LES, HYBRID LES-RANS AND SCALE-ADAPTIVE SIMULATIONS (SAS)

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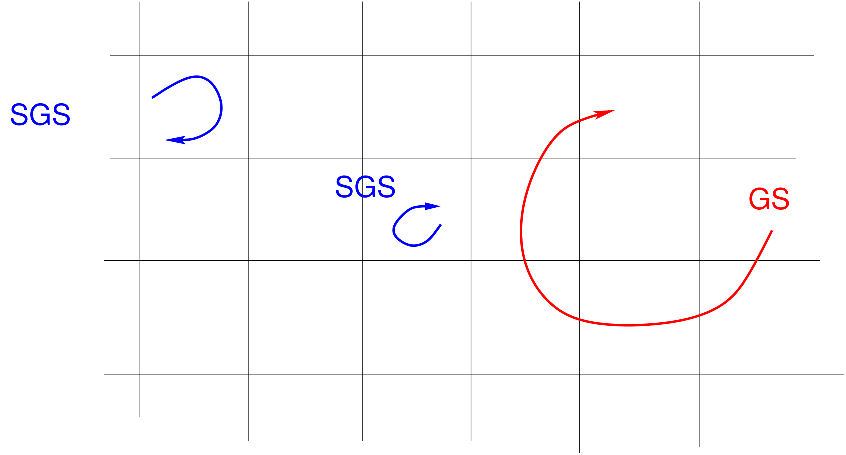
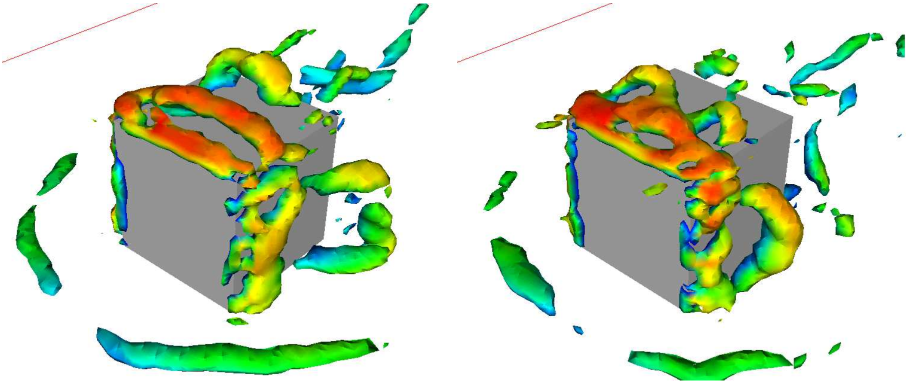
LARGE EDDY SIMULATIONS



- In LES, large (Grid) Scales (GS) are resolved and the small (Sub-Grid) Scales (SGS) are modelled.
- LES is suitable for bluff body flows where the flow is governed by large turbulent scales

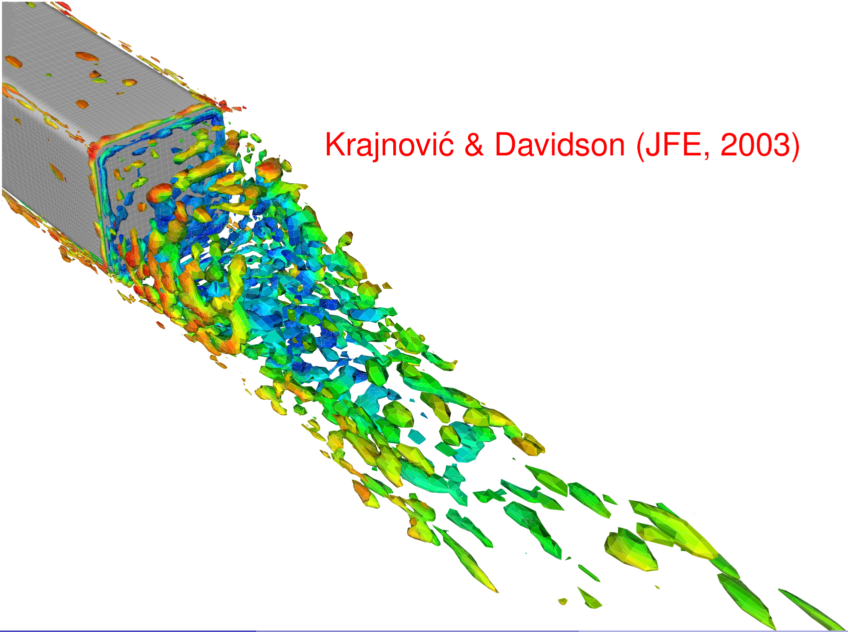
BLUFF-BODY FLOW: SURFACE-MOUNTED CUBE[1]

Krajnović & Davidson (AIAA J., 2002)



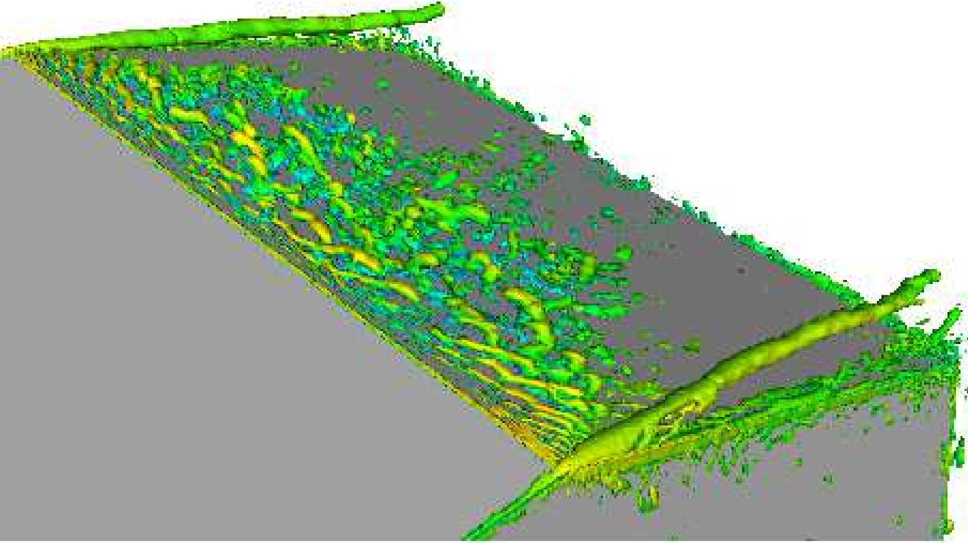
Snapshots of large turbulent scales illustrated by $Q = -\frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_i}$

BLUFF-BODY FLOW: FLOW AROUND A BUS[2]

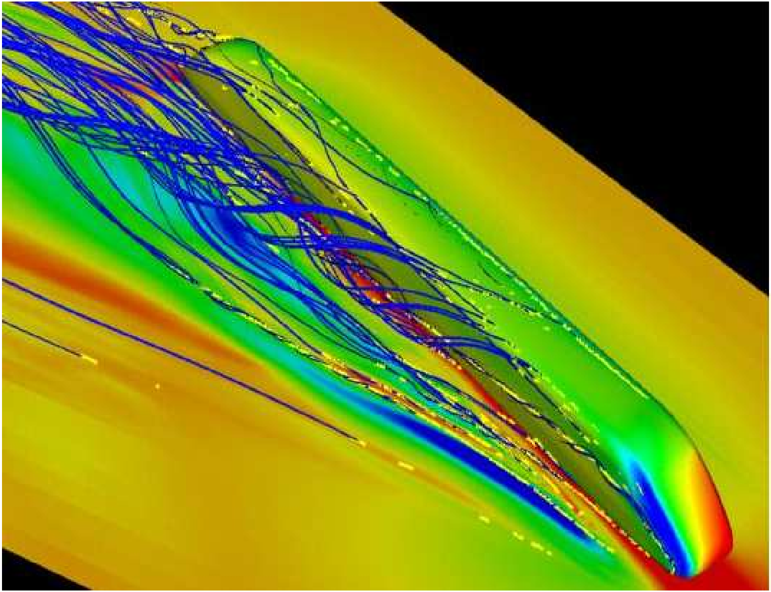


Krajnović & Davidson (JFE, 2003)

BLUFF-BODY FLOW: FLOW AROUND A CAR[3]

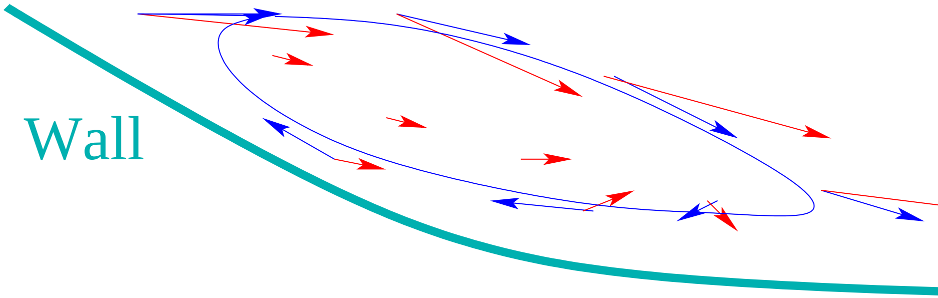


BLUFF-BODY FLOW: FLOW AROUND A TRAIN[4]



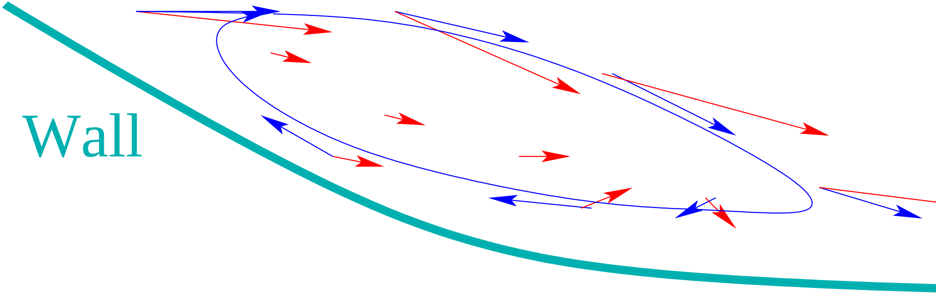
Hemida & Krajnović, 2006

SEPARATING FLOWS



- TIME-AVERAGED flow and INSTANTANEOUS flow

Separating Flows



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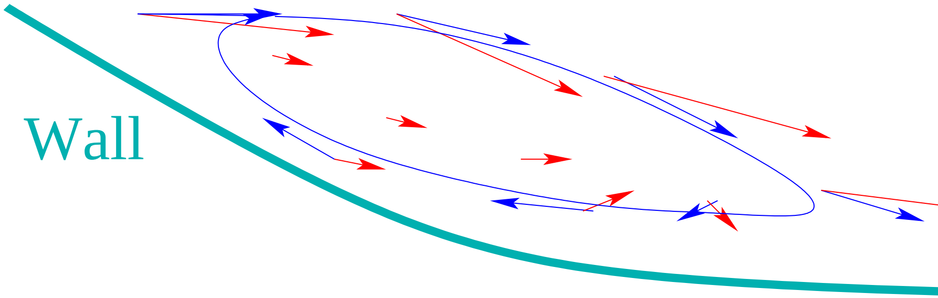

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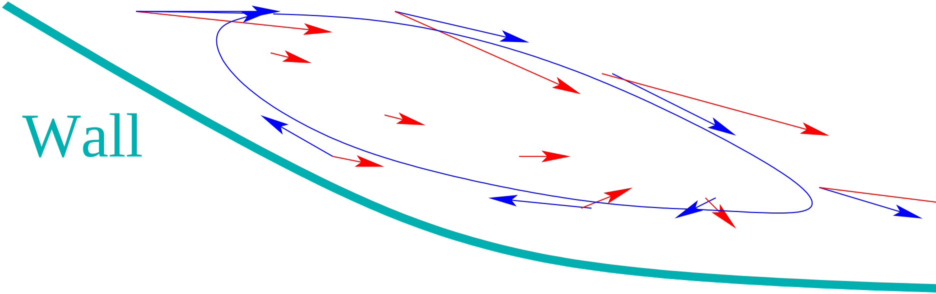

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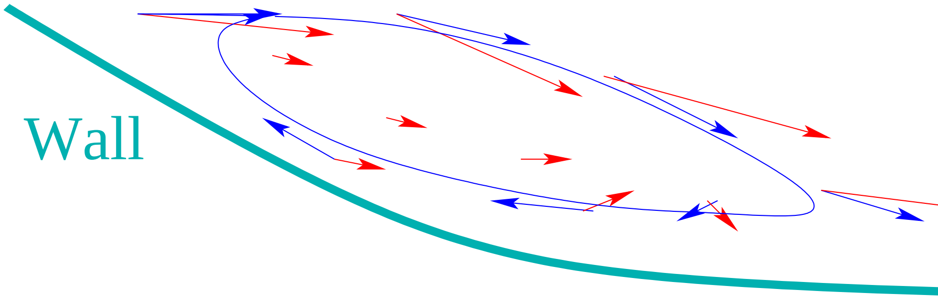

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- How easy is it to model fluctuations that are as large as the mean flow?
- Is it reasonable to require a turbulence model to fix this?
- Isn't it better to RESOLVE the large fluctuations?

NEAR-WALL TREATMENT

- Biggest problem with LES: near walls, it requires very fine mesh in all directions, not only in the near-wall direction.

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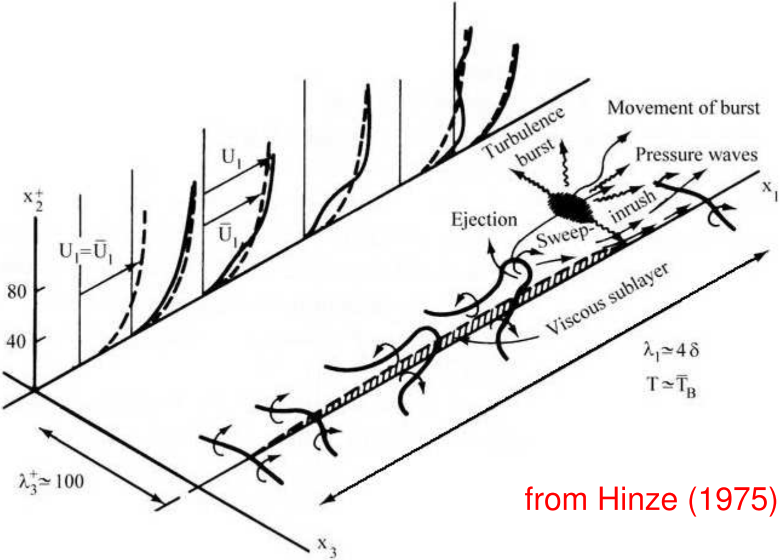
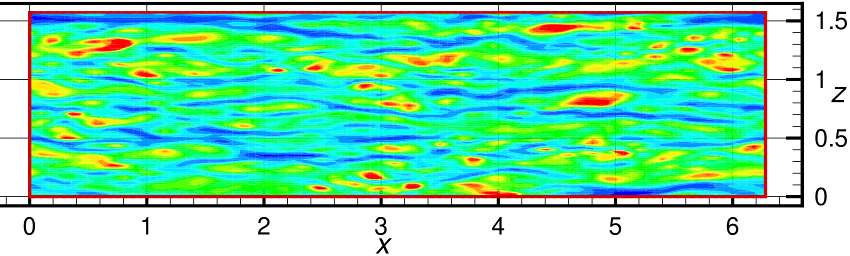
- In the presentation we use Hybrid LES-RANS for which the grid requirements are much smaller than for LES

Near-Wall Treatment



from Hinze (1975)

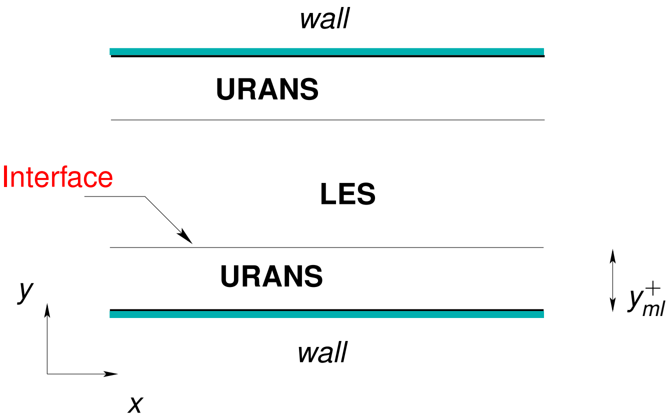
Near-Wall Treatment



- Fluctuating streamwise velocity at $y^+ = 5$. DNS of channel flow.
- We find that the structures in the spanwise direction are very small which requires a very fine mesh in z direction.

HYBRID LES-RANS

Near walls: a RANS one-eq. k or a $k-\omega$ model.
In core region: a LES one-eq. k_{SGS} model.



- Location of interface either pre-defined or automatically computed

Momentum Equations

• The Navier-Stokes, time-averaged in the near-wall regions and filtered in the core region, reads

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_i \bar{u}_j\right) = \beta \delta_{1i} - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\frac{\partial \bar{u}_i}{\partial x_j}\right]$$

$$\nu_T = \nu_t, y \leq y_{ml}$$

$$\nu_T = \nu_{sgs}, y \geq y_{ml}$$

• The equation above: URANS or LES? Same boundary conditions ⇒ same solution!

Turbulence Model

- Use one-equation model in both URANS region and LES region.

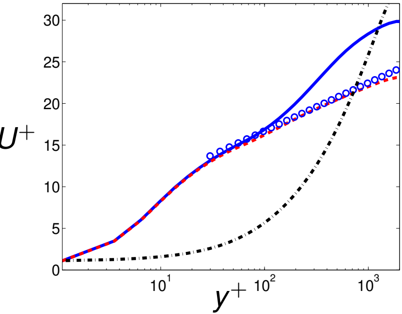
$$\frac{\partial k_T}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_j k_T) = \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\frac{\partial k_T}{\partial x_j}\right] + P_{k_T} - C_\varepsilon \frac{k_T^{3/2}}{\ell}$$

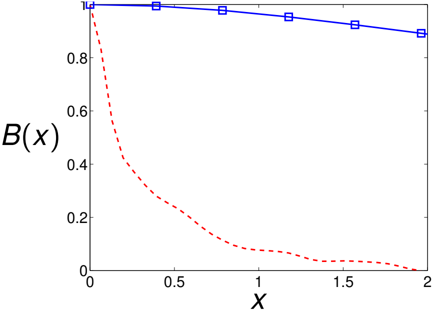
$$P_{k_T} = 2\nu_T \bar{S}_{ij}\bar{S}_{ij},\ \nu_T = C_k \ell k_T^{1/2}$$

  - LES-region: $k_T = k_{sgs}$, $\nu_T = \nu_{sgs}$, $\ell = \Delta = (\delta V)^{1/3}$
  - URANS-region: $k_T = k$, $\nu_T = \nu_t$, $\ell \equiv \ell_{RANS} = 2.5n[1 - \exp(-Ak^{1/2}y/\nu)]$, Chen-Patel model (AIAA J. 1988)
  - Location of interface can be defined by $\min(0.65\Delta, y)$, $\Delta = \max(\Delta x, \Delta y, \Delta z)$

Standard Hybrid LES-RANS

- Coarse mesh: $\Delta x^+ = 2\Delta z^+ = 785$. $\delta/\Delta x \simeq 2.5$, $\delta/\Delta z \simeq 5$.





standard LES-RANS; DNS; LES
$\circ$ $0.4\ln(y^+) + 5.2$

$$B(x) = \frac{\langle u(x_0)u(x-x_0)\rangle}{u_{rms}u_{rms}}$$

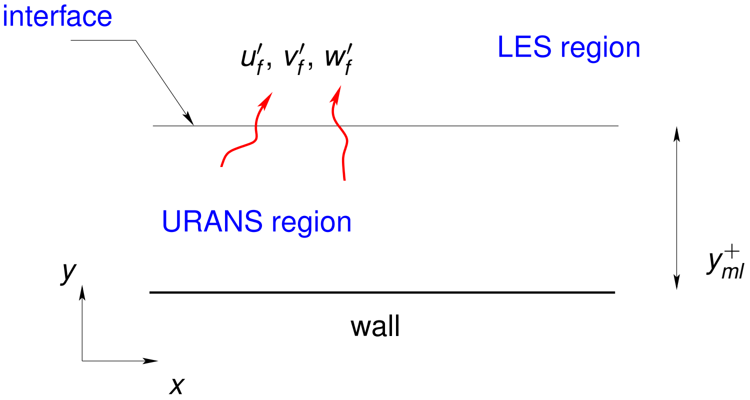
- Too high velocity because too low shear stress

Ways to Improve the RANS-LES Method[5, 6, 7]

- The reason is that LES region is supplied with bad boundary (i.e. interface) conditions by the URANS region.
- The flow going from the RANS region into the LES region has no proper turbulent length or time scales
- New approach: Synthesized isotropic turbulent fluctuations are added as momentum sources at the interface.
- The superimposed fluctuations should be regarded as forcing functions rather than boundary conditions.

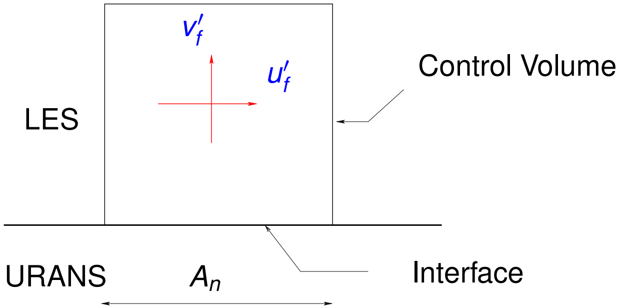
Forcing Fluctuations Added at the Interface

- Object: to trig the momentum equations into resolving large-scale turbulence



- For more info, see Davidson at al. [5, 7]

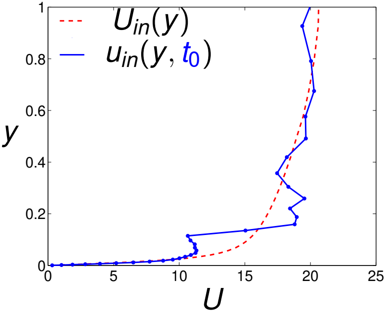
Implementation

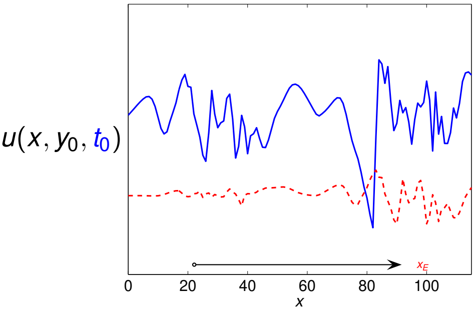


- Fluctuations u'_f, v'_f, w'_f are added as sources in all three momentum equations. The source is $-\gamma\rho u'_{i,f}u'_{2,f}A_n = -\gamma\rho u'_{i,f}u'_{2,f}V/\Delta y$ (A_n=area, V=volume of the C.V.)
- The source is scaled with $\gamma = k_T/k_{synt}$

INLET BOUNDARY CONDITIONS

U_{inlet} constant in time; u_{inlet} function of time.



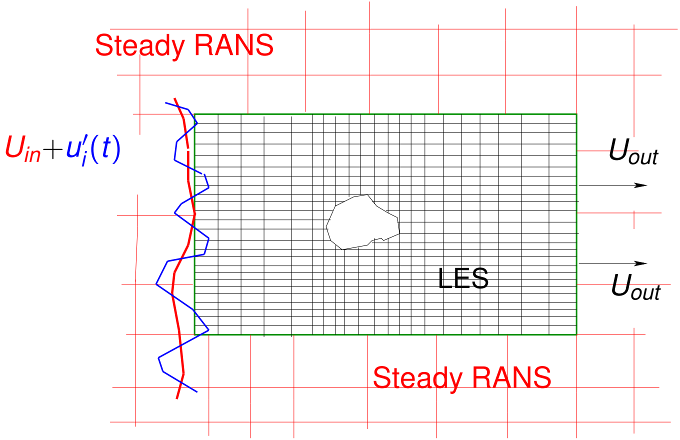


Left: Inlet boundary profiles

Right: Evolution of u velocity depending of type of inlet B.C.

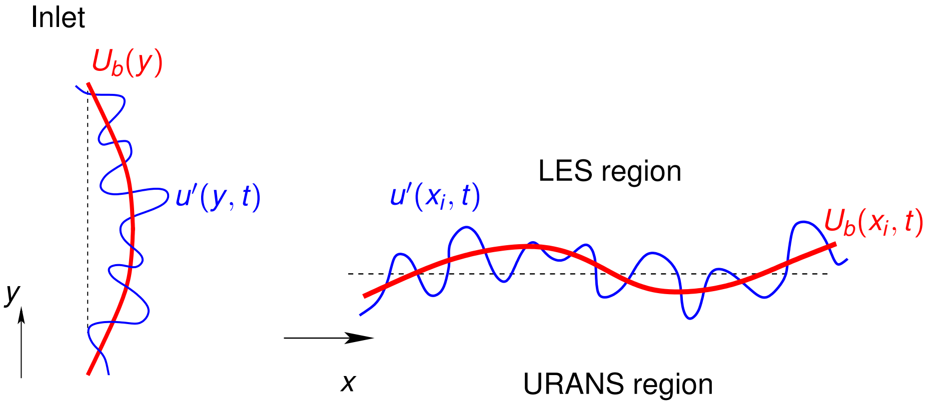
- With steady inlet B.C., u gets turbulent first at $x = x_E$.

EMBEDDED LES (BLUFF BODY FLOWS)

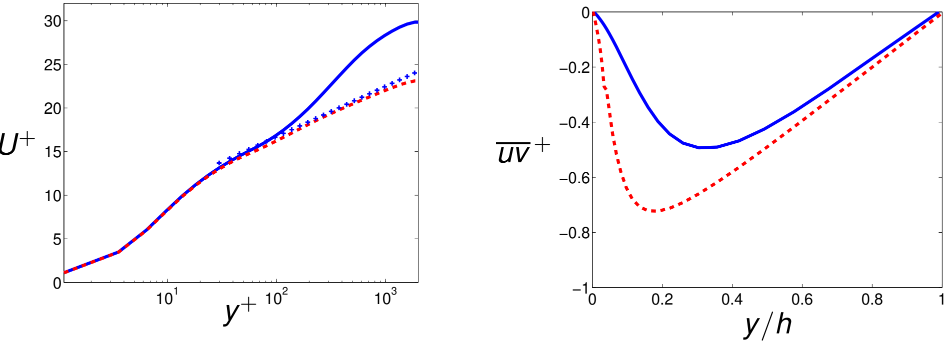


- $U_{in}+u_i'(t)$ used as B.C. for LES in the inner region.
- Examples of inner region: external mirror of a car; a flap/slat; a detail of a landing gear. Often in connection with aero-acoustics.

Inlet Boundary Conditions vs. Forcing



Fully Developed Channel Flow (periodic in x)



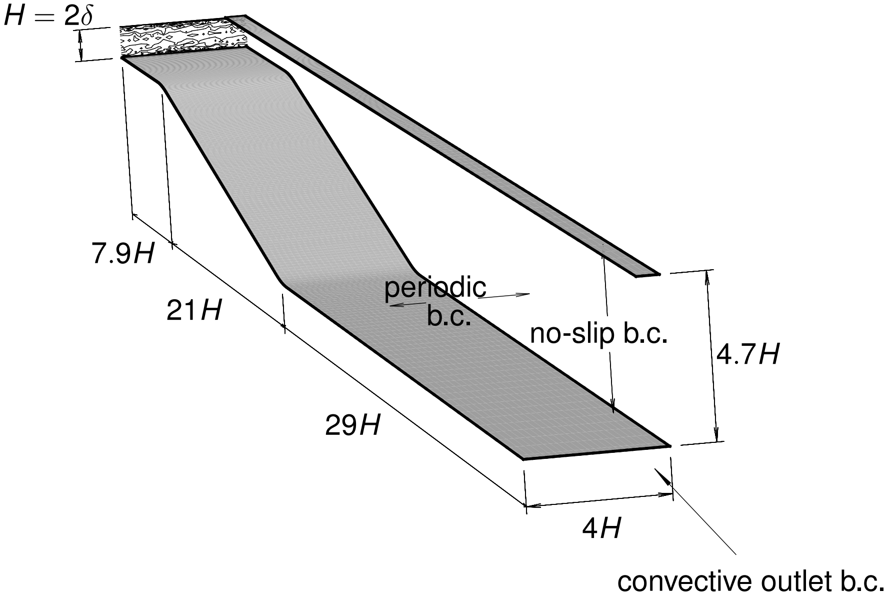
no forcing; forcing (isotropic fluctuations)

$\circ$ $0.4\ln(y^+) + 5.2$

DIFFUSER[5]

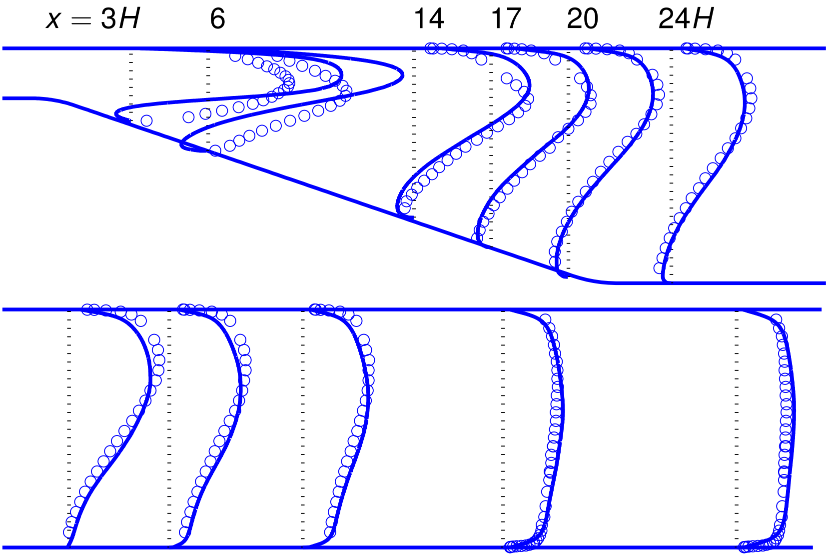
- Instantaneous inlet data from channel DNS used.
- Domain: $-8 \le x \le 48$, $0 \le y_{inlet} \le 1$, $0 \le z \le 4$.
- $x_{max} = 40$ gave return flow at the outlet
- Grid: $258 \times 66 \times 32$.
- $Re = U_{in}H/\nu = 18\,000$, angle 10^o
- The grid is much too coarse for LES (in the inlet region $\Delta z^+ \simeq 170$)
- Matching plane fixed at y_{ml} at the inlet. In the diffuser it is located along the 2D instantaneous streamline corresponding to y_{ml}.

DIFFUSER GEOMETRY. $Re = 18\ 000$, ANGLE 10^o

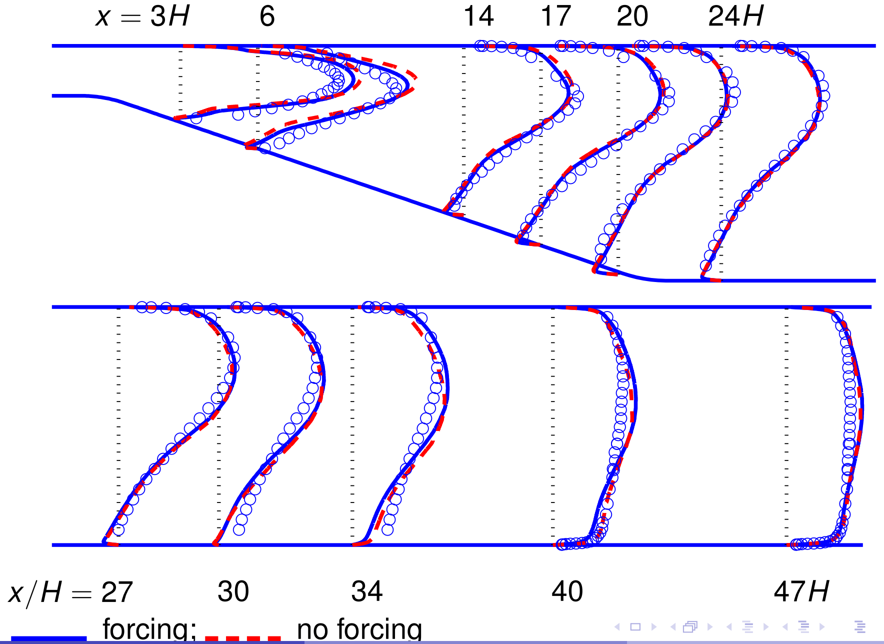


Diffuser: Results with LES

- Velocities. Markers: experiments by Buice & Eaton (1997)



DIFFUSER: RESULTS WITH NEW RANS-LES



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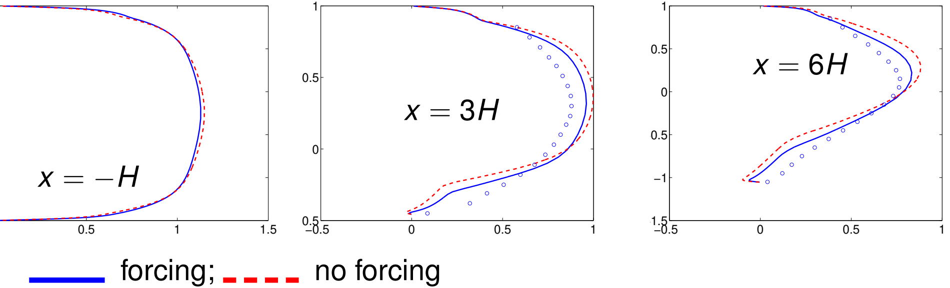
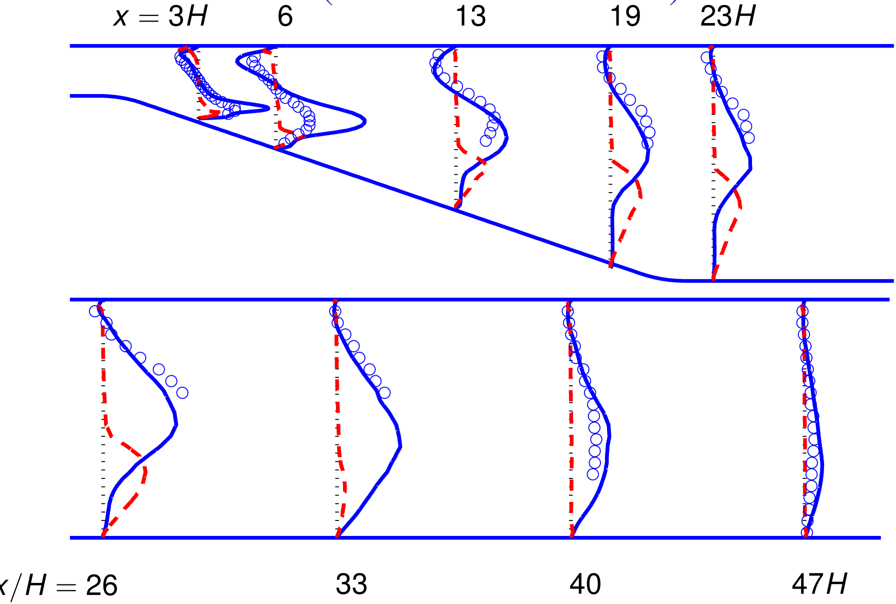
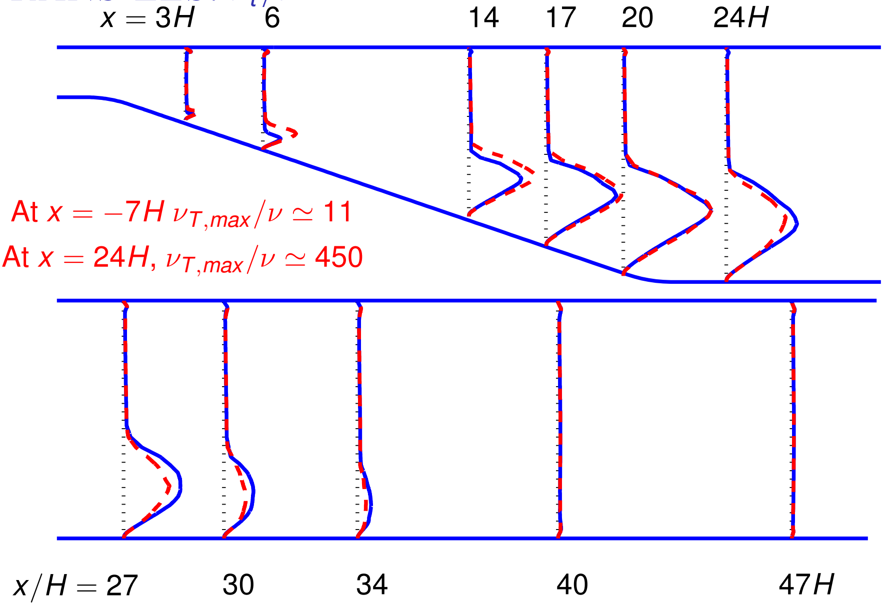


Shear Stresses (×2 In Lower Half)



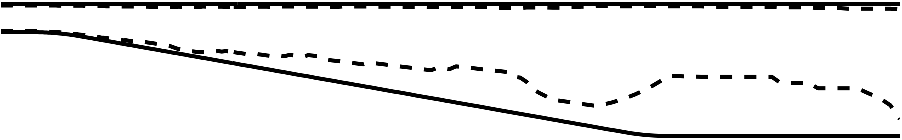
——— resolved; - - - modelled

RANS-LES: ν_t/ν



forcing; without forcing

RANS-LES: Location Of Matching Line

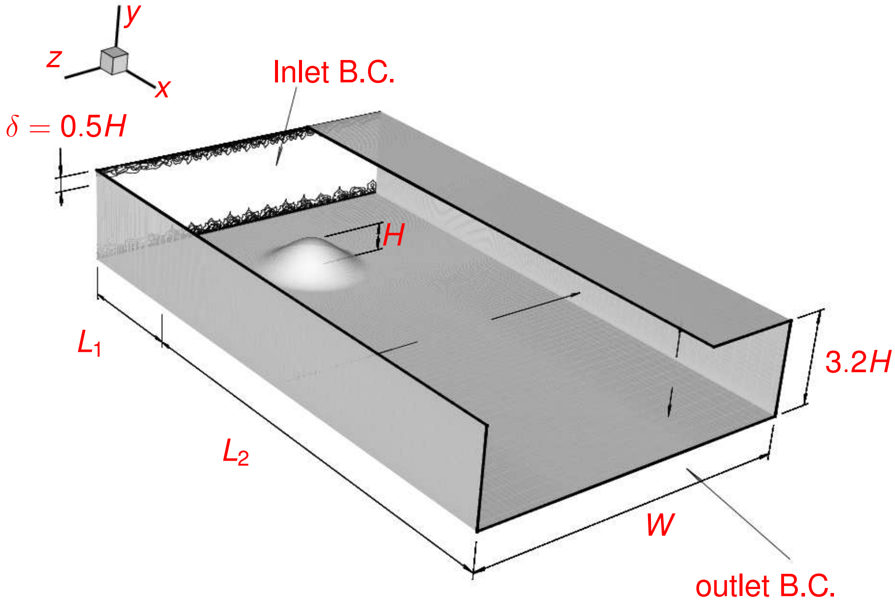


• Location of matching line. It is defined along 2D instantaneous streamline (defined by mass flow).

$$U_{b,in,k} y_{ml,in,k} \Delta z = \sum_{2}^{j_{ml,i,k}} (\bar{u}_e A_{e,x} + \bar{v}_e A_{e,y})$$

- This approach has successfully been used for asymmetric plane diffuser as well as 3D hill (Simpson & Byun)
- Other option: $\min(0.65\Delta, y)$, $\Delta = \max(\Delta x, \Delta y, \Delta z)$

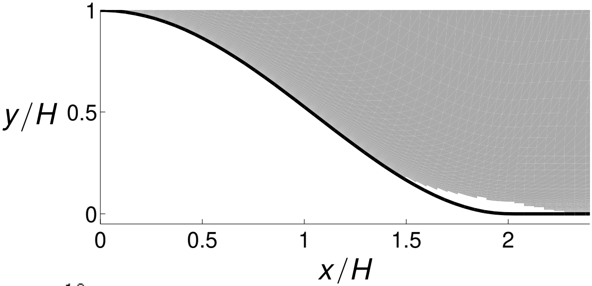
3D-HILL



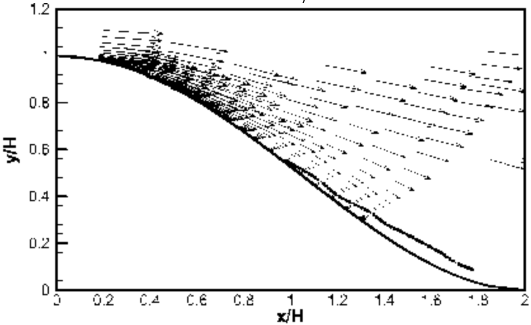
Numerical Method

- Implicit, finite volume (collocated),
- Central differencing in space and time (Crank-Nicolson ($\alpha = 0.6$))
- Efficient multigrid solver for the pressure Poisson equation
- CPU/time step 25 seconds on a single AMD Opteron 244
- Time step $\Delta t U_{in}/H = 0.026$. Mesh $160 \times 80 \times 128$
- $8\,000 + 8\,000$ time steps for fully developed+averaging ($10 + 10$ through flow or $T^* = TU_b/H = 200 + 200$)
- One simulation ($8\,000 + 8\,000$) takes one week

Symmetry Plane $z = 0$

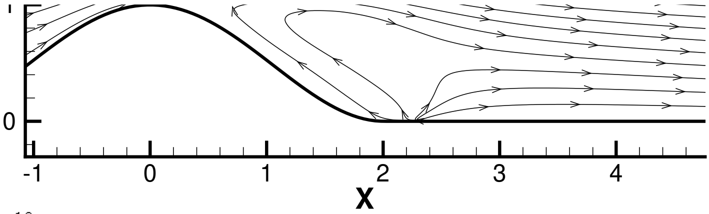


Hybrid LES-RANS

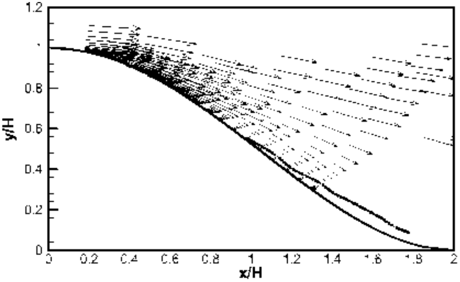


Experiments

3D HILL: RANS



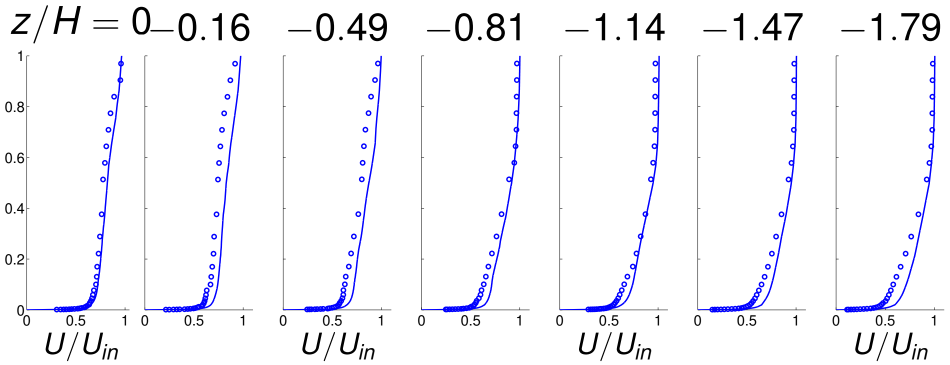
RANS, SST



Experiments

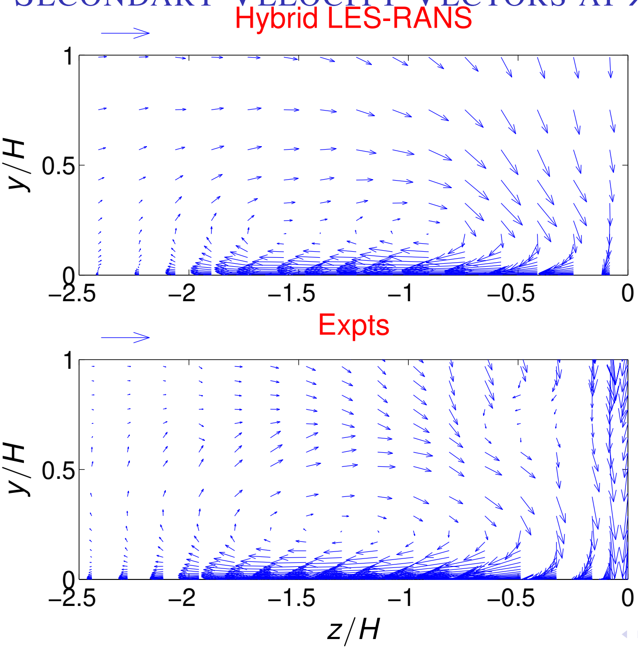
- Similar results obtained with all other RANS models ($k - \omega$, Low-Re RSM, EARSM, SA-model etc) [9].

STREAMWISE PROFILES AT $x = 3.69H$ [8]



Hybrid LES-RANS; ○ Experiments

SECONDARY VELOCITY VECTORS AT $x = 3.69H$



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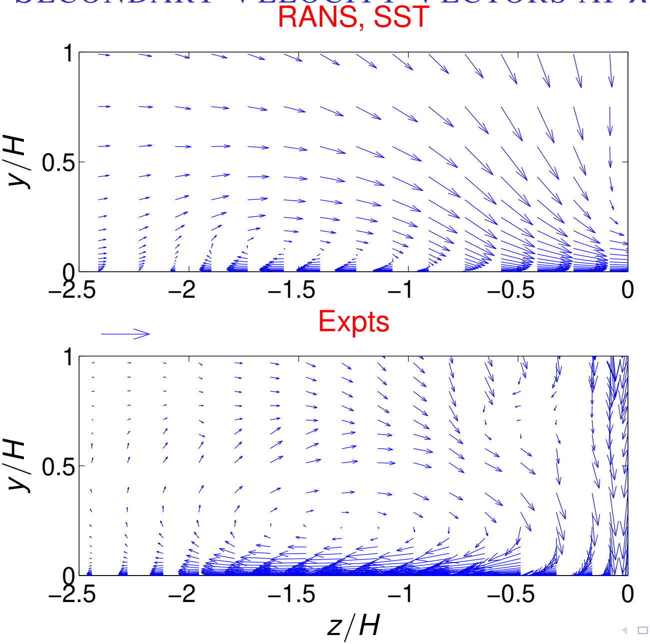
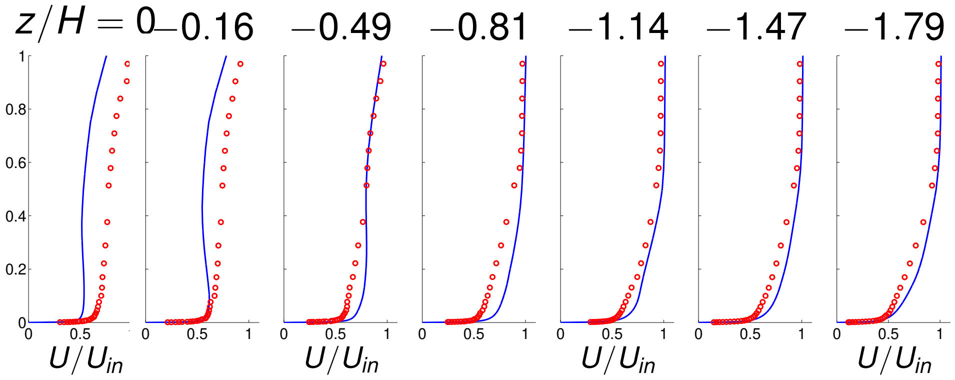


RANS SST: Streamwise Profiles at $x = 3.69H$



RANS-SST; ○ Experiments

3D Hill: Summary

- All RANS models give a completely incorrect flow field
- LES and hybrid LES-RANS in good agreement with expts.
- Mesh sizes

RANS	0.5 – 1.2 million (half of the domain)
Hybrid LES-RANS	1.7 million

- CPU times

RANS, EARSM	1 – 2 days	1-CPU DEC-Alpha
LES-RANS	1 week (10+10 T-F)*	1-CPU Opteron 244

* T-F=Through-Flows

- Hybrid LES-RANS results in Ref. [8]

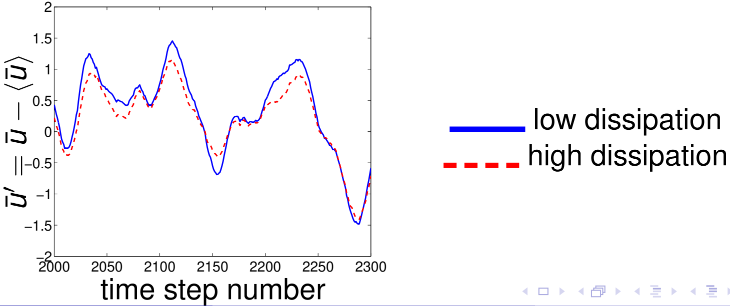
MODELLED DISSIPATION, ε_M

- The unsteady Navier-Stokes reads

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_i \bar{u}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right]$$

The turbulent viscosity, ν_T, dampens the fluctuations, via the modelled dissipation, ε_M, which reads

$$\varepsilon_M = -\tau_{ij}\frac{\partial \bar{u}_i}{\partial x_j} = \langle 2\nu_T \bar{s}_{ij}\bar{s}_{ij}\rangle, \tau_{ij} = -2\nu_t \bar{s}_{ij} + \frac{2}{3}\delta_{ij}k, \bar{s}_{ij} = 0.5\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$$

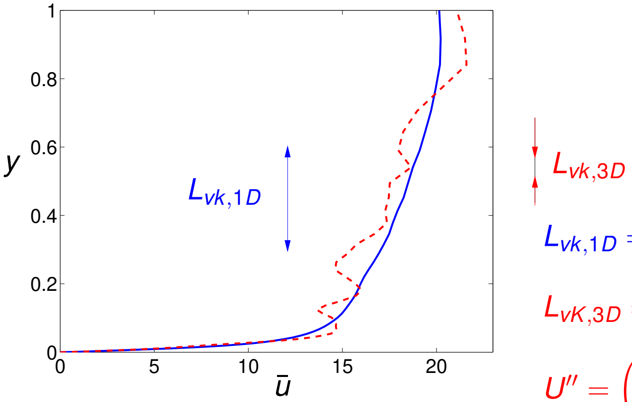


Steady vs. Unsteady Regions

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_i \bar{u}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right]$$

- OBJECT:
  - In regions of fine grid: turbulence resolved by $\bar{u}'_i$, i.e. $\dfrac{\partial \bar{u}_i}{\partial t}$
  - In regions of coarse grid: turbulence modelled by ν_T
- PROBLEM: in fine-grid regions, ν_T increases too much which kills $\bar{u}'_i$
- SOLUTION: when $\bar{u}'_i$ starts to grow, reduce ν_T

von Kármán Length Scale



$$L_{vk,1D} = \kappa \frac{\partial \langle \bar{u} \rangle / \partial y}{\partial^2 \langle \bar{u} \rangle / \partial y^2}$$

$$L_{vK,3D} = \kappa \frac{\bar{s}}{|U''|}, \quad \bar{s} = (2\bar{s}_{ij}\bar{s}_{ij})^{1/2}$$

$$U'' = \left(\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \right)^{0.5}$$

- The von Kármán detects unsteadiness (i.e. resolved turbulence, $\bar{u}'_i$) and reduces the length scale

THE SAS TURBULENCE MODEL[10, 11, 12]

$$\frac{Dk}{Dt} - \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] = \nu_t \bar{s}^2 - c_1 k\omega$$

$$\underbrace{\frac{D\omega}{Dt} - \left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]}_{\text{transport}} = c_2 \bar{s}^2 - c_3\omega^2 + P_{SAS}$$

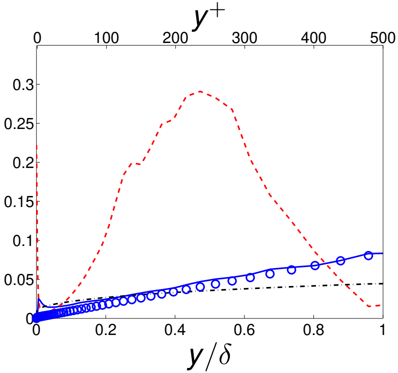
$$\nu_t = c_4\frac{k}{\omega}, \quad P_{SAS} = c_5\frac{L}{L_{vK,3D}}, \quad L_{vK,3D} = c_6\frac{\bar{s}}{U''}$$

- Fine grid $\Rightarrow$ unsteadiness $\Rightarrow$ small $L_{vK,3D}$ $\Rightarrow$ large P_{SAS} $\Rightarrow$ large ω $\Rightarrow$ small k and low ν_t

- SAS: Scale-Adapated Simulation

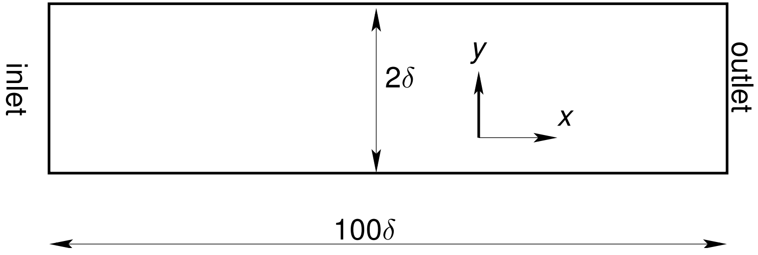
SAS: Evaluation from DNS Channel Data

- $Re_\tau = 500$, $\Delta x^+ = 50$, $\Delta z^+ = 12$, $y^+_{min} = 0.3$



$\kappa\langle \bar{s}/U'' \rangle$ $\quad$ $\kappa \left| \frac{\partial \langle U \rangle / \partial y}{\partial^2 \langle U \rangle / \partial y^2} \right|$ $\quad$ $(\Delta V)^{1/3}$ $\quad$ $\circ$ Δy

DOMAIN, $Re_\tau = u_\tau\delta/\nu = 2000$ ($Re_b \simeq 80\,000$)



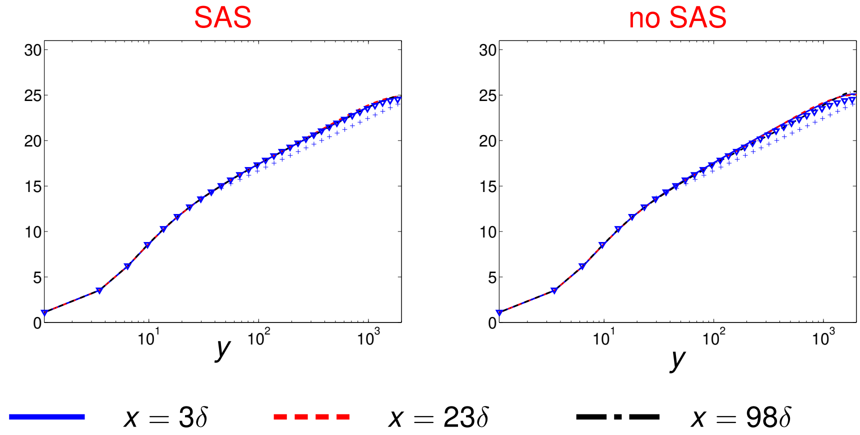
- $256 \times 64 \times 32$ (x, y, z) cells. $z_{max} = 6.3\delta$, $\Delta x^+ \simeq 785$, $\Delta z^+ \simeq 393$.
- $\delta/\Delta z \simeq 5$, $\delta/\Delta x \simeq 2.5$
- MODELS: SAS and no SAS

Channel With Inlet-Outlet

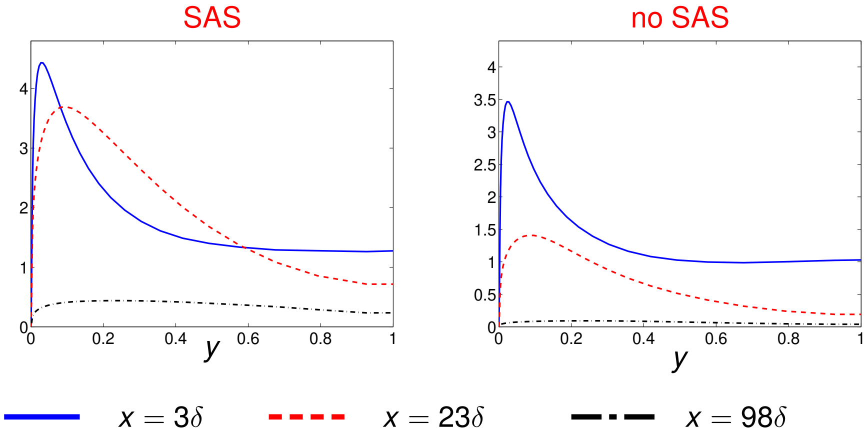
- Synthesized inlet fluctuations $(\mathcal{U}')^m$, $(\mathcal{V}')^m$, $(\mathcal{W}')^m$ with time scale $\mathcal{T} = 0.2\delta/u_\tau$ and length scale $\mathcal{L} = 0.1\delta$.

- The streamwise fluctuations are superimposed to a mean profile obtained from 1D channel flow with $k-\omega$ model

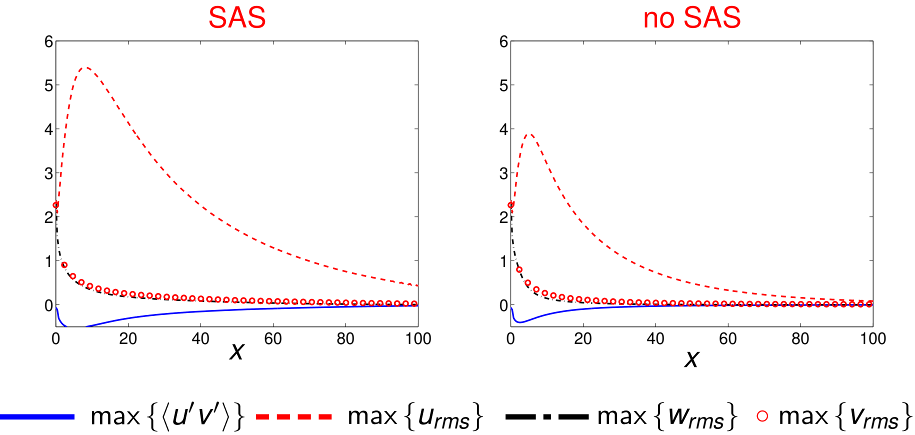
Mean Velocity



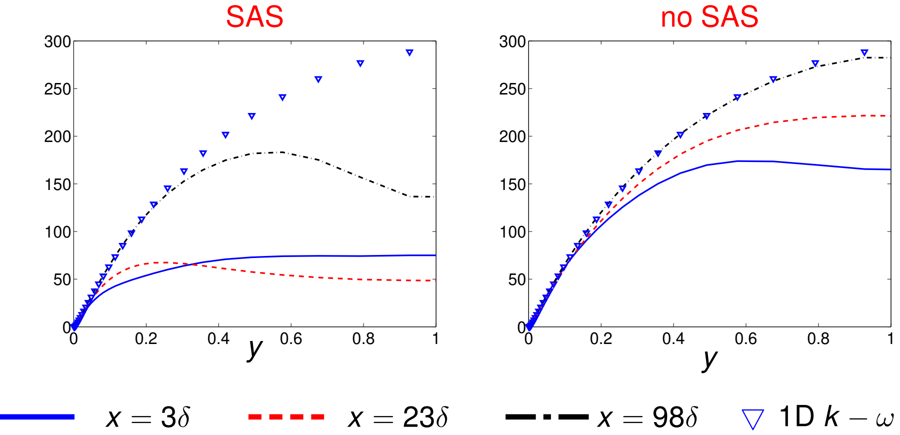
RESOLVED URMS



Peak Resolved Fluctuations



Turbulent Viscosity $\langle \nu_t \rangle / \nu$



Evaluation of the Second Derivative

- Option I: (used) compute the first derivatives at the faces

$$\left(\frac{\partial u}{\partial y}\right)_{j+1/2} = \frac{u_{j+1}-u_j}{\Delta y}, \qquad \left(\frac{\partial u}{\partial y}\right)_{j-1/2} = \frac{u_j-u_{j-1}}{\Delta y}$$

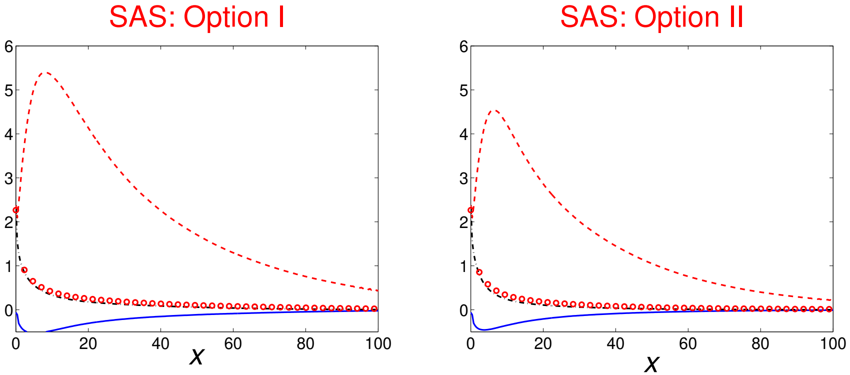
$$\Rightarrow \left(\frac{\partial^2 u}{\partial y^2}\right)_j = \frac{u_{j+1}-2u_j+u_{j-1}}{(\Delta y)^2} + \frac{(\Delta y)^2}{12}\frac{\partial^4 u}{\partial y^4}$$

- Option II: compute the first derivatives at the centre

$$\left(\frac{\partial u}{\partial y}\right)_{j+1} = \frac{u_{j+2}-u_j}{2\Delta y}, \qquad \left(\frac{\partial u}{\partial y}\right)_{j-1} = \frac{u_j-u_{j-2}}{2\Delta y}$$

$$\Rightarrow \left(\frac{\partial^2 u}{\partial y^2}\right)_j = \frac{u_{j+2}-2u_j+u_{j-2}}{4(\Delta y)^2} + \frac{(\Delta y)^2}{3}\frac{\partial^4 u}{\partial y^4}$$

SECOND DERIVATIVES



max $\{\langle u'v'\rangle\}$ max $\{u_{rms}\}$ max $\{w_{rms}\}$ ○ max $\{v_{rms}\}$

SAS: Conclusions

- SAS: A model which controls the modelled dissipation, ε_M, has been presented
- It detects unsteadiness and then reduces ε_M
- In this way the model let the equations resolve the turbulence instead of modelling it
- The results is improved accuracy because of less modelling
- More details in [13]

Conclusions

- Flows with large turbulence fluctuations difficult to model with RANS models because $u' \simeq \bar{u}$
- Unsteady methods (URANS, DES, SAS, Hybrid LES-RANS, LES) are increasingly being used in universities as well as in industry
- LES is a suitable method for bluff body flows
- Methods based on a mixture of LES and RANS are likely to be the methods of the future
- For boundary layers ($Re_x \to \infty$) some kind of forcing needed when going from (U)RANS region to LES region
- Fluctuating inlet boundary conditions can be regarded as a special case of forcing

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