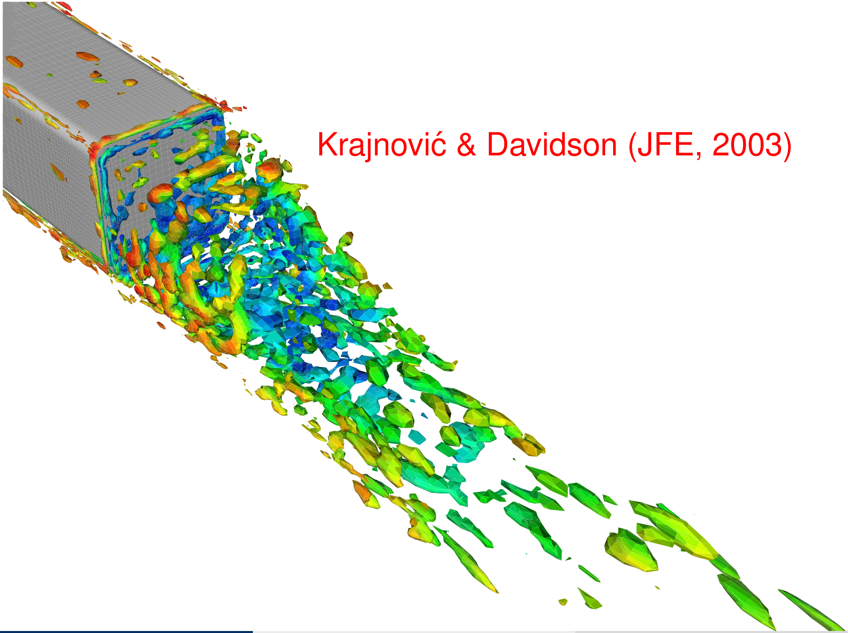
BLUFF-BODY FLOW: SURFACE-MOUNTED CUBE[14]

Krajnović & Davidson (AIAA J., 2002)



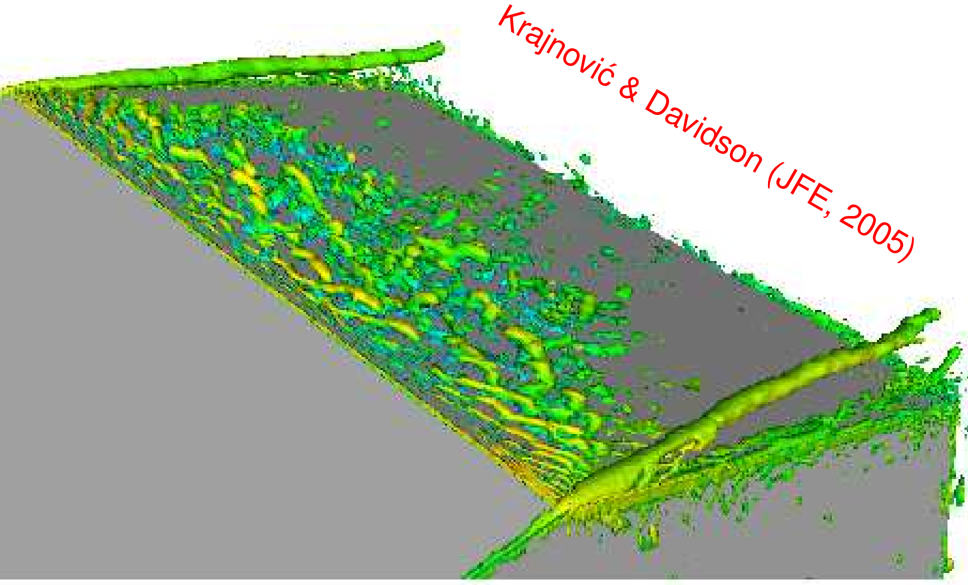
Snapshots of large turbulent scales illustrated by $Q = -\frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_j}{\partial x_i}$

BLUFF-BODY FLOW: FLOW AROUND A BUS[15]



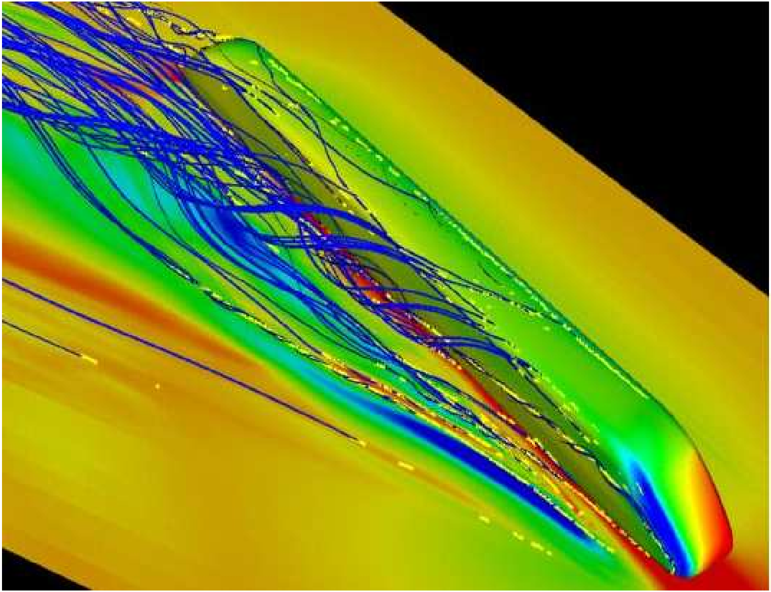
Krajnović & Davidson (JFE, 2003)

BLUFF-BODY FLOW: FLOW AROUND A CAR[16]



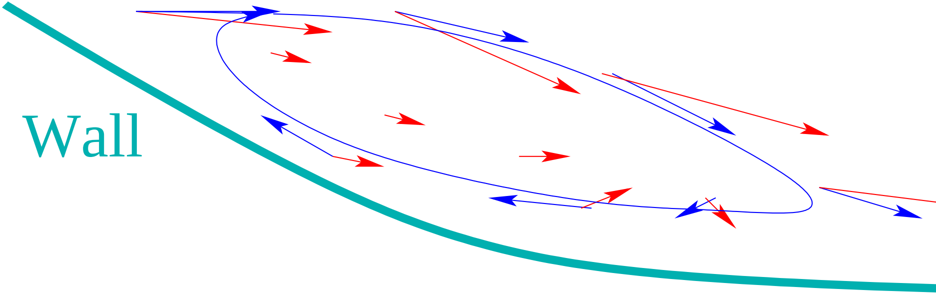
Krajnović & Davidson (JFE, 2005)

BLUFF-BODY FLOW: FLOW AROUND A TRAIN[12]



Hemida & Krajnović, 2006

SEPARATING FLOWS



- TIME-AVERAGED flow and INSTANTANEOUS flow

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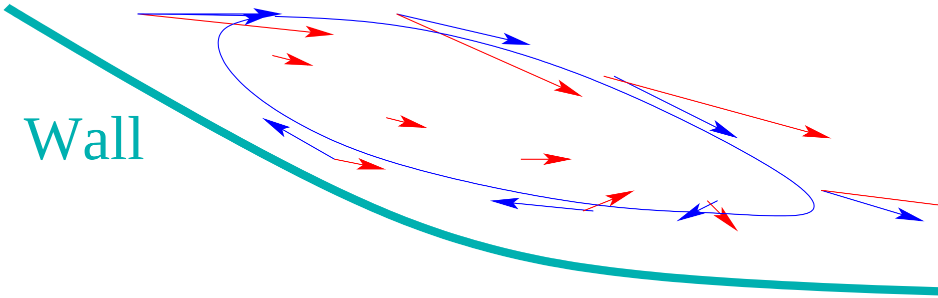
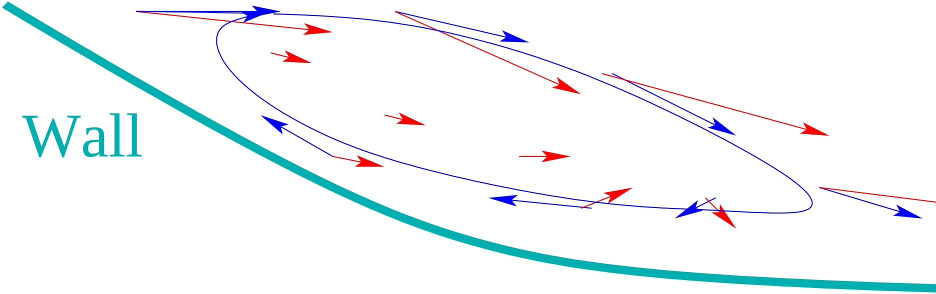


- TIME-AVERAGED flow and INSTANTANEOUS flow
- In average there is backflow (negative velocities). Instantaneous, the negative velocities are often positive.

Separating Flows



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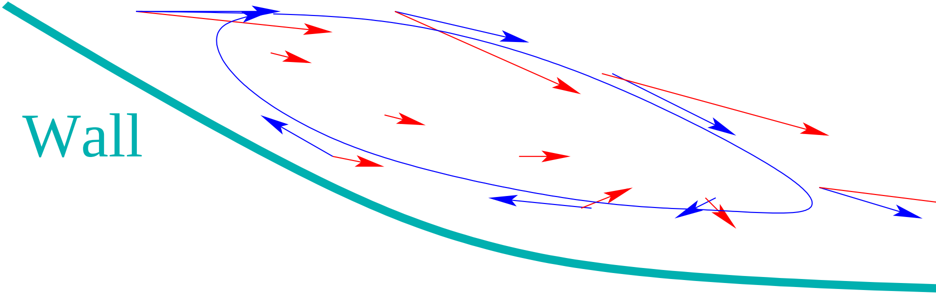

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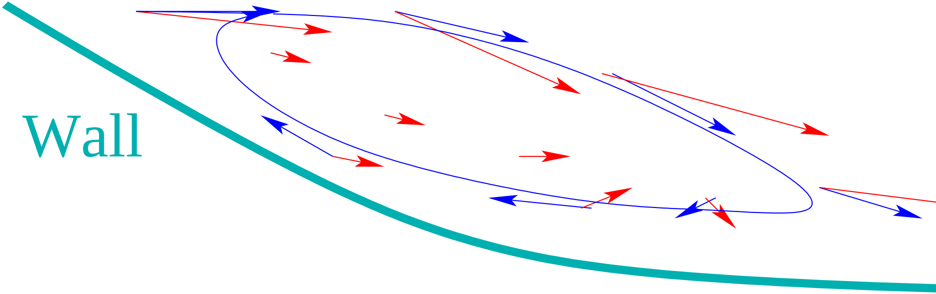

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- In average there is backflow (negative velocities). Instantaneous, the negative velocities are often positive.
- How easy is it to model fluctuations that are as large as the mean flow?
- Is it reasonable to require a turbulence model to fix this?
- Isn't it better to RESOLVE the large fluctuations?

TIME AVERAGING AND FILTERING

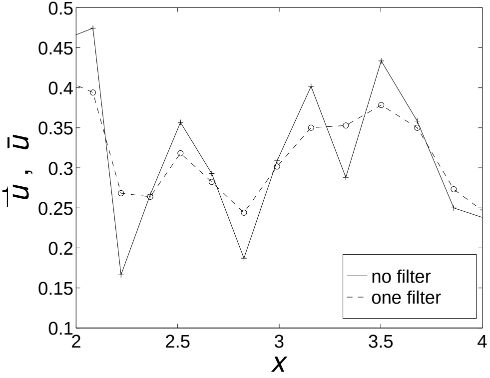
RANS: time average. This is called Reynolds time averaging:

$$\langle \Phi \rangle = \frac{1}{2T}\int_{-T}^{T} \Phi(t)dt, \; \Phi = \langle \Phi \rangle + \Phi'$$

In LES we filter (volume average) the equations. In 1D we get:

$$\bar{\Phi}(x,t) = \frac{1}{\Delta x}\int_{x-0.5\Delta x}^{x+0.5\Delta x} \Phi(\xi,t)d\xi \quad (1$$

$$\Phi = \bar{\Phi} + \Phi''$$



EQUATIONS

- The filtering is defined by the discretization (nothing is done)
- The filtered Navier-Stokes (N-S) eqns, i.e. the LES eqns, read

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_i \bar{u}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \quad \frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{2}$$

where the subgrid stresses are given by

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$

Contrary to Reynolds time averaging where $\langle u_i' \rangle = 0$, we have here

$$\overline{u_i''} \neq 0 \quad \bar{\bar{u}}_i \neq \bar{u}_i$$

FILTERING: HOW IS EQ. 2 OBTAINED?

- The N-S eqns are filtered (=discretized) using Eq. 1
- The pressure gradient term, for example, reads

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV$$

- Now we want to move the derivative out of the integral. It is allowed if V is constant.
- The filtering volume, V=grid cell which is not constant
- Fortunately, the error is proportional to V^2, i.e. it is 2nd-order error

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left(V^2\right) = \frac{\partial}{\partial x_i}(\bar{p}) + \mathcal{O}\left(V^2\right)$$

All linear terms are treated in the same way.

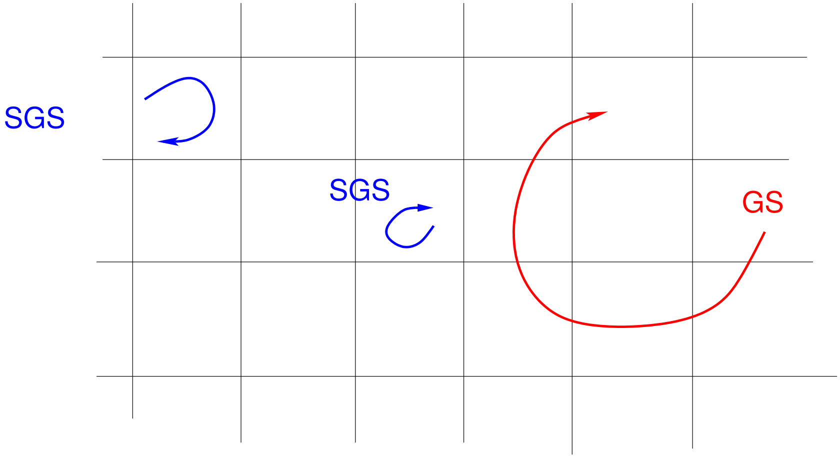
NON-LINEAR TERM

- First we filter the term and move the derivative out of the integral

$$\overline{\frac{\partial u_i u_j}{\partial x_j}} = \frac{\partial}{\partial x_j}\left(\frac{1}{V}\int_V u_i u_j dV\right) + \mathcal{O}\left(V^2\right) = \frac{\partial}{\partial x_j}(\overline{u_i u_j}) + \mathcal{O}\left(V^2\right)$$

- We have $\frac{\partial}{\partial x_j}\overline{u_i u_j}$; we want $\frac{\partial}{\partial x_j}\bar{u}_i\bar{u}_j$
- Let's add want we want (on both LHS ans RHS) and subtract want we don't want
- This is how we end up with the convective term and the SGS term in Eq. 2, i.e. $-\frac{\partial \tau_{ij}}{\partial x_j} = -\frac{\partial}{\partial x_j}(\overline{u_i u_j} - \bar{u}_i\bar{u}_j)$

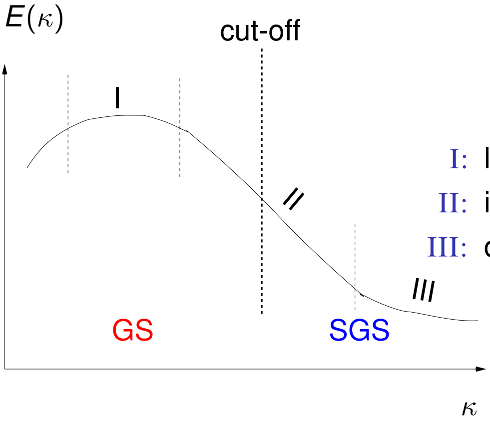
LARGE EDDY SIMULATIONS



- Large scales (GS) are resolved; small scales (SGS) are modelled.

ENERGY SPECTRUM

The limit (cut-off) between GS and SGS is supposed to take place in the inertial subrange (II)



I: large scales

II: inertial subrange, $-5/3$-range

III: dissipation subrange

SUBGRID MODEL

- We need a subgrid model for the SGS turbulent scales
- The simplest model is the Smagorinsky model [23]:

$$\begin{aligned}\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} &= -2\nu_{sgs}\bar{s}_{ij}\\ \nu_{sgs} &= (C_S\Delta)^2\sqrt{2\bar{s}_{ij}\bar{s}_{ij}} \equiv (C_S\Delta)^2\left|\bar{s}\right|\\ \bar{s}_{ij} &= \frac{1}{2}\left(\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial\bar{u}_j}{\partial x_i}\right), \quad \Delta = (\Delta V_{IJK})^{1/3}\end{aligned} \tag{3}$$

- A damping function f_μ is added to ensure that $\nu_{sgs} \Rightarrow 0$ as $y \Rightarrow 0$

$$f_\mu = 1 - \exp(-y^+/26)$$

- A more convenient way to dampen the SGS viscosity near the wall is

$$\Delta = \min\left\{(\Delta V_{IJK})^{1/3}, \kappa y\right\}$$

where y is the distance to the nearest wall.

SMAGORINSKY MODEL VS. MIXING-LENGTH MODEL

- The eddy viscosity in the mixing length model reads in boundary-layer flow [13, 22]

$$\nu_t = \ell^2 \left| \frac{\partial U}{\partial y} \right| .$$

- Generalized to three dimensions, we have

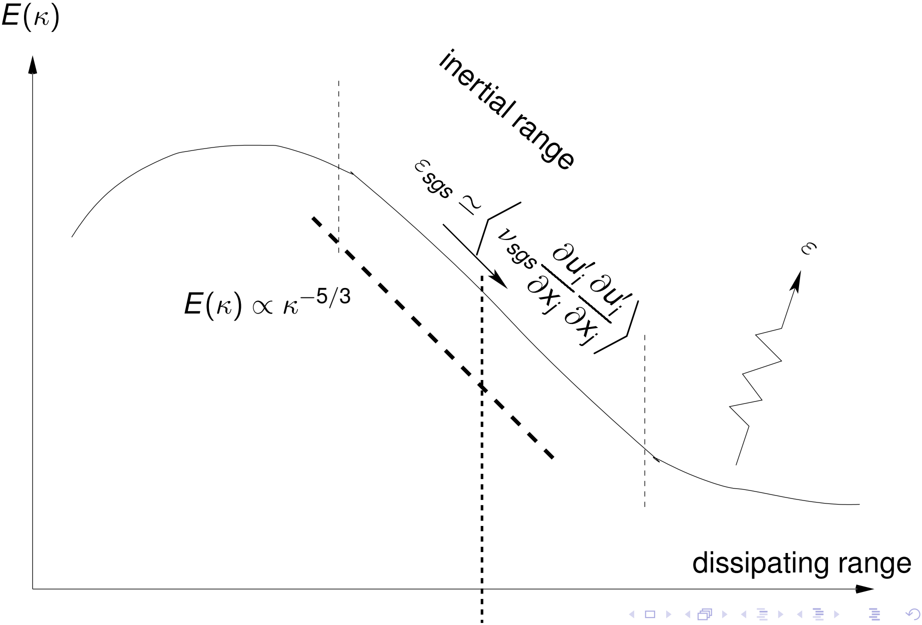
$$\nu_t = \ell^2 \left[\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} \right]^{1/2} = \ell^2 \left(2 S_{ij} S_{ij} \right)^{1/2} \equiv \ell^2 |S|.$$

- In the Smagorinsky model the SGS length scale $\ell = C_S \Delta$ i.e.

$$\nu_{sgs} = (C_S \Delta)^2 |\bar{s}|$$

which is the same as Eq. 3

ENERGY PATH


$E(\kappa)$
inertial range
$\varepsilon_{sgs} \simeq \left\langle \nu_{sgs} \frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j} \right\rangle$
$E(\kappa) \propto \kappa^{-5/3}$
ε
dissipating range

LES vs. RANS

LES can handle many flows which RANS (Reynolds Averaged Navier Stokes) cannot; the reason is that in LES large, turbulent scales are resolved. Examples are:

- *o* Flows with large separation
- *o* Bluff-body flows (e.g. flow around a car); the wake often includes large, unsteady, turbulent structures
- *o* Transition

- In RANS all turbulent scales are modelled ⇒ inaccurate

- In LES only small, isotropic turbulent scales are modelled ⇒ accurate

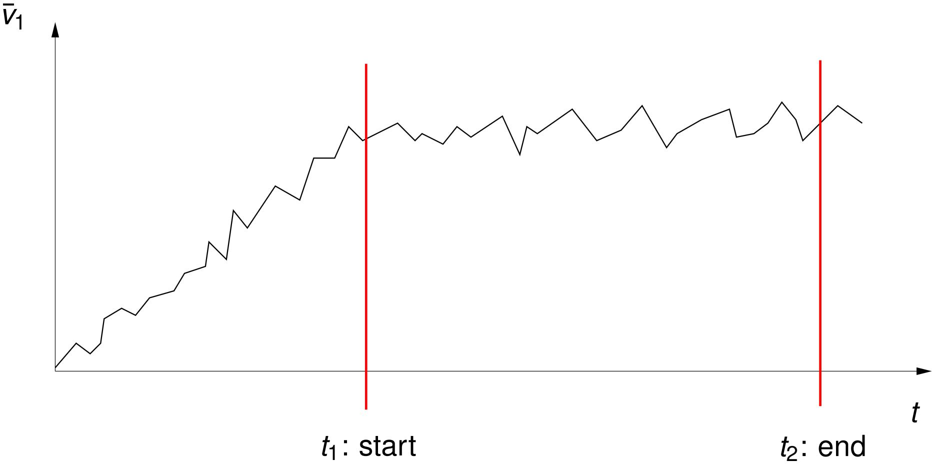
LES is *very* much more expensive than RANS.

FINITE VOLUME RANS AND LES CODES.

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	central differencing
Time discretization	1st order	2nd order (e.g. C-N)
Turbulence model	$\geq$ two-equations	zero- or one-eq

TIME AVERAGING IN LES

- t_1: Start time averaging
- t_2: Stop time averaging



NEAR-WALL RESOLUTION

- Biggest problem with LES: near walls, it requires very fine mesh in all directions, not only in the near-wall direction.

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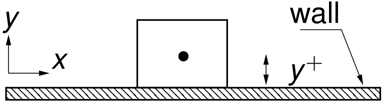
- In the presentation we use Hybrid LES-RANS for which the grid requirements are much smaller than for LES

NEAR-WALL RESOLUTION CONT'D

- In RANS when using wall-functions, $30 < y^+ < 100$ for the wall-adjacent cells



NEAR-WALL RESOLUTION CONT'D

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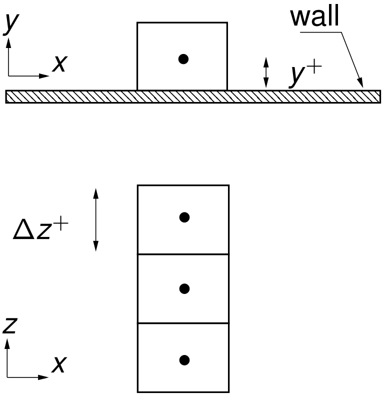
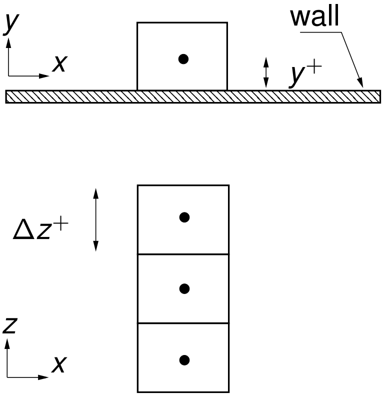


NEAR-WALL RESOLUTION CONT'D

- In RANS when using wall-functions, $30 < y^+ < 100$ for the wall-adjacent cells
- In LES, $\Delta z^+ \simeq 30$ EVERYWHERE



NEAR-WALL RESOLUTION CONT'D

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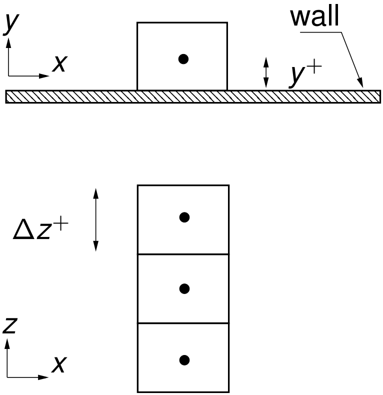
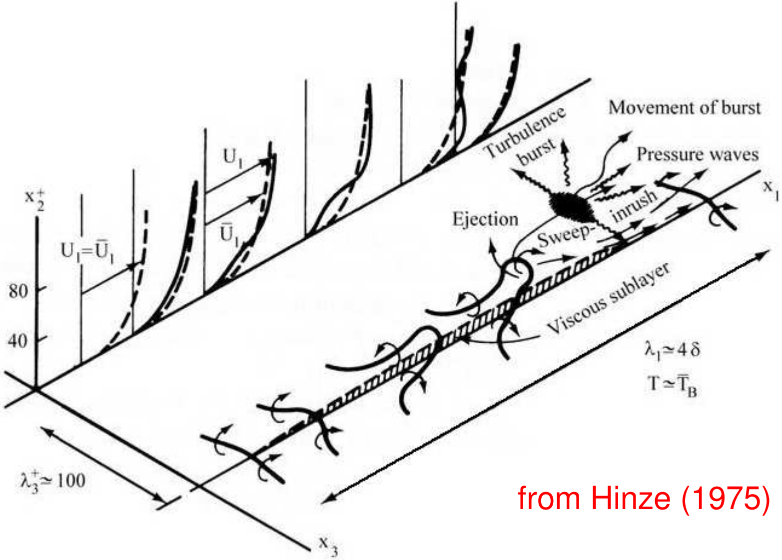
- AND $\Delta x^+ \simeq 100$, $\Delta y^+_{min} \simeq 1$



NEAR-WALL TREATMENT



from Hinze (1975)

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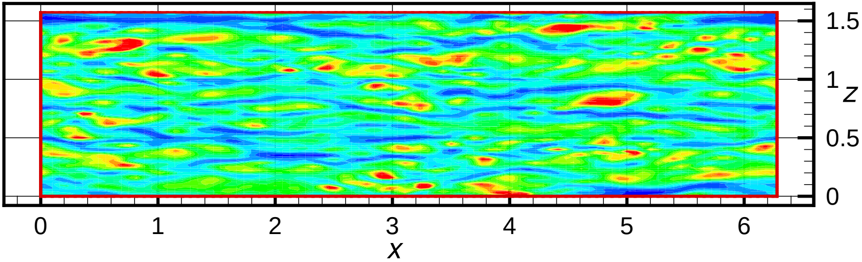

- Fluctuating streamwise velocity at $y^+ = 5$. DNS of channel flow.
- We find that the structures in the spanwise direction are very small which requires a very fine mesh in z direction.

ZONAL PANS MODEL

L. Davidson
A New Approach of Zonal Hybrid RANS-LES Based on a Two-equation $k-\varepsilon$ Model [7]
ETMM9, Thessaloniki, 7-9 June 2012

Financed by the EU project
ATAAC (Advanced Turbulence Simulation for Aerodynamic Application Challenges)

DLR, Airbus UK, Alenia, ANSYS, Beijing Tsinghua University, CFS Engineering, Chalmers, Dassault Aviation, EADS, Eurocopter Deutschland, FOI, Imperial College, IMFT, LFK, NLR, NTS, Numeca, ONERA, Rolls-Royce Deutschland, TU Berlin, TU Darmstadt, UniMAN

PANS LOW REYNOLDS NUMBER MODEL [17]

$$\frac{\partial k}{\partial t}+\frac{\partial(kU_j)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_{ku}}\right)\frac{\partial k}{\partial x_j}\right]+(P_k-\varepsilon)$$

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$C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_k, σ_ε and C_μ same values as [1]. $f_\varepsilon = 1$. f_2 and f_μ read

$$f_2=\left[1-\exp\left(-\frac{y^*}{3.1}\right)\right]^2\left\{1-0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}$$

$$f_\mu=\left[1-\exp\left(-\frac{y^*}{14}\right)\right]^2\left\{1+\frac{5}{R_t^{3/4}}\exp\left[-\left(\frac{R_t}{200}\right)^2\right]\right\}$$

- Baseline model: $f_k = 0.4$. Range of $0.2 < f_k < 0.6$ is evaluated

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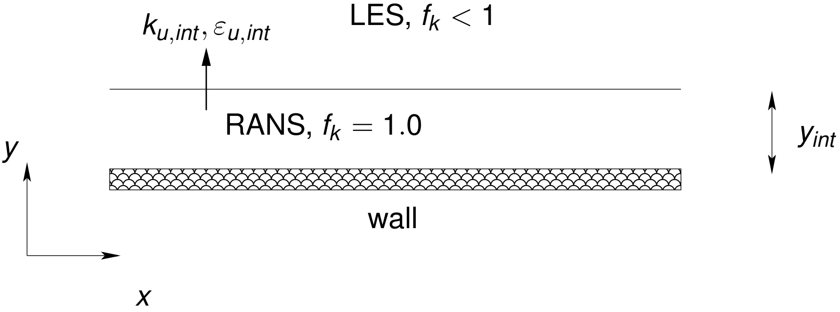
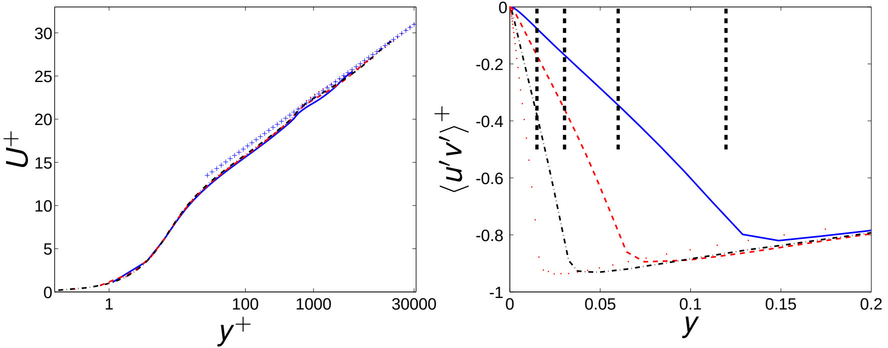
CHANNEL FLOW: ZONAL RANS-LES



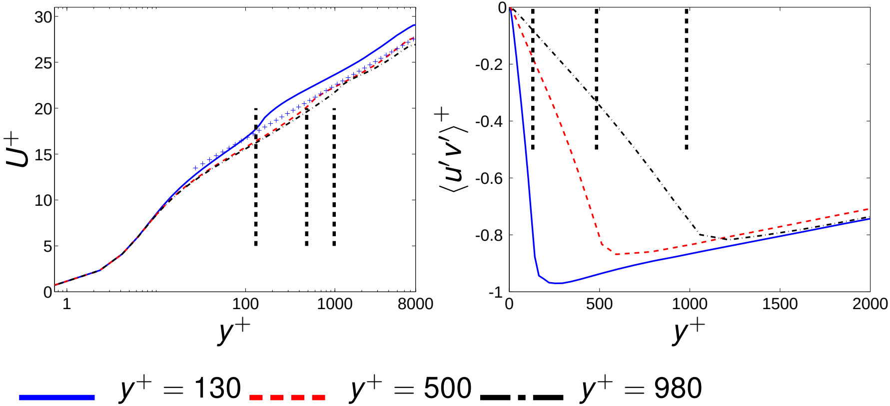
- Interface: how to treat k and ε over the interface? They should be reduced from their RANS values to suitable LES values
- The usual convection and diffusion across the interface is cut off, and new “interface boundary” conditions are prescribed
- $k_{u,int} = f_k k_{RANS}$
- Nothing is done for ε
- $x_{max} = 3.2$ (64 cells), $z_{max} = 1.6$ (64 cells), y dir: $80 - 128$ cells
- CDS in entire region

$(N_x \times N_z) = (64 \times 64).\ y^+_{int} = 500$

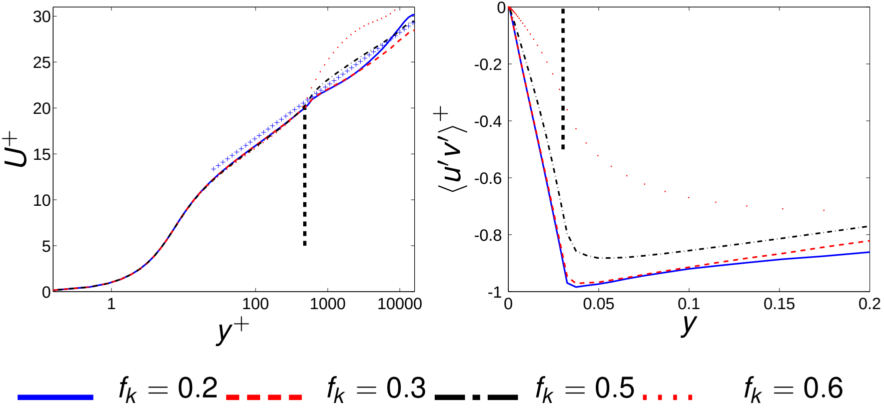


—— $Re_\tau = 4\,000$ - - - - $Re_\tau = 8\,000$ — · — $Re_\tau = 16\,000$;
···· $Re_\tau = 32\,000$.

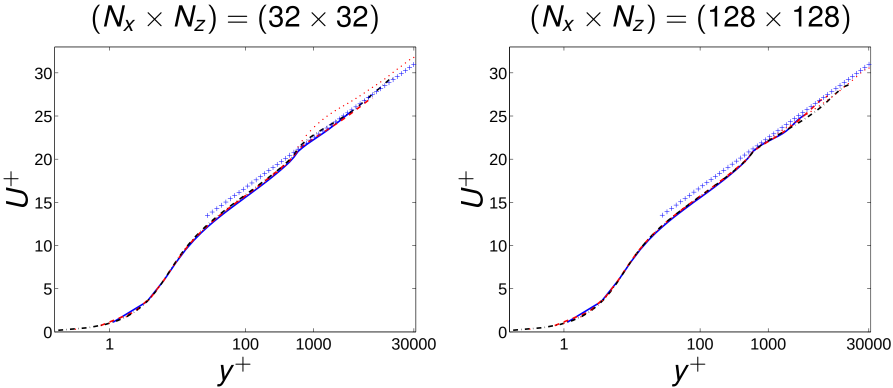
INTERFACE LOCATION. $Re_\tau = 8\,000$.



EFFECT OF f_k. $Re_\tau = 16\,000$. $y^+_{int} = 500$



EFFECT OF RESOLUTION: VELOCITY

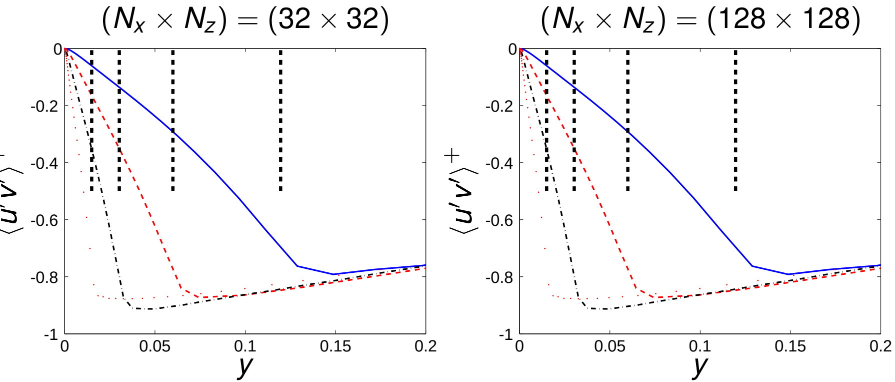


$(N_x \times N_z) = (32 \times 32)$

$(N_x \times N_z) = (128 \times 128)$

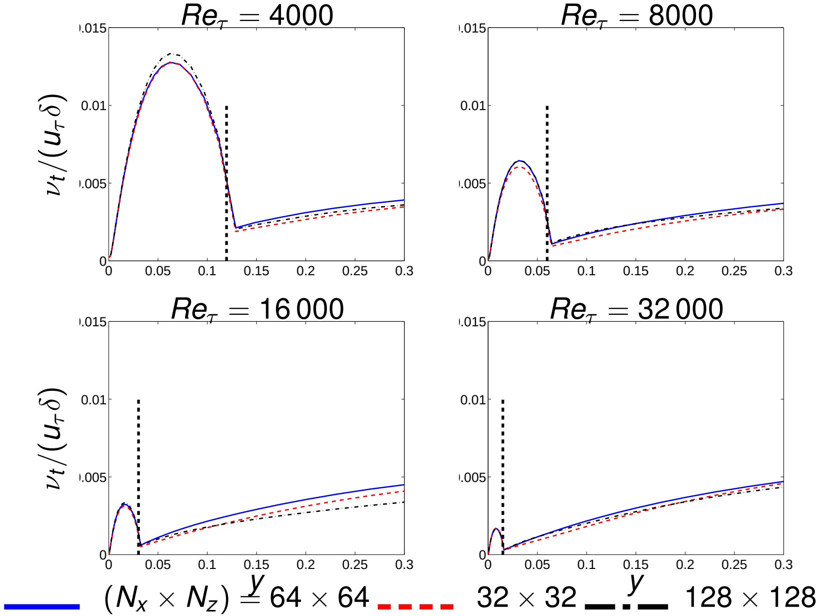
— $Re_\tau = 4\,000$ - - - $Re_\tau = 8\,000$ — · — $Re_\tau = 16\,000$;
· · · · $Re_\tau = 32\,000$.

EFFECT OF RESOLUTION: RESOLVED SHEAR STRESS

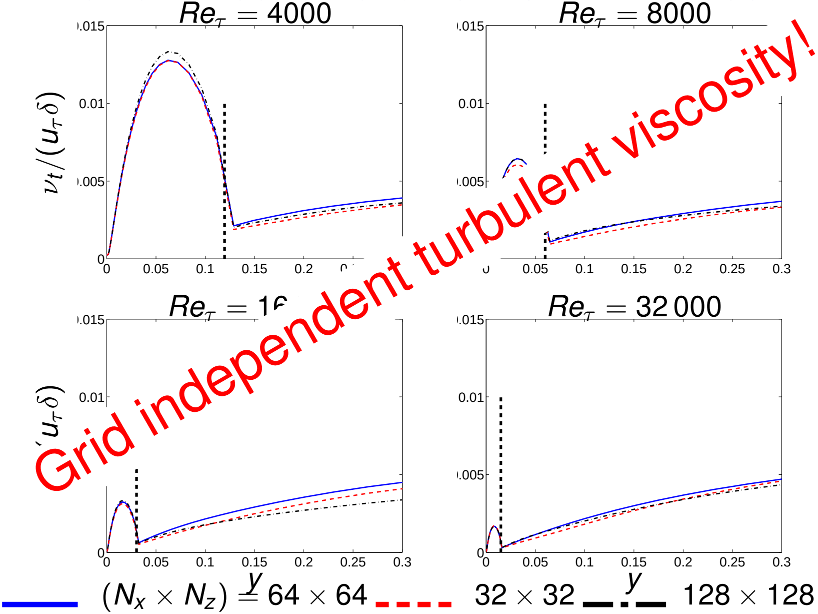


— $Re_\tau = 4\,000$ - - - - $Re_\tau = 8\,000$ — · — $Re_\tau = 16\,000$;
· · · · $Re_\tau = 32\,000$.

EFFECT OF RESOLUTION: TURBULENT VISCOSITY



EFFECT OF RESOLUTION: TURBULENT VISCOSITY



Grid independent turbulent viscosity!

SGS MODELS BASED ON GRID SIZE

- When the grid is refined, ν_t gets smaller

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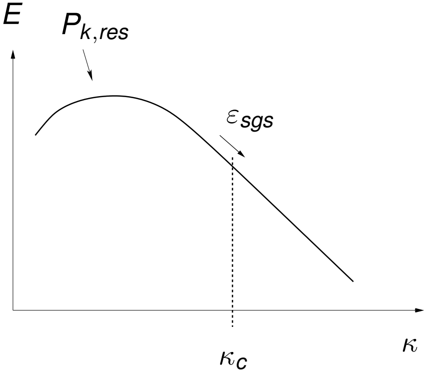

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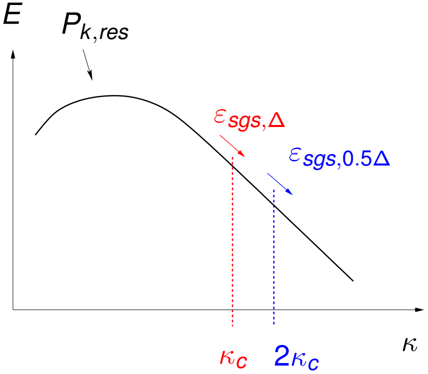

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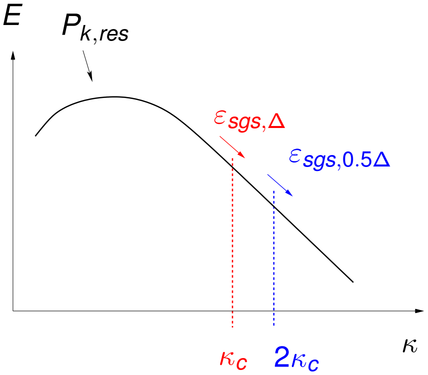
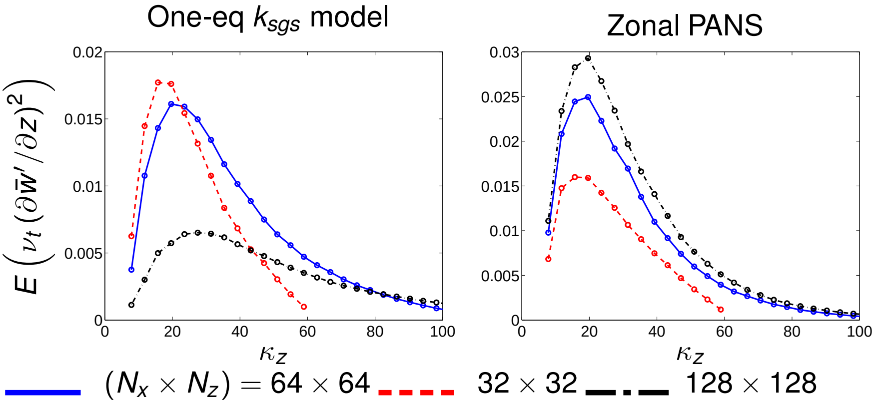


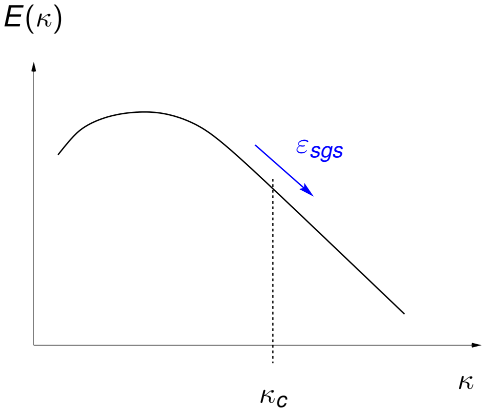
- $\varepsilon_{sgs,\Delta} = \varepsilon_{sgs,0.5\Delta}$
- $\varepsilon_{sgs} = 2\langle \nu_t \bar{s}_{ij}\bar{s}_{ij}\rangle - \langle \tau_{12,t}\rangle \dfrac{\partial\langle \bar{u}\rangle}{\partial y}$
- Grid refinement ⇒ must be accompanied with larger $\bar{s}_{ij}\bar{s}_{ij}$
- ⇒ $\bar{s}_{ij}\bar{s}_{ij}$ must take place at higher wavenumbers
- if not ⇒ grid dependent

POWER DENSITY SPECTRA OF $\nu_t^{0.5}\frac{\partial \bar{w}'}{\partial z}$



SGS DISSIPATION VS. WAVENUMBER

- Energy spectra of the SGS dissipation show that the peak takes place at surprisingly low wavenumber (length scale corresponding to 10 cells or more).



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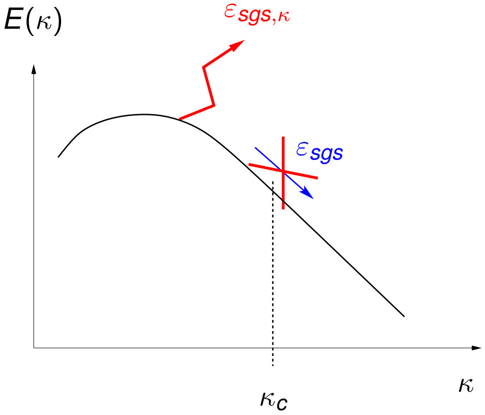
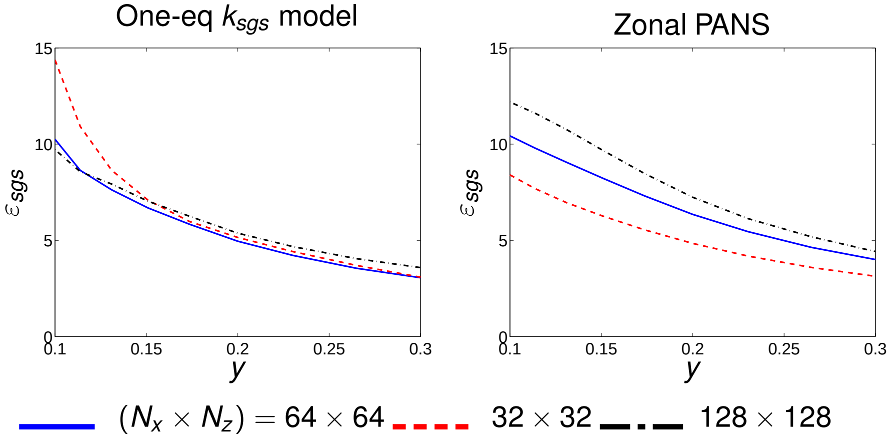
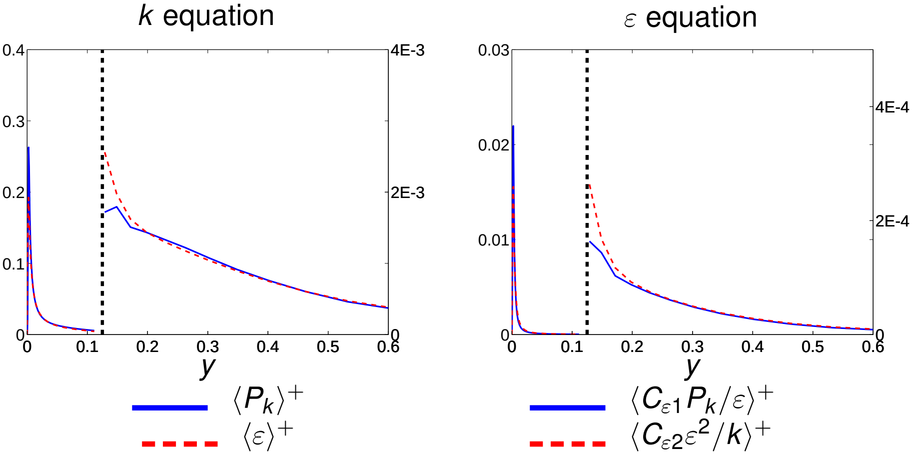


SGS DISSIPATION, $Re_\tau = 8000$

- SGS dissipation in the $\bar{u}'_i\bar{u}'_i/2$ eq, $\varepsilon_{sgs} = 2\langle \nu_t \bar{s}_{ij}\bar{s}_{ij}\rangle - \langle \tau_{12,t}\rangle \dfrac{\partial\langle\bar{u}\rangle}{\partial y}$



LOCAL EQUILIBRIUM. $Re_\tau = 4000$, $N_x \times N_z = 64 \times 64$.



Left vertical axes: URANS region; right vertical axes: LES region.

LOCAL EQUILIBRIUM IN ε EQUATION.

- How can both the k eq. and ε be in local equilibrium??

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If

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$$C_1 \frac{\langle \varepsilon \rangle}{\langle k \rangle} \langle P_k \rangle \neq C_2^* \frac{\langle \varepsilon \rangle^2}{\langle k \rangle}, \text{because} \quad C_1 \neq C_2^*$$

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However, the previous slide shows

$$C_1 \left\langle \frac{\varepsilon}{k} P_k \right\rangle = C_2^* \left\langle \frac{\varepsilon^2}{k} \right\rangle$$

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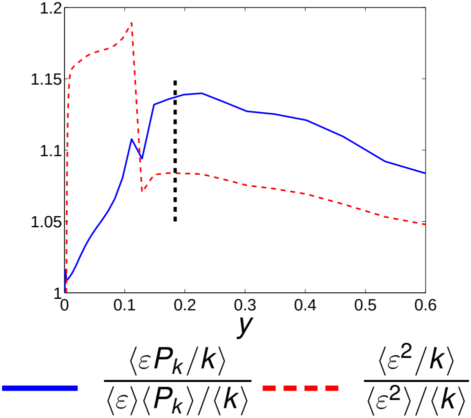
However, the previous slide shows

$$C_1 \left\langle \frac{\varepsilon}{k} P_k \right\rangle = C_2^* \left\langle \frac{\varepsilon^2}{k} \right\rangle$$

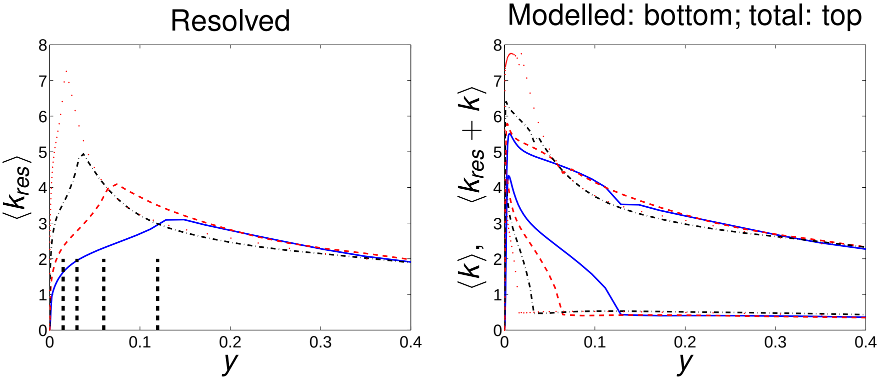
- Answer: when time-averaging $\langle ab \rangle \neq \langle a \rangle \langle b \rangle$

LOCAL EQUILIBRIUM IN ε EQUATION.

- The answer is because of time averaging ($\langle ab \rangle < \langle a \rangle \langle b \rangle$, (see below)



RESOLVED AND MODELLED TURBULENT KINETIC ENERGY.



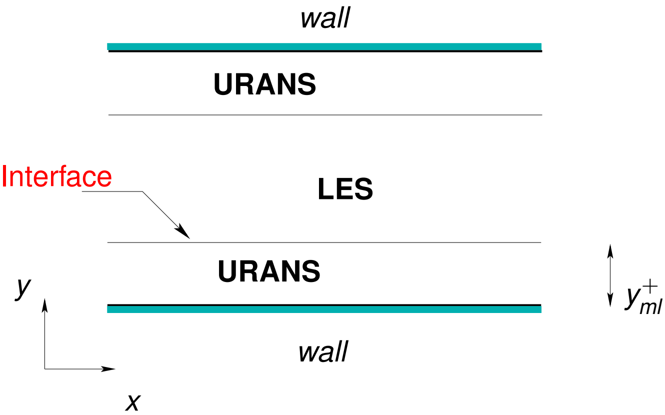
—— $Re_\tau = 4\,000$ - - - - $Re_\tau = 8\,000$ —·— $Re_\tau = 16\,000$;
· · · · $Re_\tau = 32\,000$.

CONCLUDING REMARKS

- LRN PANS works well as zonal LES-RANS model for very high Re_τ ($> 32\,000$)
- The model gives grid independent results
- The location of the interface is not important (it should not be too close to the wall)
- Values of $0.2 < f_k < 0.5$ have little impact on the results

HYBRID LES-RANS

Near walls: a RANS one-eq. k or a $k-\omega$ model.
In core region: a LES one-eq. k_{SGS} model.



- Location of interface either pre-defined or automatically computed

MOMENTUM EQUATIONS

- The Navier-Stokes, time-averaged in the near-wall regions and filtered in the core region, reads

$$\frac{\partial \bar{u}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\bar{u}_i\bar{u}_j\right)=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left[(\nu+\nu_T)\frac{\partial \bar{u}_i}{\partial x_j}\right]$$

$$\nu_T=\nu_t, y\leq y_{ml}$$

$$\nu_T=\nu_{sgs}, y\geq y_{ml}$$

- The equation above: URANS or LES? Same boundary conditions ⇒ same solution!

TURBULENCE MODEL

- Use one-equation model in both URANS region and LES region.

$$\frac{\partial k_T}{\partial t}+\frac{\partial}{\partial x_j}(\bar{u}_j k_T)=\frac{\partial}{\partial x_j}\left[(\nu+\nu_T)\frac{\partial k_T}{\partial x_j}\right]+P_{k_T}-C_\varepsilon\frac{k_T^{3/2}}{\ell}$$

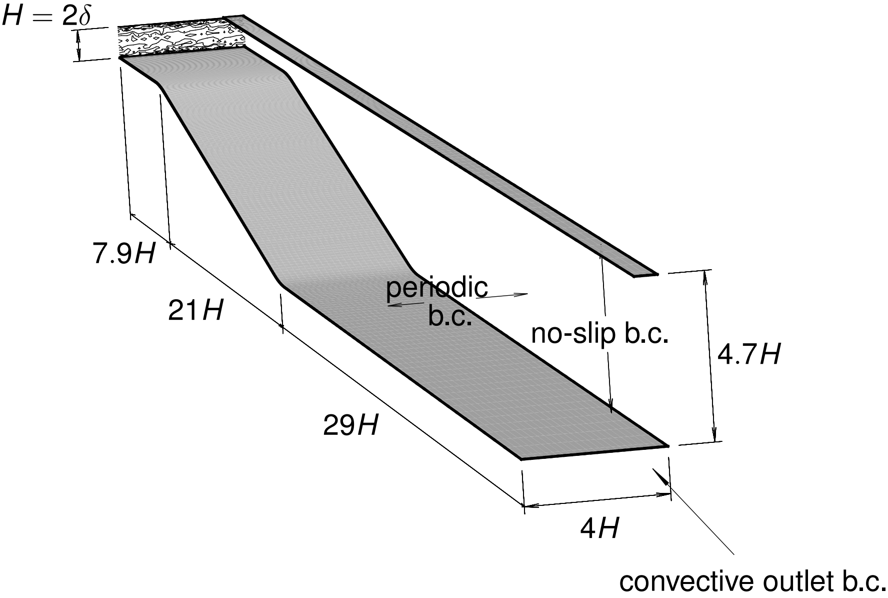
$$P_{k_T}=2\nu_T\bar{S}_{ij}\bar{S}_{ij},\ \nu_T=C_k\ell k_T^{1/2}$$

  - LES-region: $k_T=k_{sgs}$, $\nu_T=\nu_{sgs}$, $\ell=\Delta=(\delta V)^{1/3}$
  - URANS-region: $k_T=k$, $\nu_T=\nu_t$, $\ell\equiv\ell_{RANS}=2.5n[1-\exp(-Ak^{1/2}y/\nu)]$, Chen-Patel model (AIAA J. 1988)
  - Location of interface can be defined by $\min(0.65\Delta, y)$, $\Delta=\max(\Delta x,\Delta y,\Delta z)$

DIFFUSER[9]

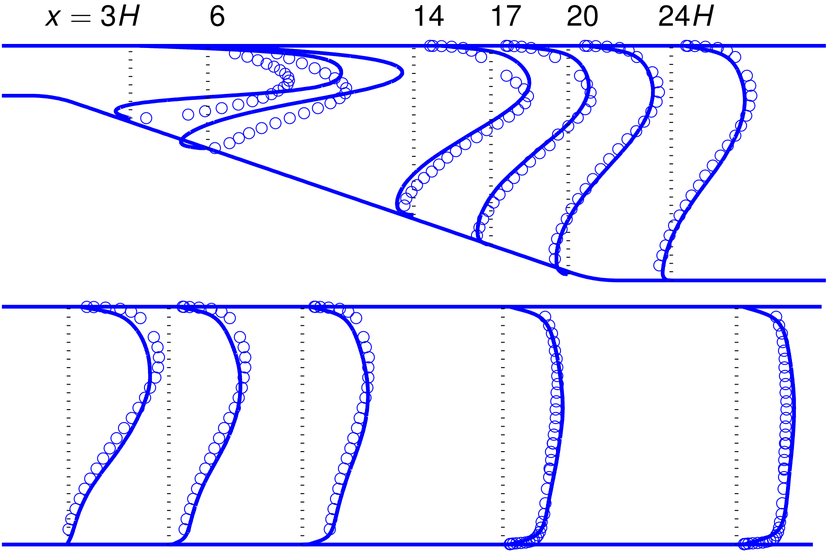
- Instantaneous inlet data from channel DNS used.
- Domain: $-8 \leq x \leq 48$, $0 \leq y_{inlet} \leq 1$, $0 \leq z \leq 4$.
- $x_{max} = 40$ gave return flow at the outlet
- Grid: $258 \times 66 \times 32$.
- $Re = U_{in}H/\nu = 18\,000$, angle 10^o
- The grid is much too coarse for LES (in the inlet region $\Delta z^+ \simeq 170$)
- Matching plane fixed at y_{ml} at the inlet. In the diffuser it is located along the 2D instantaneous streamline corresponding to y_{ml}.

DIFFUSER GEOMETRY. $Re = 18\,000$, ANGLE 10^o

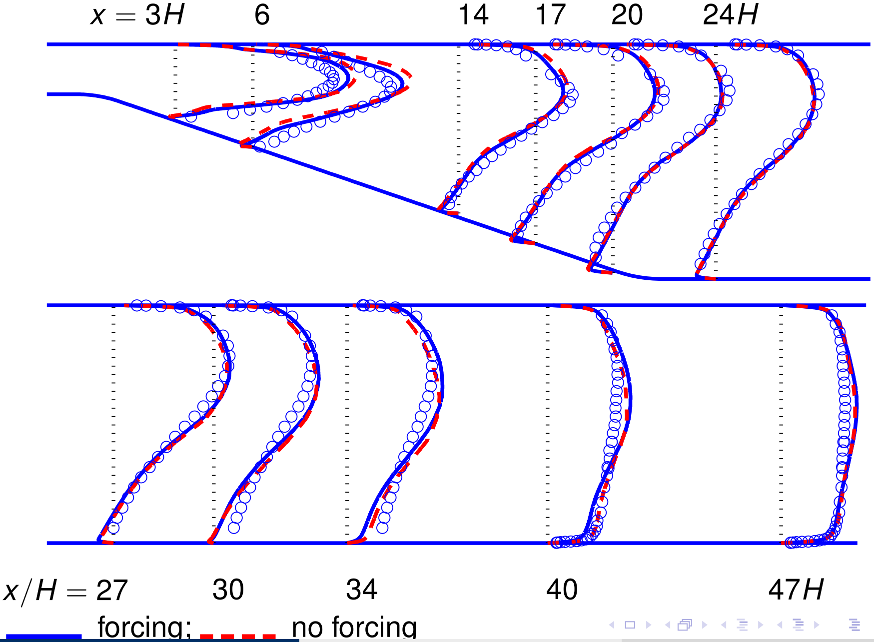


DIFFUSER: RESULTS WITH LES

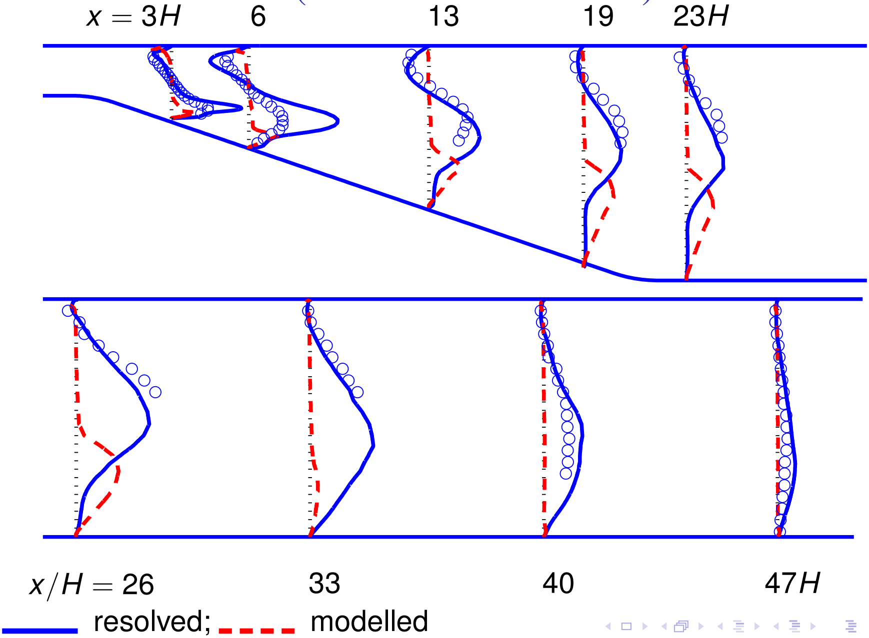
- Velocities. Markers: experiments by Buice & Eaton (1997)



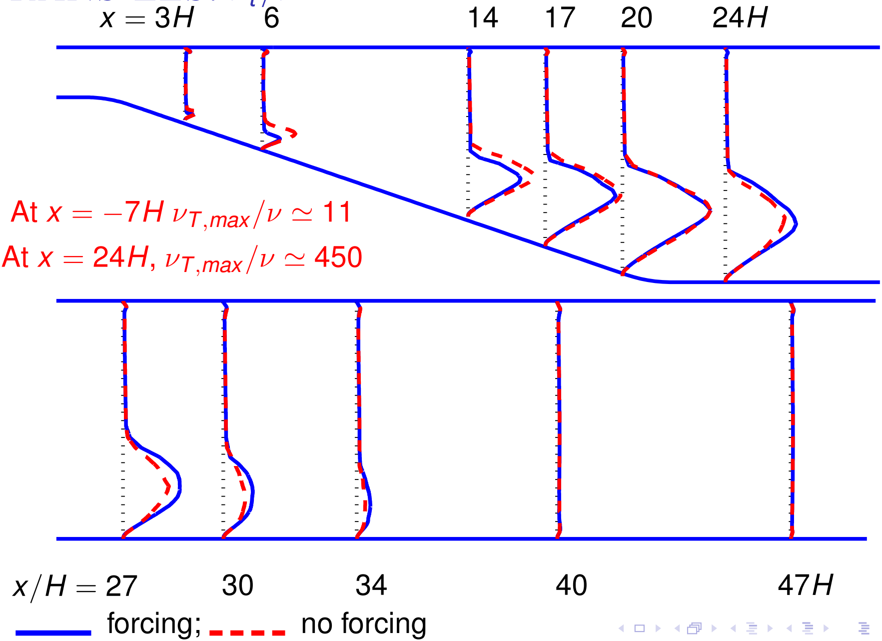
DIFFUSER: RESULTS WITH RANS-LES



Shear Stresses (×2 In Lower Half)



RANS-LES: ν_t/ν



At $x = -7H$ $\nu_{T,max}/\nu \simeq 11$

At $x = 24H$, $\nu_{T,max}/\nu \simeq 450$

$k-\omega$ MODEL SST-DES

- DES [24]: Detached Eddy Simulation
- SST [18, 19]: A combination of the $k-\varepsilon$ and the $k-\omega$ model

$$\frac{\partial k}{\partial t}+\frac{\partial}{\partial x_j}(\bar{u}_j k)=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k-\beta^* k\omega$$

$$\frac{\partial \omega}{\partial t}+\frac{\partial}{\partial x_j}(\bar{u}_j \omega)=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]+\alpha\frac{P_k}{\nu_t}-\beta\omega^2+\ldots$$

- The dissipation term in the k equation is modified as [19, 25]

$$\beta^* k\omega \to \beta^* k\omega F_{DES}, \quad F_{DES}=\max\left\{\frac{L_t}{C_{DES}\Delta},1\right\}$$

$$\Delta=\max\left\{\Delta x_1,\Delta x_2,\Delta x_3\right\}, \quad L_t=\frac{k^{1/2}}{\beta^*\omega}$$

⇒ RANS near walls and LES away from walls

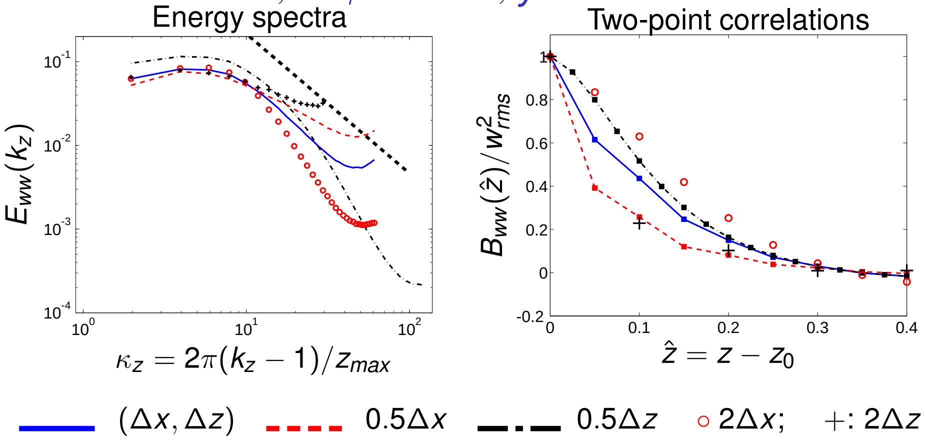
RESOLUTION

- For the near-wall region, we know how fine the mesh should be in terms of viscous units (see Slide 22)
- An appropriate resolution for the fully turbulent part of the boundary layer is $\delta/\Delta x \simeq 10 - 20$ and $\delta/\Delta z \simeq 20 - 40$
- This may be relevant also for jets and shear layers

HOW TO ESTIMATE RESOLUTION IN GENERAL? [4, 5]

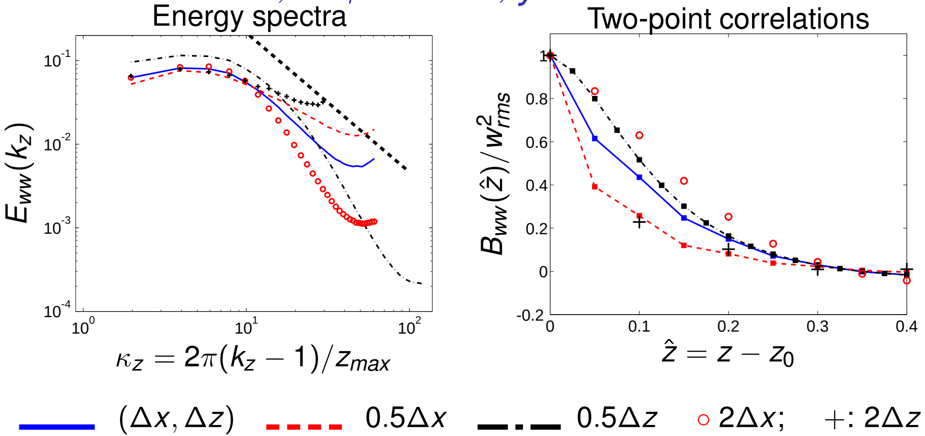
- Energy spectra (both in spanwise direction and time)
- Two-point correlations
- Ratio of SGS turbulent kinetic energy $\langle k_{sgs} \rangle$ to resolved $0.5\langle u'u' + v'v' + w'w' \rangle$
- Ratio of SGS shear stress $\langle \tau_{sgs,12} \rangle$ to resolved $\langle u'v' \rangle$
- Ratio of SGS viscosity, $\langle \nu_{sgs} \rangle$ to molecular, ν
- Energy spectra of SGS dissipation
- Comparison of SGS dissipation due to $\partial u'_i/\partial x_j$ and $\partial \langle \bar{u}_i \rangle/\partial x_j$

CHANNEL FLOW, $Re_\tau = 4000$, $y^+ = 440$



The $(\Delta x, \Delta z)$ mesh is $(\delta/\Delta x, \delta/\Delta z) = (10, 20)$

CHANNEL FLOW, $Re_\tau = 4000$, $y^+ = 440$



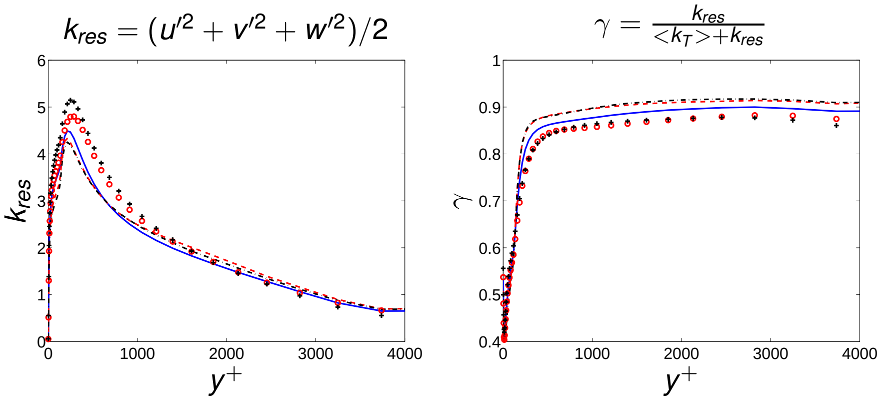
Energy spectra

Two-point correlations

The $(\Delta x, \Delta z)$ mesh is $(\delta/\Delta x, \delta/\Delta z) = (10, 20)$

- Two-point correlation is better
- Shows that $2\Delta z$ and $2\Delta x$ (two-point corr in x) are too coarse.

CHANNEL FLOW, $Re_\tau = 4000$, $y^+ = 440$



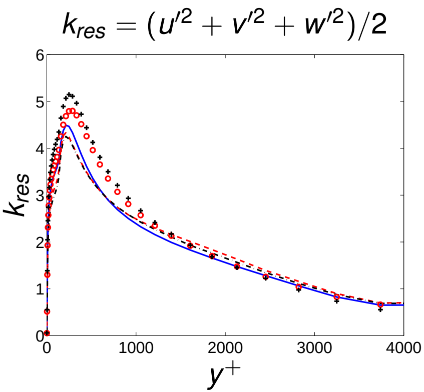
$k_{res} = (u'^2 + v'^2 + w'^2)/2$

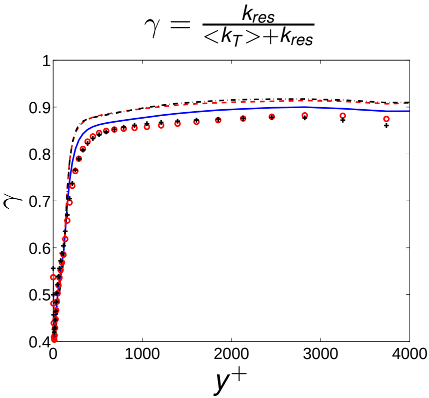
$\gamma = \frac{k_{res}}{<k_T>+k_{res}}$

- Pope [20] suggests $\gamma > 0.8$ indicates well resolved flow

—— $(\Delta x, \Delta z)$ - - - - $0.5\Delta x$ —·— $0.5\Delta z$ ○ $2\Delta x$; $+$: $2\Delta z$

CHANNEL FLOW, $Re_\tau = 4000$, $y^+ = 440$





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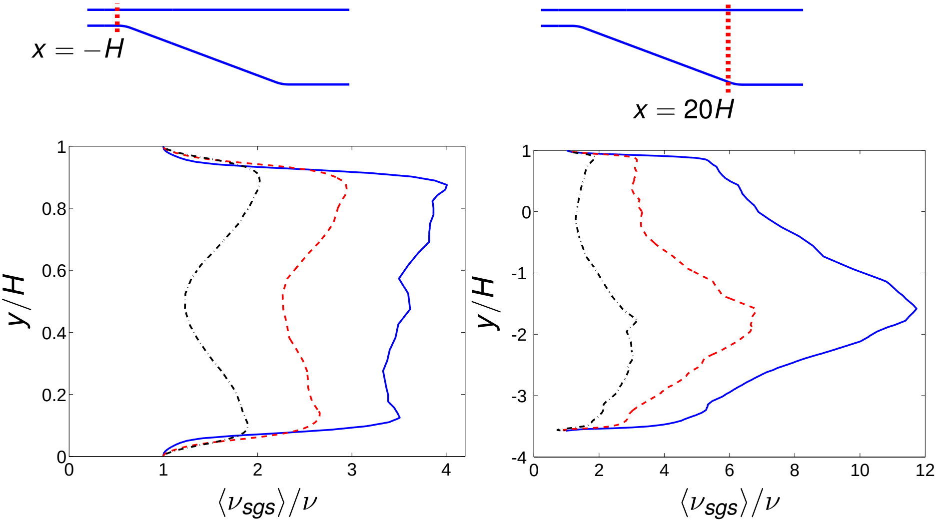
▬▬ $(\Delta x, \Delta z)$ - - - - $0.5\Delta x$ ▬ · ▬ $0.5\Delta z$ ○ $2\Delta x$; $+$: $2\Delta z$

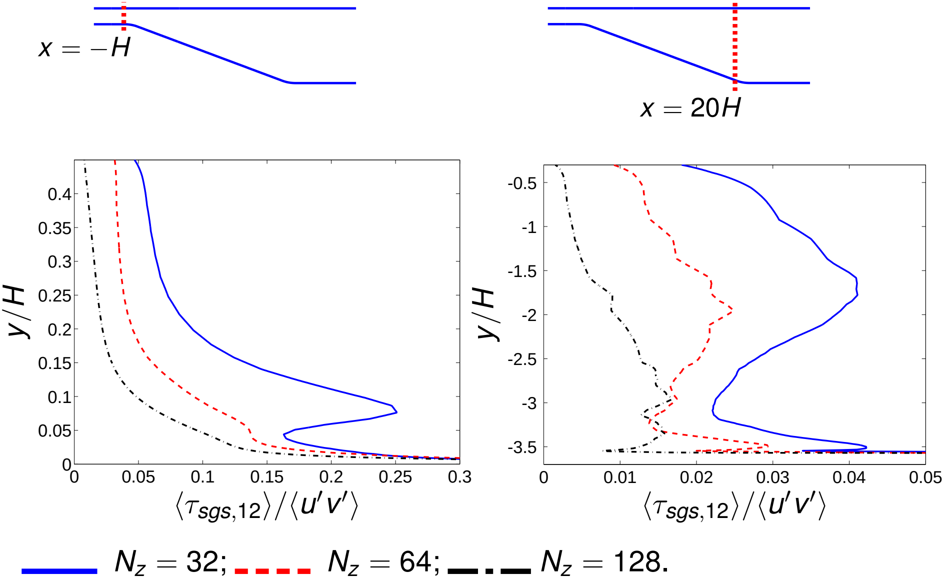
- Pope criterion does not work here

SGS VS. MOLECULAR VISCOSITY [5]



—— $N_z = 32$; - - - $N_z = 64$; —·— $N_z = 128$.

SGS VS. RESOLVED SHEAR STRESSES



THE PANS MODEL

- The PANS model is a modified $k - \varepsilon$ model
- It can operate both in RANS mode and LES mode
- In the present work a low-Reynolds turbulence version of the PANS is used
- A method how to implement embedded LES is proposed
- It is evaluated for channel flow and hump flow

Embedded LES Using PANS [10, 11]

Lars Davidson[1] and Shia-Hui Peng[1,2]
Davidson& Peng

[1]Department of Applied Mechanics
Chalmers University of Technology, SE-412 96 Gothenburg, SWEDEN
[2]FOI, Swedish Defence Research Agency, SE-164 90, Stockholm, SWEDEN

PANS LOW REYNOLDS NUMBER MODEL [17]

$$\frac{\partial k_u}{\partial t} + \frac{\partial (k_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{ku}}\right)\frac{\partial k_u}{\partial x_j}\right] + (P_u - \varepsilon_u)$$

$$\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial (\varepsilon_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon_u}{\partial x_j}\right] + C_{\varepsilon 1} P_u \frac{\varepsilon_u}{k_u} - C^*_{\varepsilon 2}\frac{\varepsilon_u^2}{k_u}$$

$$\nu_u = C_\mu f_\mu \frac{k_u^2}{\varepsilon_u}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

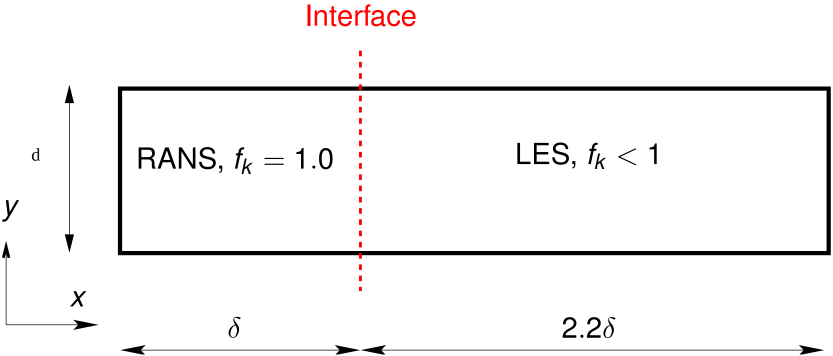
$C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_k, σ_ε and C_μ same values as [1]. $f_\varepsilon = 1$. f_2 and f_μ read

$$f_2 = \left[1 - \exp\left(-\frac{y^*}{3.1}\right)\right]^2 \left\{1 - 0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}$$

$$f_\mu = \left[1 - \exp\left(-\frac{y^*}{14}\right)\right]^2 \left\{1 + \frac{5}{R_t^{3/4}}\exp\left[-\left(\frac{R_t}{200}\right)^2\right]\right\}$$

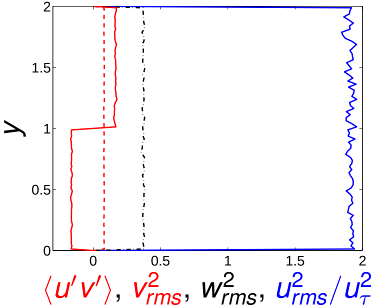
- Baseline model: $f_k = 0.4$. Range of $0.2 < f_k < 0.6$ is evaluated

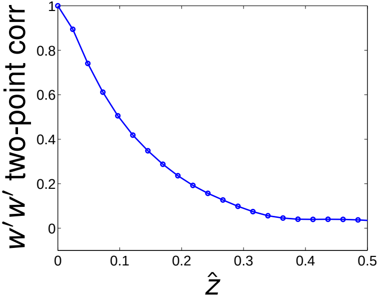
CHANNEL FLOW: DOMAIN



- Interface: Synthetic turbulent fluctuations are introduced as additional convective fluxes in the momentum equations and the continuity equation
- $f_k = 0.4$ is the baseline value for LES [17]

INLET FLUCTUATIONS





- Anisotropic synthetic fluctuations, u', v', w',
- Integral length scale $\mathcal{L} \simeq 0.13$ (see 2-p point correlation)
- Asymmetric time filter $(\mathcal{U}')^m = a(\mathcal{U}')^{m-1} + b(u')^m$ with $a = 0.954, b = (1 - a^2)^{1/2}$ gives a time integral scale $\mathcal{T} = 0.015$ ($\Delta t = 0.00063$)

INTERFACE CONDITIONS FOR k_u AND ε_u

- For k_u & ε_u we prescribe “inlet” boundary conditions at the interface.
- First, the usual convective and diffusive fluxes at the interface are set to zero
- Next, new convective fluxes are added. Which “inlet” values should be used at the interface?
  - $k_{u,int} = f_k k_{RANS}(x = 0.5\delta)$, $\varepsilon_{u,int} = C_\mu^{3/4} k_{u,int}^{3/2}/\ell_{sgs}$, $\ell_{sgs} = C_s\Delta$, $\Delta = V^{1/3}$
  -

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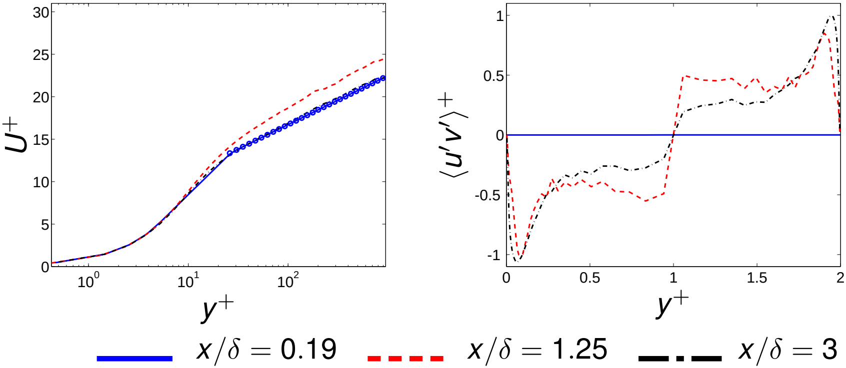
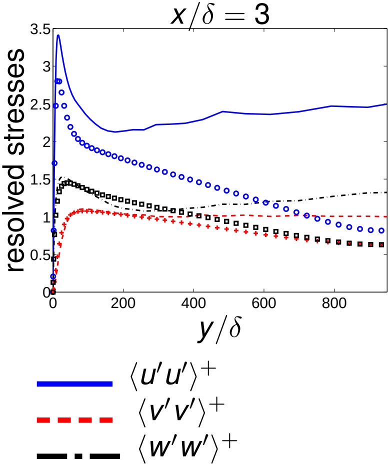
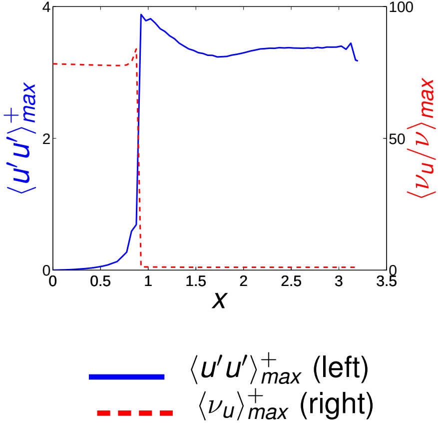
  - Baseline $C_s = 0.07$; different C_s values are tested

CHANNEL FLOW: VELOCITY AND SHEAR STRESSES



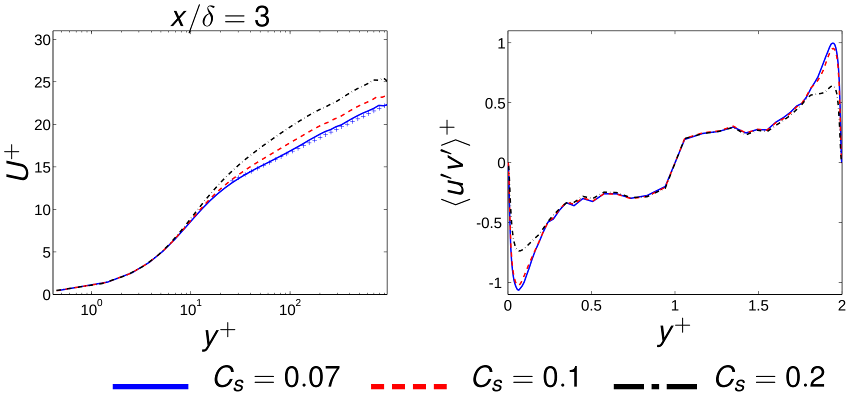
Channel Flow: stresses and peak values vs. x





CHANNEL FLOW: DIFFERENT C_s VALUE FOR $\varepsilon_{interface}$

- $k_{u,int} = f_k k_{RANS}$
- $\varepsilon_{u,int} = C_\mu^{3/4} k_{u,int}^{3/2} / \ell_{sgs}$, $\ell_{sgs} = C_s \Delta$



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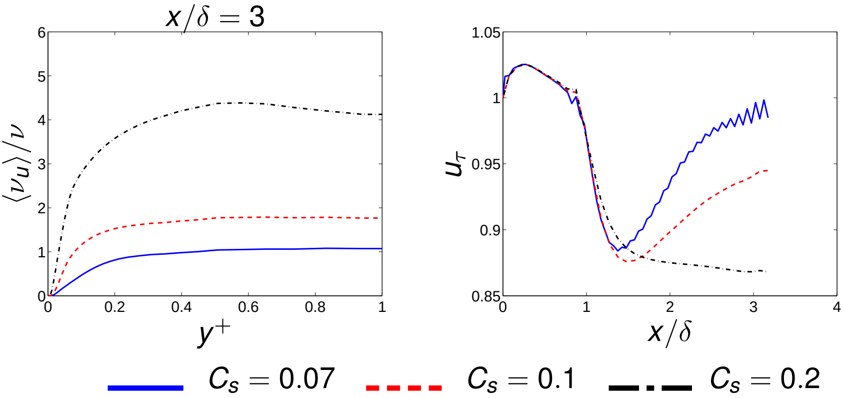
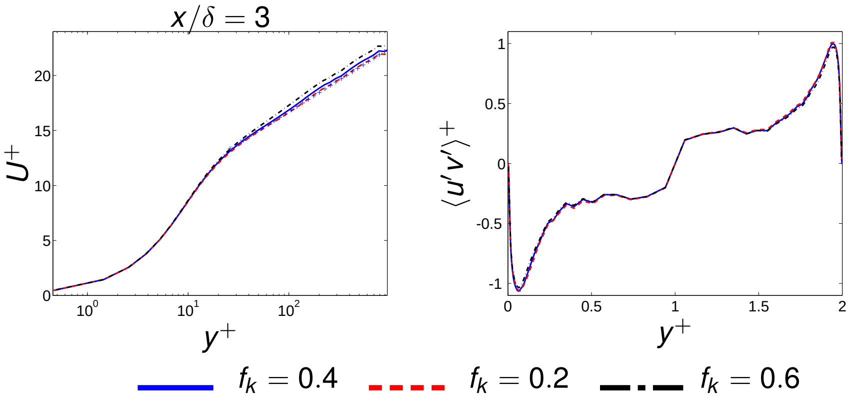
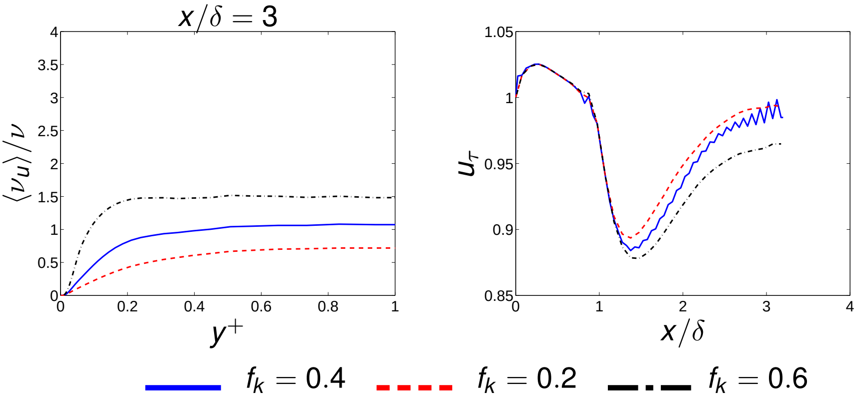


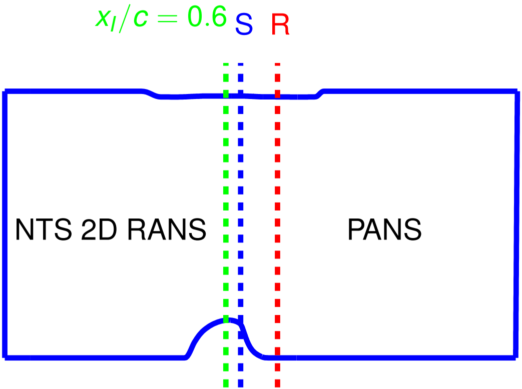
CHANNEL FLOW: DIFFERENT f_k VALUES



CHANNEL FLOW: DIFFERENT f_k VALUES

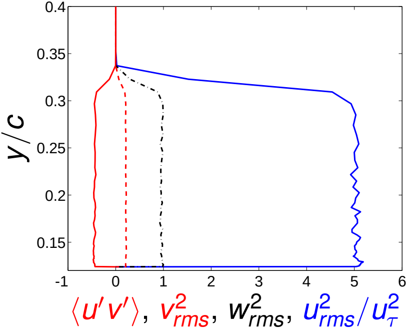


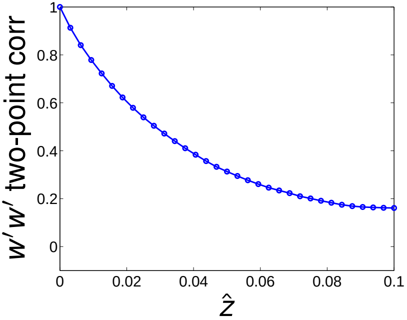
HUMP FLOW



- Inlet, Separation $x_S/c = 0.65$; reattachment $x_R/c = 1.1$
- $Re_c = 936\,000 \frac{U_{ij}c}{\nu}$ ($U_{in} = c = \rho = 1$, $\nu = 1/Re_c$
- $H/c = 0.91$, $h/c = 0.128$, $x/c = [0.6, 4.2]$
- Mesh: $312 \times 120 \times 64$, $Z_{max} = 0.2c$ (baseline)

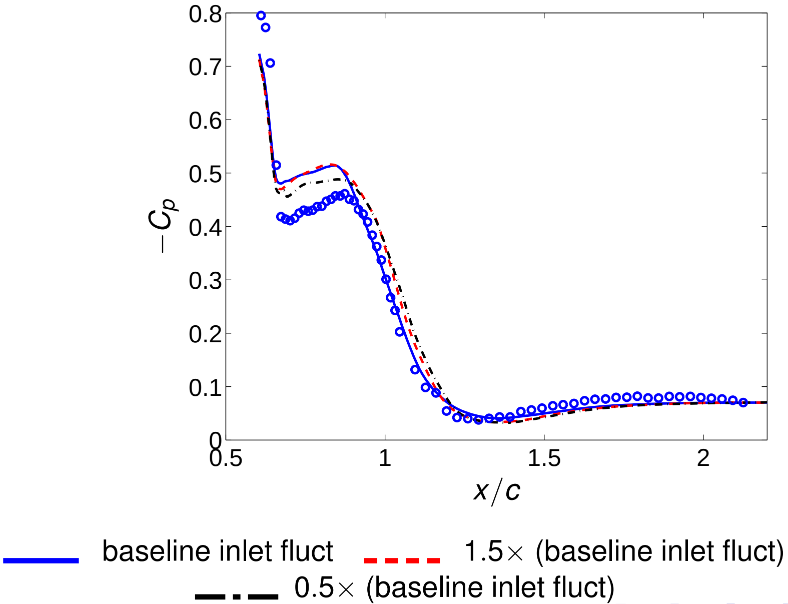
BASELINE INLET FLUCTUATIONS



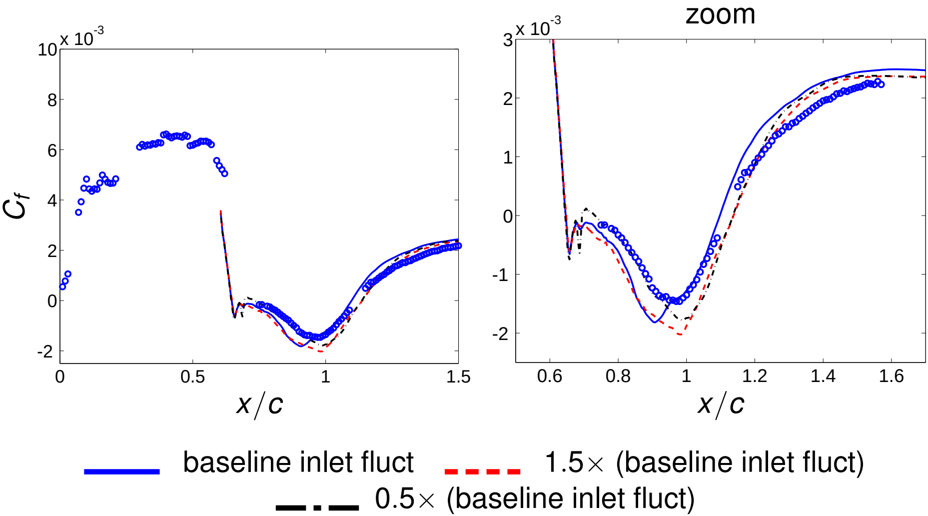


- Integral length scale $\mathcal{L} \simeq 0.04$ (see 2-p point correlation)
- Asymmetric time filter $(\mathcal{U}')^m = a(\mathcal{U}')^{m-1} + b(u')^m$ with $a = 0.954, b = (1 - a^2)^{1/2}$ gives a time integral scale $\mathcal{T} = 0.038$
- $\Delta t = 0.002$. $7500 + 7500$ time steps (100 hours one core)
- Fluctuations multiplied by $f_{bl} = \max\{0.5\,[1 - \tanh(y - y_{bl} - y_{wall})/b]\,, 0.02\}$, $y_{bl} = 0.2$, $b = 0.01$.

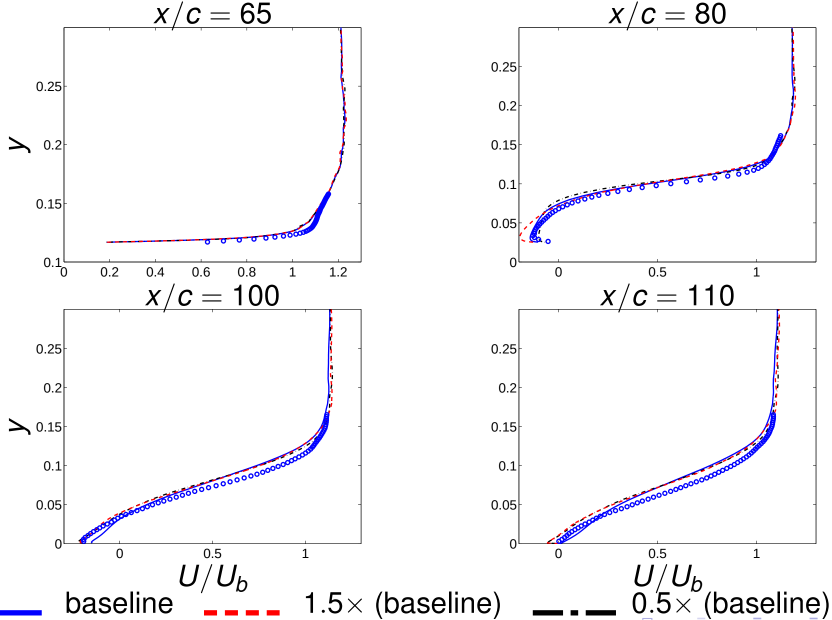
PRESSURE: AMPLITUDES OF INLET FLUCT



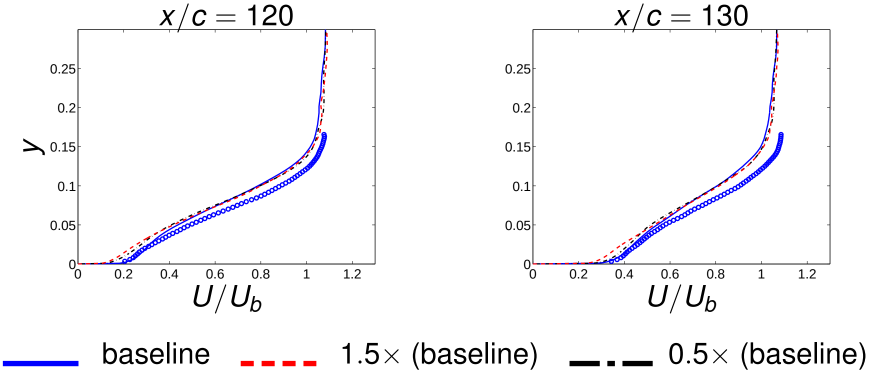
SKIN FRICTION: AMPLITUDES OF INLET FLUCT



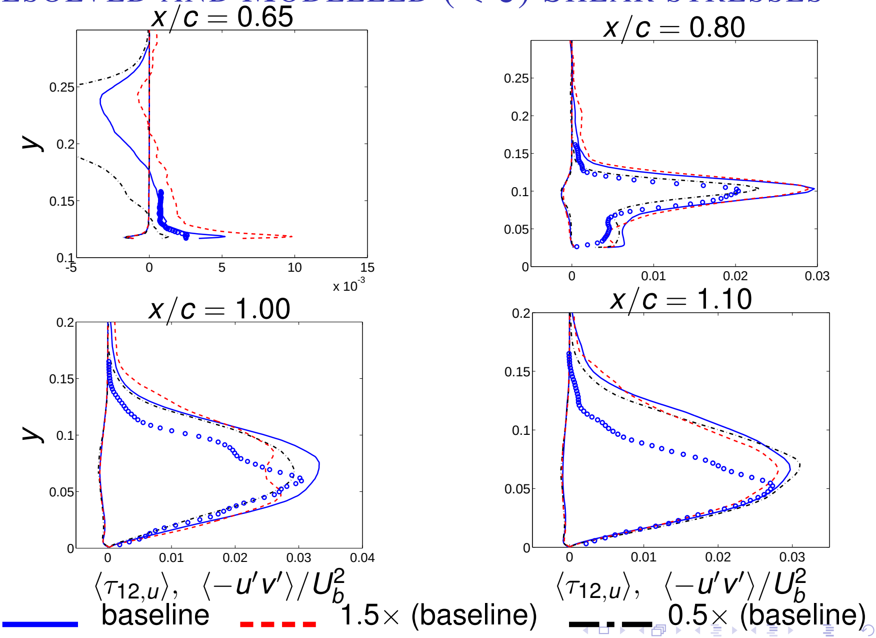
VELOCITIES: AMPLITUDES OF INLET FLUCT



VELOCITIES: AMPLITUDES OF INLET FLUCT

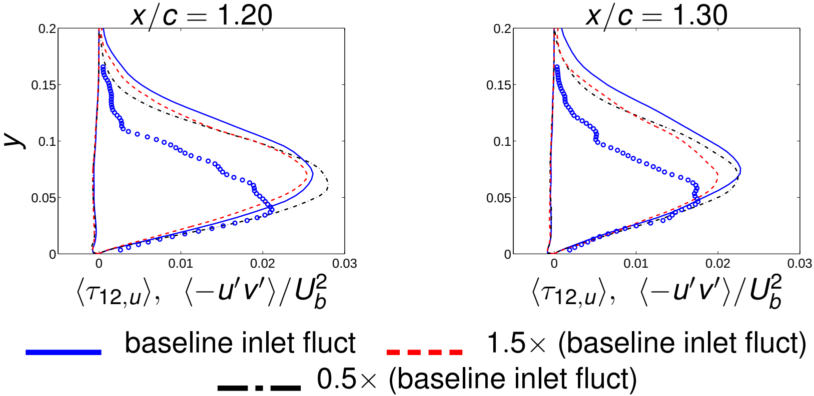


RESOLVED AND MODELLED (< 0) SHEAR STRESSES

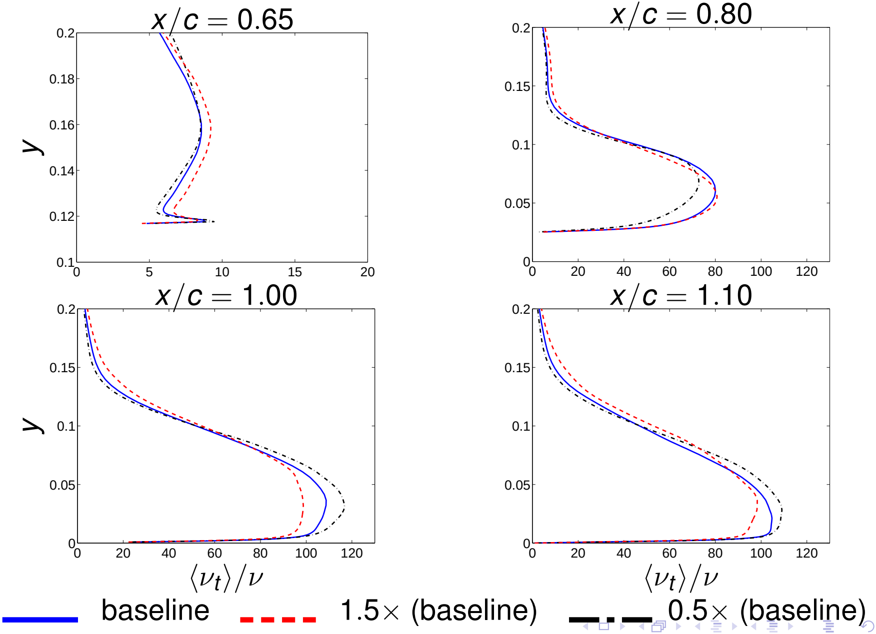


SHEAR STRESSES: AMPLITUDES OF INLET FLUCT

- Resolved and Modelled (< 0) Shear stresses



TURB VISCOSITY: AMPLITUDES OF INLET FLUCT



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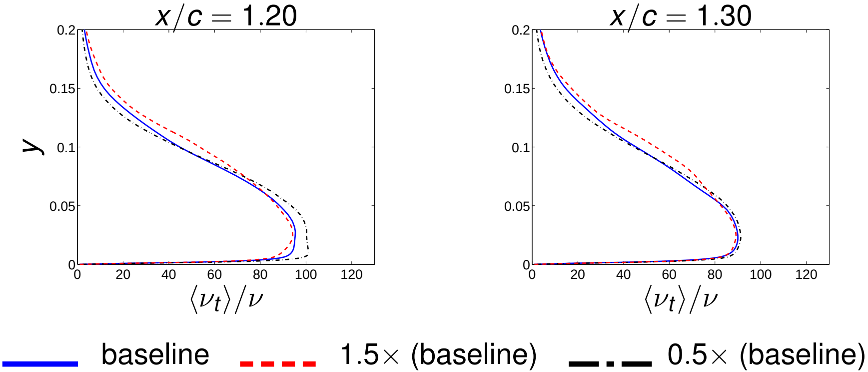
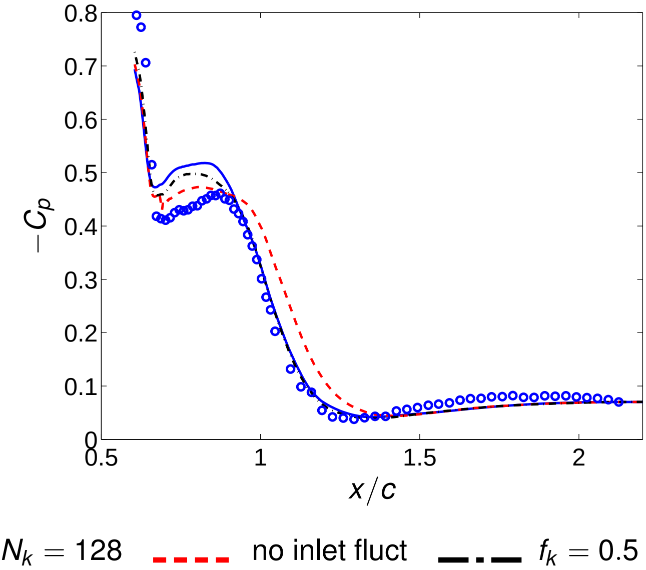
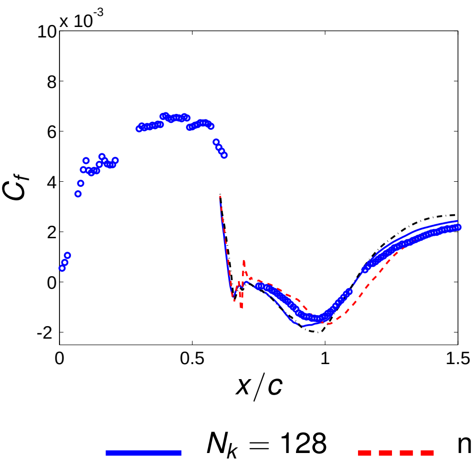


PRESSURE: $f_k = 0.5$; NO INLET FLUCT; $N_k = 128$



SKIN FRICTION: $f_k = 0.5$; NO INLET FLUCT; $N_k = 128$



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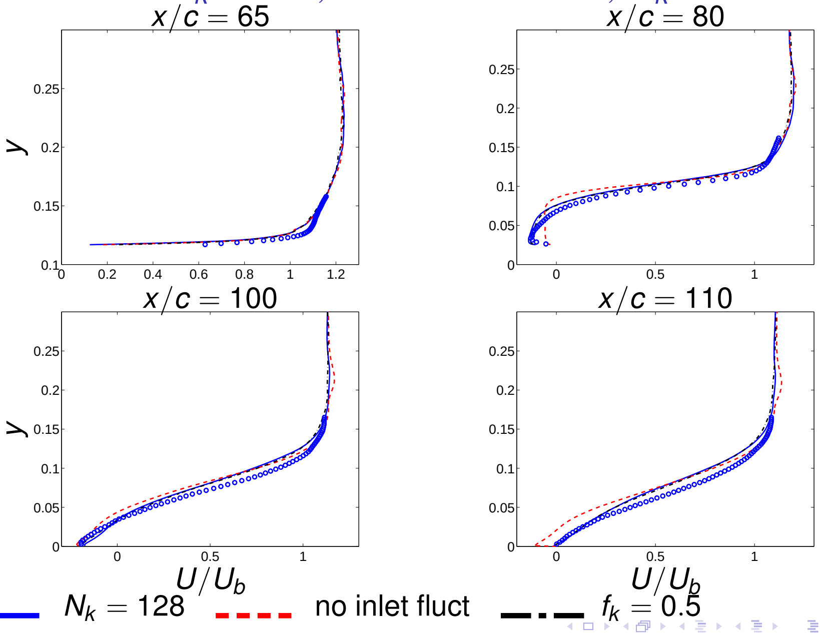

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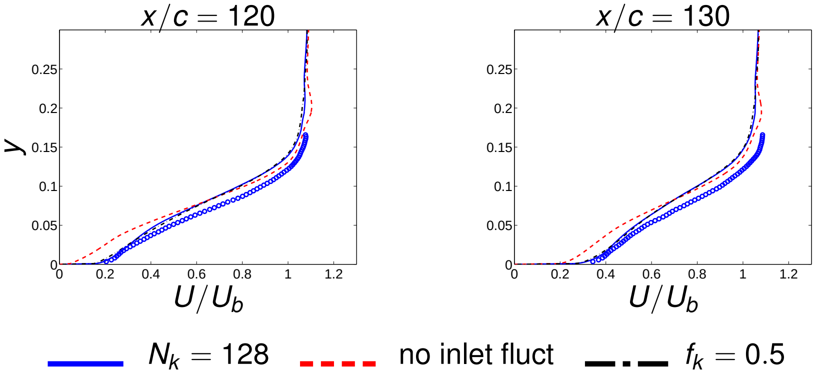
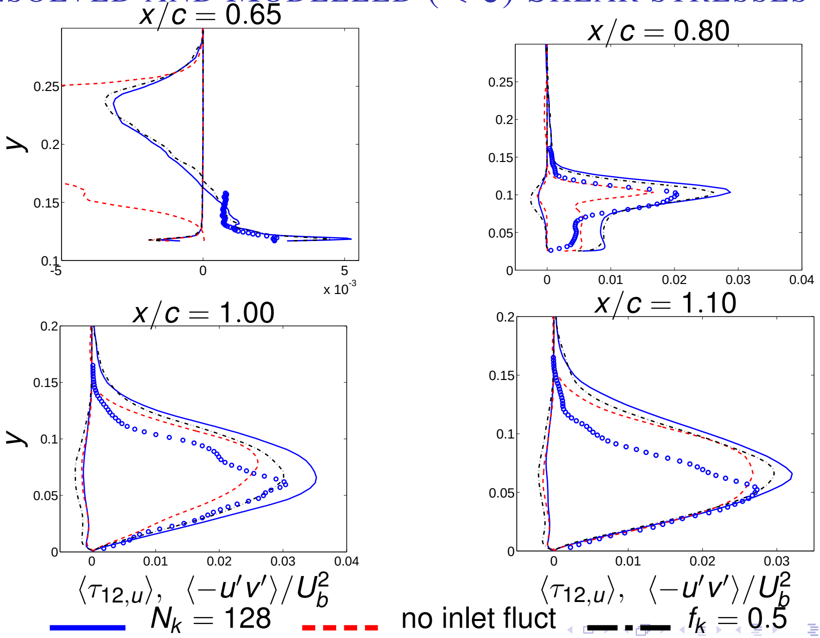


RESOLVED AND MODELLED (< 0) SHEAR STRESSES



SHEAR STRESSES: $f_k = 0.5$; NO INLET FLUCT; $N_k = 128$

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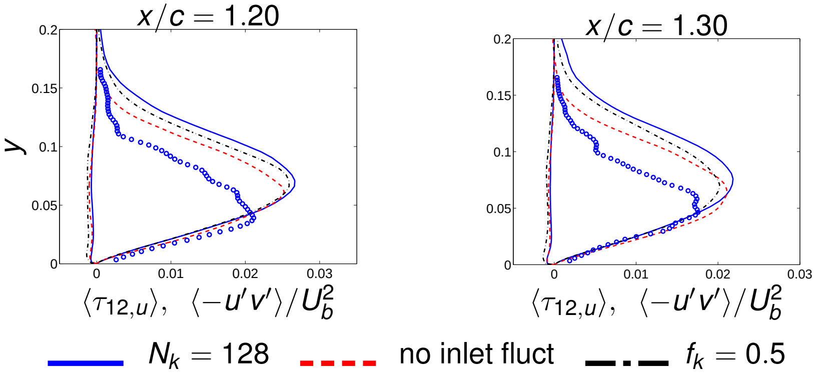

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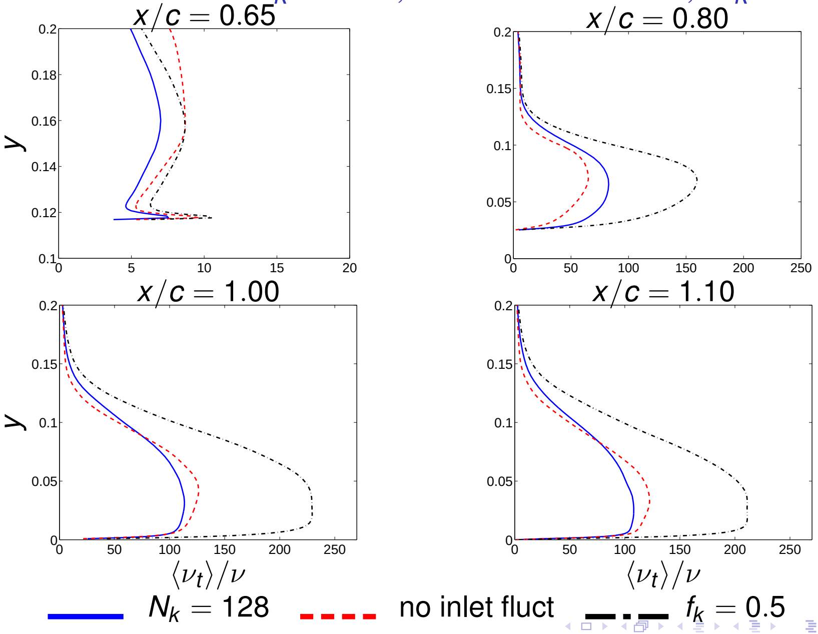

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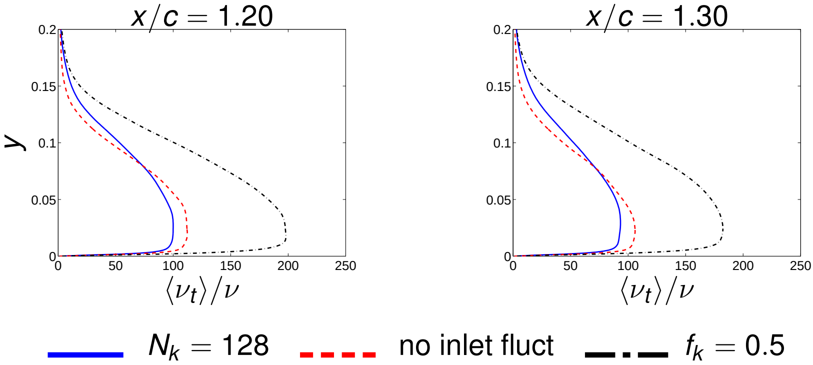

CONCLUDING REMARKS

- LRN PANS has been shown to work well as an embedded LES method
- Channel flow: At two δ downstream the interface, the resolved turbulence in good agreement with DNS data and the wall friction velocity has reached 99% of its fully developed value.
- Channel flow: The treatment of the modelled k_u and ε_u across the interface is important.
- LRN PANS predicts the hump flow well but the recover rate sligtly too slow
- Hump flow: large (small) inlet fluctuations gives a smaller (larger) recirculation

PANS: CONCLUDING REMARKS

- Embedded LES with $k-\varepsilon$ PANS and Synthetic b.c.

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- Embedded LES with $k-\varepsilon$ PANS and Synthetic b.c.
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  - Strong dependence on interface k_u & ε_u values
  - No strong dependence on amplitude, L or $\mathcal{T}$ of fluctuations

PANS: CONCLUDING REMARKS CONT'D

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  - With embedded anisotropic fluctuations, good results are obtained, still poor C_f
  - On-going work . . .

Large Eddy Simulation of Heat Transfer in Boundary layer and Backstep Flow Using PANS [6]

Lars Davidson
THMT-12, Palermo, Sept 2012

PANS LOW REYNOLDS NUMBER MODEL [17]

$$\frac{\partial k_u}{\partial t} + \frac{\partial (k_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{ku}}\right)\frac{\partial k_u}{\partial x_j}\right] + (P_u - \varepsilon_u)$$

$$\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial (\varepsilon_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon_u}{\partial x_j}\right] + C_{\varepsilon 1} P_u \frac{\varepsilon_u}{k_u} - C^*_{\varepsilon 2}\frac{\varepsilon_u^2}{k_u}$$

$$\nu_u = C_\mu f_\mu \frac{k_u^2}{\varepsilon_u}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

$C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_k, σ_ε and C_μ same values as [1]. $f_\varepsilon = 1$. f_2 and f_μ read

$$f_2 = \left[1 - \exp\left(-\frac{y^*}{3.1}\right)\right]^2 \left\{1 - 0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}$$

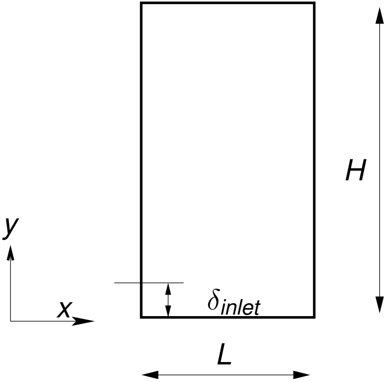
$$f_\mu = \left[1 - \exp\left(-\frac{y^*}{14}\right)\right]^2 \left\{1 + \frac{5}{R_t^{3/4}}\exp\left[-\left(\frac{R_t}{200}\right)^2\right]\right\}$$

- Baseline model: $f_k = 0.4$.

NUMERICAL METHOD

- Incompressible finite volume method
- Pressure-velocity coupling treated with fractional step
- Differencing scheme for momentum eqns:
    - 95% 2^{nd} order central and 5% 2^{nd} order upwind differencing scheme (baseline) OR
    - 100% 2^{nd} order central differencing
- Hybrid 1^{st} order upwind/2^{nd} order central scheme k & ε eqns.
- 2^{nd}-order Crank-Nicholson for time discretization

BOUNDARY LAYER FLOW: DOMAIN



- Inlet: $\delta_{inlet} = 1$ (covered by 45 cells), $Re_\theta = 3\,600$, $U_{in} = \rho = 1$. Stretching 1.12 up to $y/\delta \simeq 1$.
- Domain: $L/\delta_{in} = 3.2$, $H/\delta_{in} = 15.6$, $Z_{max} = 1.5\delta_{in}$
- Resolution: $\Delta z^+_{in} \simeq 27$, $\Delta x^+_{in} \simeq 54$
- Grid: $66 \times 96 \times 64$ (x, y, z)

ANISOTROPIC SYNTHETIC FLUCTUATIONS: I [3, 2, 8]

- Prescribe an homogeneous Reynolds tensor, $\overline{u_i u_j}$ (here from DNS)
-
-

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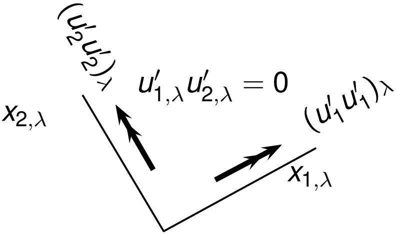

- Prescribe an homogeneous Reynolds tensor, $\overline{u_iu_j}$ (here from DNS)
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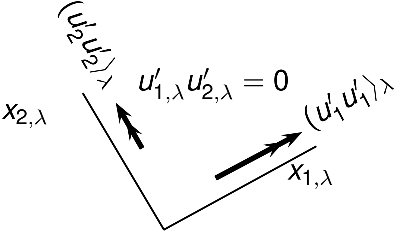
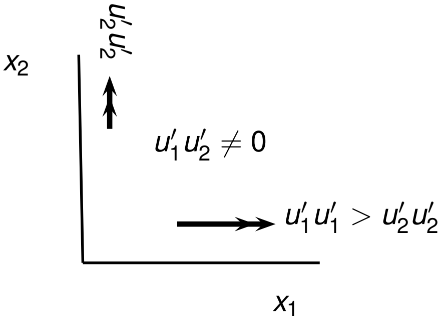


- Prescribe an homogeneous Reynolds tensor, $\overline{u_iu_j}$ (here from DNS)
- isotropic fluctuations in principal directions, $(u'_1u'_1)_\lambda = (u'_2u'_2)_\lambda$, $u'_{1,\lambda}u'_{2,\lambda} = 0$
- re-scale the normal components, $(u'_1u'_1)_\lambda > (u'_2u'_2)_\lambda$, $u'_{1,\lambda}u'_{2,\lambda} = 0$

ANISOTROPIC SYNTHETIC FLUCTUATIONS: II

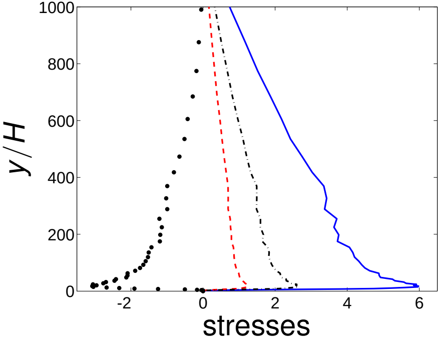


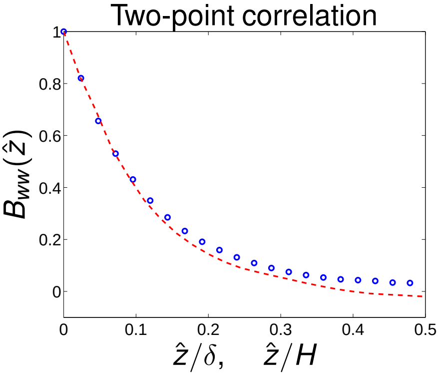
- Transform from $(x_{1,\lambda}, x_{2,\lambda})$ to (x_1, x_2)
- $\frac{u_1'^2}{u_2'^2} = 23$, $\frac{u_1'^2}{u_3'^2} = 5$ from $(u_1'u_1')_{peak}$ in DNS channel flow, $Re_\tau = 500$

INLET CONDITIONS FOR k_u AND ε_u AS IN [10]

- A pre-cursor RANS simulation using the AKN model (i.e. PANS with $f_k = 1$) is carried out. At $Re_\theta = 3\,600$, U_{RANS}, V_{RANS}, k_{RANS} are taken.
- $\bar{u}_{in} = U_{RANS} + u'_{synt}$, $\bar{v}_{in} = V_{RANS} + v'_{synt}$, $\bar{w}_{in} = w'_{synt}$
- Anisotropic synthetic fluctuations are used. The fluctuations are scaled with $k_u/k_{u,max}$.
- $k_{u,in} = f_k k_{RANS}$, $\varepsilon_{u,in} = C_\mu^{3/4} k_{u,in}^{3/2}/\ell_{sgs}$, $\ell_{sgs} = C_s\Delta$, $\Delta = V^{1/3}$, $C_s = 0.05$

INLET TURB. FLUCTUATION, TWO-POINT CORRELATIONS





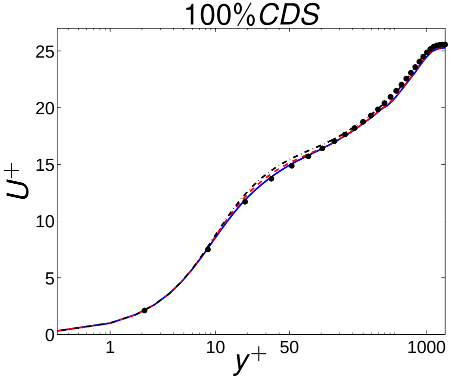
— u^{+2}_{rms}, - - - v^{+2}_{rms},

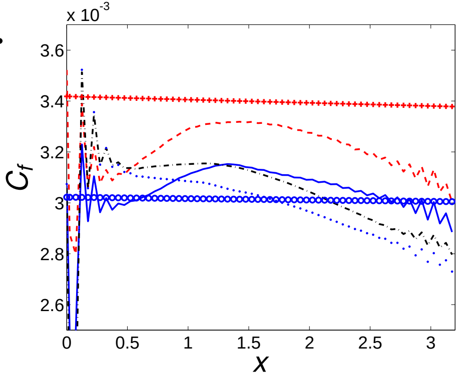
— · — w^{+2}_{rms}

· · · $\langle u'v'\rangle^{+}$

○: inlet; - - - $x = 3\delta_{in}$

BOUNDARY LAYER: VELOCITY AND SKIN FRICTION

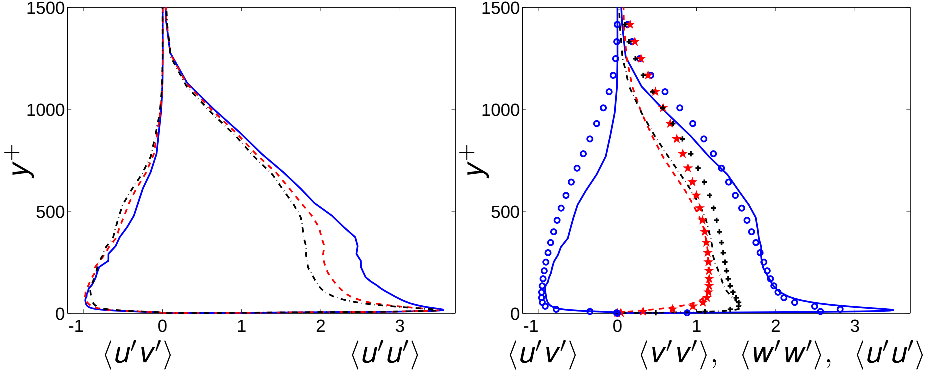




▬ $x = \delta_{in}$; - - - $x = 2\delta_{in}$; — · — $x = 3\delta_{in}$; ■: DNS [21]

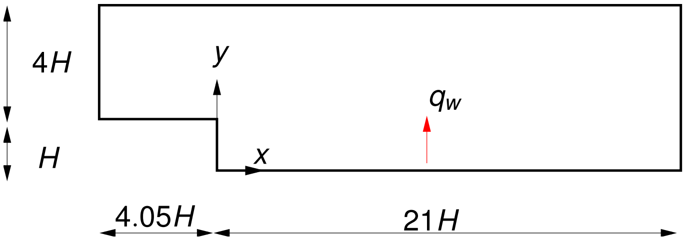
▬ 100% CDS; - - - 100% CDS, U_{in} from AKN; — · — 25% larger inlet fluct.; . . . . 25% larger inlet fluct., $C_s = 0.07$; markers: $0.37\,(log_{10}Re_x)^{-2.584}$ (+: AKN; ○: DNS)

REYNOLDS STRESSES



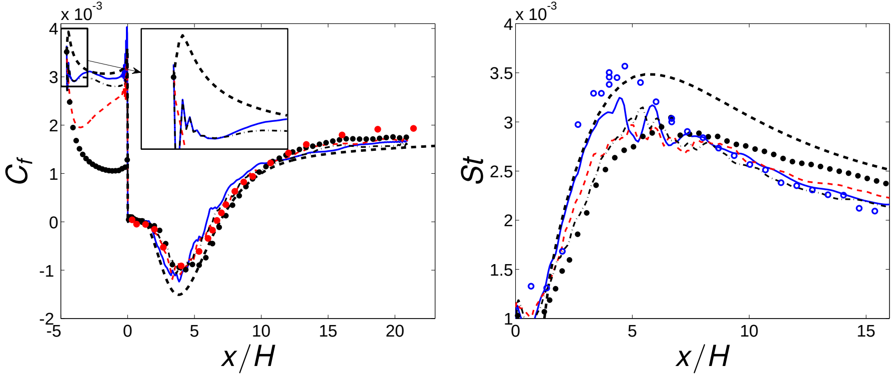
$x = \delta_{in}$; $x = 2\delta_{in}$; $x = 3\delta_{in}$; Markers: DNS [21]
$x = 3\delta_{in}$.

BACKWARD FACING STEP: DOMAIN



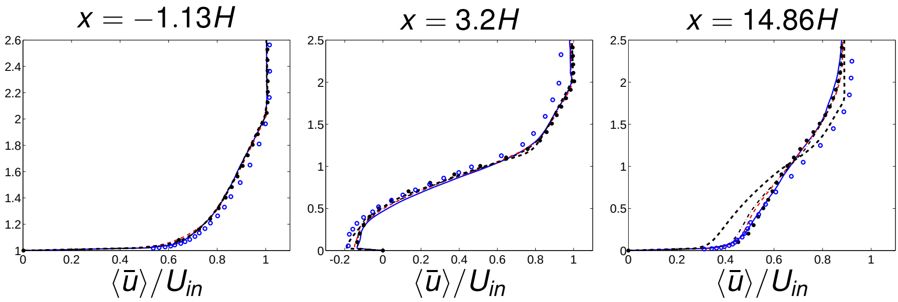
- $Re_H = 28\,000$ Experiments by Vogel & Eaton [26]
- Mean inlet profiles from RANS (same as in boundary layer)
- Grid: 336 × 120 in $x \times y$ plane. $Z_{max} = 1.6H$, $N_k = 64$, $\Delta z^+_{in} = 31$.
- Anisotropic synthetic fluctuations, u', v', w' (same as for boundary layer flow); no fluctuations for t'
- Constant heat flux, q_w, on lower wall.

BACKSTEP FLOW. SKIN FRICTION AND STANTON NUMBER



—— PANS; - - - - PANS, 50% smaller inlet fluctuations; — · — WALE; ●: PANS, no inlet fluctuations; - - - : 2D RANS; ○,●: experiments [26].

BACKSTEP FLOW: VELOCITIES.

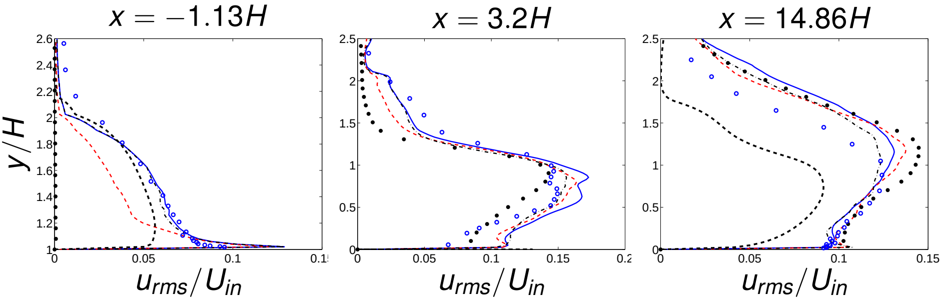


—— PANS; – – – – PANS, 50% smaller inlet fluctuations;

— · — WALE;

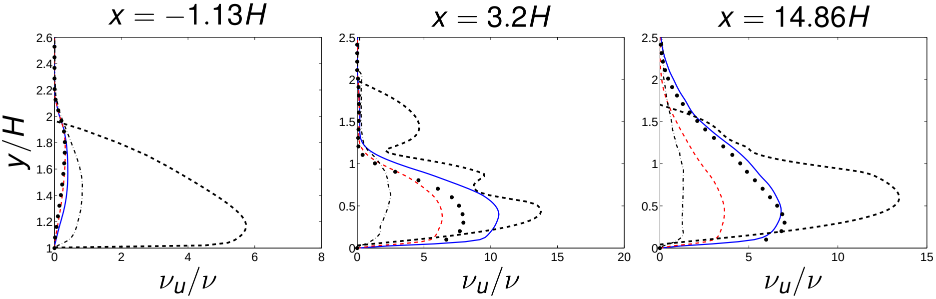
●: PANS, no inlet fluctuations; - - - : 2D RANS; ○: experiments [26].

BACKSTEP FLOW: RESOLVED STREAMWISE STRESS.



—— PANS; - - - - PANS, 50% smaller inlet fluctuations;
—·— WALE;
●: PANS, no inlet fluctuations; - - - : 2D RANS; ○: experiments [26].

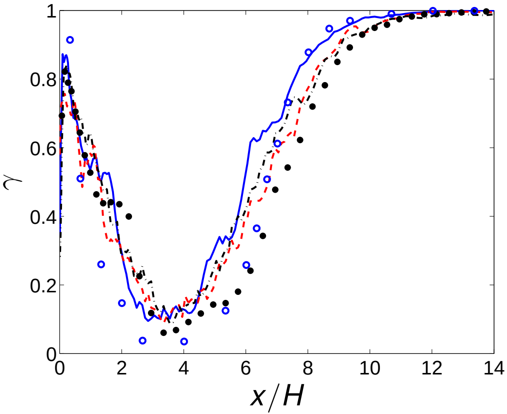
BACKSTEP FLOW: TURBULENT VISCOSITIES.



—— PANS; - - - - PANS, 50% smaller inlet fluctuations;
—·— WALE;
•: PANS, no inlet fluctuations; - - - : 2D RANS/10;

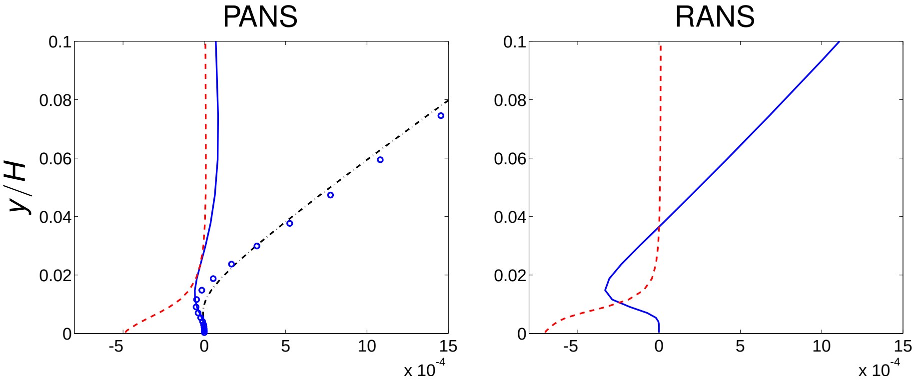
FORWARD/BACKWARD FLOW

- Fraction of time, γ, when the flow along the bottom wall is in the downstream direction.



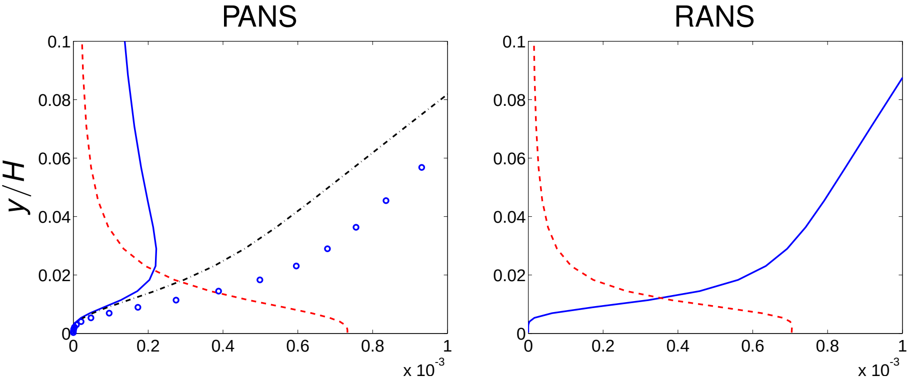
—— PANS; - - - - PANS, 50% smaller inlet fluctuations;
— · — WALE;
●: PANS, no inlet fluctuations; ○: experiments [26].

SHEAR STRESSES. $x = 3.2H$



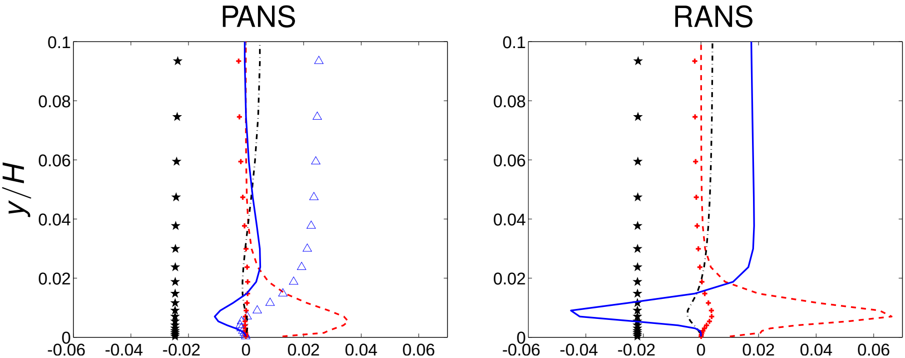
— $2\langle\nu_t\bar{s}_{12}\rangle$; - - - $\nu\frac{\partial\langle\bar{u}\rangle}{\partial y}$; —·— $-\langle\langle u'v'\rangle\rangle$; ○: $2\langle\nu_t\bar{s}_{12}\rangle - \langle\langle u'v'\rangle\rangle$.

SHEAR STRESSES. $x = 14.86$



— $2\langle\nu_t\bar{s}_{12}\rangle$; - - - $\nu\dfrac{\partial\langle\bar{u}\rangle}{\partial y}$; — · — $-\langle\langle u'v'\rangle\rangle$; ○: $2\langle\nu_t\bar{s}_{12}\rangle-\langle\langle u'v'\rangle\rangle$.

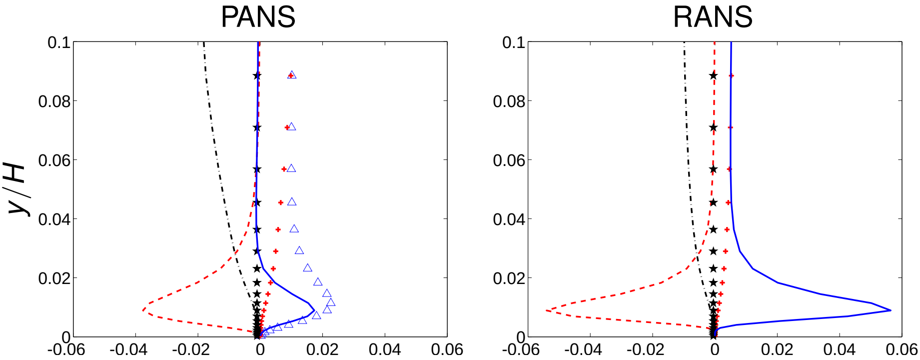
TERMS IN THE $\langle \bar{u} \rangle$ EQUATION. $x = 3.2H$



— $\frac{\partial}{\partial y}\left(2\langle \nu_t \bar{s}_{12} \rangle\right)$; - - - $\nu \frac{\partial^2 \langle \bar{u} \rangle}{\partial y^2}$; — · — $-\frac{\partial \langle \bar{u} \rangle \langle \bar{u} \rangle}{\partial x}$; +: $-\frac{\partial \langle \bar{u} \rangle \langle \bar{v} \rangle}{\partial y}$;

★: $-\frac{\partial \langle \bar{p} \rangle}{\partial x}$, △: $-\frac{\partial \langle \langle u' v' \rangle \rangle}{\partial y}$.

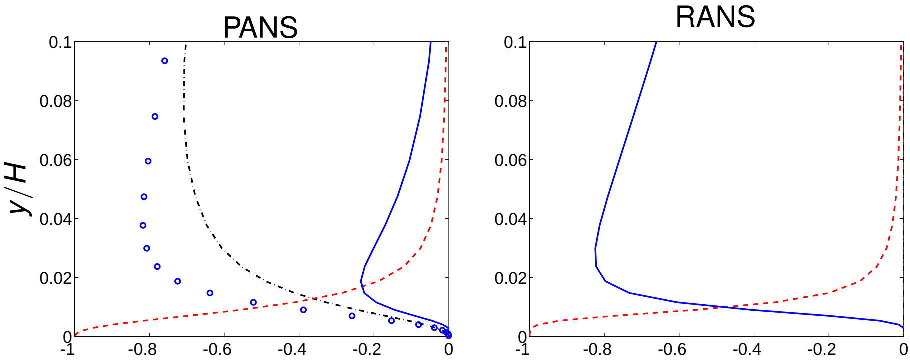
TERMS IN THE $\langle \bar{u} \rangle$ EQUATION. $x = 14.86H$



— $\dfrac{\partial}{\partial y}(2\langle \nu_t \bar{s}_{12} \rangle)$; - - - $\nu \dfrac{\partial^2 \langle \bar{u} \rangle}{\partial y^2}$; — · — $-\dfrac{\partial \langle \bar{u} \rangle \langle \bar{u} \rangle}{\partial x}$; +: $-\dfrac{\partial \langle \bar{u} \rangle \langle \bar{v} \rangle}{\partial y}$;

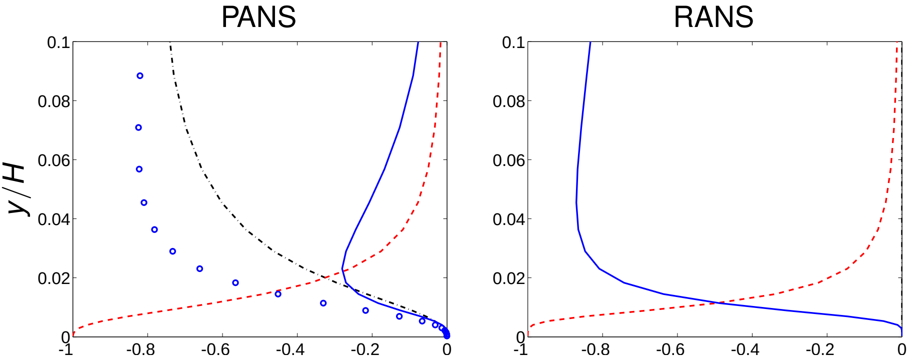
⋆: $-\dfrac{\partial \langle \bar{p} \rangle}{\partial x}$, △: $-\dfrac{\partial \langle \langle u'v' \rangle \rangle}{\partial y}$.

HEAT FLUXES. $x = 3.2H$



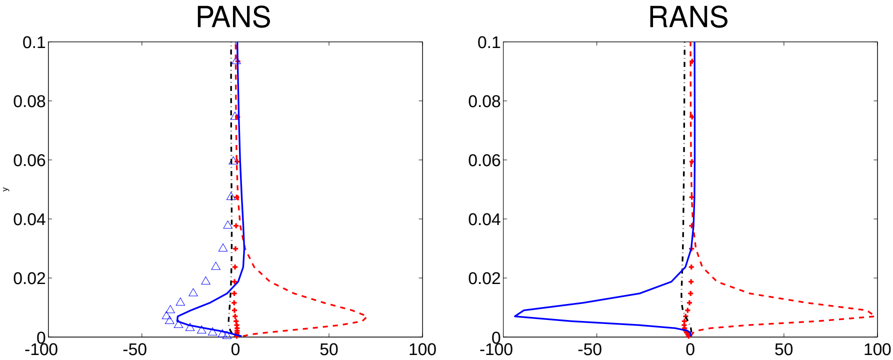
— $\left\langle \dfrac{\nu_t}{\sigma_t}\dfrac{\partial \bar{T}}{\partial y}\right\rangle$; - - - $\dfrac{\nu}{\sigma_\ell}\dfrac{\partial \langle \bar{T}\rangle}{\partial y}$; —·— $-\langle\overline{v\theta}\rangle$. ○: $\left\langle \dfrac{\nu_t}{\sigma_t}\dfrac{\partial \bar{T}}{\partial y}\right\rangle - \langle\overline{v\theta}\rangle$.

HEAT FLUXES. $x = 14.86H$



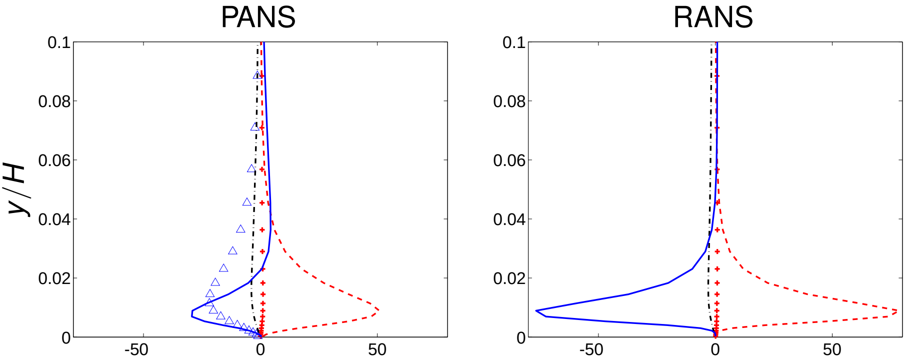
— $\left\langle \frac{\nu_t}{\sigma_t}\frac{\partial \bar{T}}{\partial y}\right\rangle$; - - - - $\frac{\nu}{\sigma_\ell}\frac{\partial\langle\bar{T}\rangle}{\partial y}$; —·— $-\langle\overline{v\theta}\rangle$. ○: $\left\langle \frac{\nu_t}{\sigma_t}\frac{\partial \bar{T}}{\partial y}\right\rangle - \langle\overline{v\theta}\rangle$.

TERMS IN THE $\langle \bar{T} \rangle$ EQUATION. $x = 3.2H$



— $\frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_t}\frac{\partial \langle \bar{T} \rangle}{\partial y}\right)$; - - - $\frac{\nu}{\sigma_\ell}\frac{\partial^2 \langle \bar{T} \rangle}{\partial y^2}$; —·— $-\frac{\partial \langle \bar{u} \rangle \langle \bar{T} \rangle}{\partial x}$; +: $-\frac{\partial \langle \bar{v} \rangle \langle \bar{T} \rangle}{\partial y}$; △: $-\frac{\partial \langle \overline{v\theta} \rangle}{\partial y}$.

TERMS IN THE $\langle\bar{T}\rangle$ EQUATION. $x = 14.86H$



— $\frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_t}\frac{\partial\langle\bar{T}\rangle}{\partial y}\right)$; - - - $\frac{\nu}{\sigma_\ell}\frac{\partial^2\langle\bar{T}\rangle}{\partial y^2}$; —·— $-\frac{\partial\langle\bar{u}\rangle\langle\bar{T}\rangle}{\partial x}$; +: $-\frac{\partial\langle\bar{v}\rangle\langle\bar{T}\rangle}{\partial y}$; △: $-\frac{\partial\langle\overline{v\theta}\rangle}{\partial y}$.

CONCLUDING REMARKS

- Developing boundary layer
  - Synthetic fluctuations give fully developed conditions after a couple of boundary layer thicknesses
  - 5% upwinding dampens resolved fluctuations; can be compensated by 25% larger inlet fluctuations
- Backstep flow
  - Very good agreement with experiments
  - 2D RANS predicts turbulent diffusion surprisingly well
  - Synthetic inlet fluctuations give an improved Stanton number; otherwise small effect in the reciculation region
  - LRN PANS and WALE equally good
  - 5% upwinding has a negligble effect in the recirculation region

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