Inlet boundary conditions for two-equation hybrid LES-RANS models [2]
Lars Davidson

Lars Davidson, www.tfd.chalmers.se/~lada

Go4Hybrid, Final meeting, Berlin, 2015

Research Question

1. I want to use a $k - \omega$ DES model
    1.1 How do I prescribe inlet values on k and ω?
    1.2 What about the URANS region? Should I prescribe k and ω from a steady RANS solution?
2. The proposed method is to add commutation terms in the k and ω equations.
3. The commutation terms read (Δ goes from Δ_{RANS} to Δ_{LES})

Research Question

1. I want to use a $k - \omega$ DES model
   1.1 How do I prescribe inlet values on k and ω?
   1.2 What about the URANS region? Should I prescribe k and ω from a steady RANS solution?
2. The proposed method is to add commutation terms in the k and ω equations.
3. The commutation terms read (Δ goes from Δ_{RANS} to Δ_{LES})
   - k equation: $-\frac{\partial \Delta}{\partial x_1}\frac{\partial \bar{u}_1 k}{\partial \Delta}$ (sink term)

Research Question

1. I want to use a $k - \omega$ DES model
   1.1 How do I prescribe inlet values on k and ω?
   1.2 What about the URANS region? Should I prescribe k and ω from a steady RANS solution?
2. The proposed method is to add commutation terms in the k and ω equations.
3. The commutation terms read (Δ goes from Δ_{RANS} to Δ_{LES})
   - k equation: $-\frac{\partial \Delta}{\partial x_1} \frac{\partial \bar{u}_1 k}{\partial \Delta}$ (sink term)
   - ω equation: $\frac{\omega}{k} \frac{\partial \Delta}{\partial x_1} \frac{\partial \bar{u}_1 k}{\partial \Delta}$ (source term)

Research Question

1. I want to use a $k-\omega$ DES model
   1.1 How do I prescribe inlet values on k and ω?
   1.2 What about the URANS region? Should I prescribe k and ω from a steady RANS solution?
2. The proposed method is to add commutation terms in the k and ω equations.
3. The commutation terms read (Δ goes from Δ_{RANS} to Δ_{LES})
   - k equation: $-\frac{\partial \Delta}{\partial x_1}\frac{\partial \bar{u}_1 k}{\partial \Delta}$ (sink term)
   - ω equation: $\frac{\omega}{k}\frac{\partial \Delta}{\partial x_1}\frac{\partial \bar{u}_1 k}{\partial \Delta}$ (source term)
4. The method can also be used in embedded LES (i.e. at the RANS-LES interface)

The Zonal $k-\omega$ Hybrid RANS-LES PDH Model

- In the LES region, the model reads

$$\frac{\partial k}{\partial t}+\frac{\partial \bar{v}_i k}{\partial x_i}=P^k-f_k\frac{k^{3/2}}{\ell_t}+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{\partial \omega}{\partial t}+\frac{\partial \bar{v}_i \omega}{\partial x_i}=C_{\omega_1}f_\omega\frac{\omega}{k}P^k-C_{\omega 2}\omega^2+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]+C_\omega\frac{\nu_t}{k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

$$\nu_t=f_\mu\frac{k}{\omega},\quad P^k=\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_j},\quad \ell_t=C_{LES}\Delta_{dw}$$

$$\Delta_{dw}=\min\left(\max\left[C_{dw}d_w,C_w\Delta_{max},\Delta_{nstep}\right],\Delta_{max}\right)$$

The Zonal $k-\omega$ Hybrid RANS-LES PDH Model

- In the LES region, the model reads

$$\frac{\partial k}{\partial t}+\frac{\partial \bar{v}_i k}{\partial x_i}=P^k-f_k\frac{k^{3/2}}{\ell_t}+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{\partial \omega}{\partial t}+\frac{\partial \bar{v}_i \omega}{\partial x_i}=C_{\omega_1}f_\omega\frac{\omega}{k}P^k-C_{\omega 2}\omega^2+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]+C_\omega\frac{\nu_t}{k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

$$\nu_t=f_\mu\frac{k}{\omega},\quad P^k=\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_j},\quad \ell_t=C_{LES}\Delta_{dw}$$

$$\Delta_{dw}=\min\left(\max\left[C_{dw}d_w, C_w\Delta_{max}, \Delta_{nstep}\right], \Delta_{max}\right)$$

- The length scale, Δ_{dw}, is taken from the IDDES model [9].

The Zonal $k-\omega$ Hybrid RANS-LES PDH Model

- In the LES region, the model reads

$$\frac{\partial k}{\partial t}+\frac{\partial \bar{v}_i k}{\partial x_i}=P^k-f_k\frac{k^{3/2}}{\ell_t}+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{\partial \omega}{\partial t}+\frac{\partial \bar{v}_i \omega}{\partial x_i}=C_{\omega_1}f_\omega\frac{\omega}{k}P^k-C_{\omega 2}\omega^2+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]+C_\omega\frac{\nu_t}{k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

$$\nu_t=f_\mu\frac{k}{\omega},\quad P^k=\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_j},\quad \ell_t=C_{LES}\Delta_{dw}$$

$$\Delta_{dw}=\min\left(\max\left[C_{dw}d_w,C_w\Delta_{max},\Delta_{nstep}\right],\Delta_{max}\right)$$

- The length scale, Δ_{dw}, is taken from the IDDES model [9].
- In the RANS regions, $\ell_t=k^{1/2}/(C_k\omega)$.

The Zonal $k-\omega$ Hybrid RANS-LES PDH Model

- In the LES region, the model reads

$$\frac{\partial k}{\partial t}+\frac{\partial \bar{v}_i k}{\partial x_i}=P^k-f_k\frac{k^{3/2}}{\ell_t}+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{\partial \omega}{\partial t}+\frac{\partial \bar{v}_i \omega}{\partial x_i}=C_{\omega_1}f_\omega\frac{\omega}{k}P^k-C_{\omega 2}\omega^2+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]+C_\omega\frac{\nu_t}{k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

$$\nu_t=f_\mu\frac{k}{\omega},\quad P^k=\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_j},\quad \ell_t=C_{LES}\Delta_{dw}$$

$$\Delta_{dw}=\min\left(\max\left[C_{dw}d_w,C_w\Delta_{max},\Delta_{nstep}\right],\Delta_{max}\right)$$

- The length scale, Δ_{dw}, is taken from the IDDES model [9].
- In the RANS regions, $\ell_t=k^{1/2}/(C_k\omega)$.
- The interface between LES and RANS regions is chosen at a fixed grid line ($y^+\simeq 500$)

Varying filter size

- When filter size in LES varies in space, an additional term appears in the momentum equation.
- The reason? the spatial derivatives and the filtering do not commute.
- For the convective term in Navier-Stokes, for example, we get

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

- Ghosal & Moin [4] showed that the error is proportional to $(\Delta x)^2$; hence it is usually neglected.

Commutation error in k equation

- In zonal[1] hybrid RANS-LES, the length scale at the RANS-LES interface changes abruptly from a RANS length scale to a LES length scale.
- Hamda [5] found that the commutation error at RANS-LES interfaces is large.
- For the k equation the commutation term reads

$$\overline{\frac{\partial u_i k}{\partial x_i}} = \frac{\partial \bar{u}_i k}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i k}{\partial \Delta}$$

[1] the interface is chosen at a location where the RANS and LES length scales differ

Commutation term: physical meaning

$$\overline{\frac{\partial u_i k}{\partial x_i}} = \frac{\partial \bar{u}_i k}{\partial x_i} - \frac{\partial \Delta}{\partial x_i} \frac{\partial \bar{u}_i k}{\partial \Delta}$$

- Consider a fluid particle in a RANS region moving in the x_1 direction and passing across a RANS-LES interface.

Commutation term: physical meaning

$$\overline{\frac{\partial u_i k}{\partial x_i}} = \frac{\partial \bar{u}_i k}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i k}{\partial \Delta}$$

- Consider a fluid particle in a RANS region moving in the x_1 direction and passing across a RANS-LES interface.
- The filterwidth decreases across the interface, i.e. $\partial \Delta / \partial x_1 < 0$

Commutation term: physical meaning

$$\overline{\frac{\partial u_i k}{\partial x_i}} = \frac{\partial \bar{u}_i k}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i k}{\partial \Delta}$$

- Consider a fluid particle in a RANS region moving in the x_1 direction and passing across a RANS-LES interface.
- The filterwidth decreases across the interface, i.e. $\partial \Delta / \partial x_1 < 0$
- k decreases when going from RANS to LES $\Rightarrow$ $\partial \bar{u}_1 k / \partial \Delta = \underbrace{(k_{LES} - k_{RANS})}_{<0} / \underbrace{(\Delta_{LES} - \Delta_{RANS})}_{<0} > 0$

Commutation term: physical meaning

$$\overline{\frac{\partial u_i k}{\partial x_i}} = \frac{\partial \bar{u}_i k}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i k}{\partial \Delta}$$

- Consider a fluid particle in a RANS region moving in the x_1 direction and passing across a RANS-LES interface.
- The filterwidth decreases across the interface, i.e. $\partial \Delta / \partial x_1 < 0$
- k decreases when going from RANS to LES $\Rightarrow$ $\partial \bar{u}_1 k / \partial \Delta = \underbrace{(k_{LES} - k_{RANS})}_{<0} / \underbrace{(\Delta_{LES} - \Delta_{RANS})}_{<0} > 0$
- $\Rightarrow$ The commutation term > 0

Commutation term: physical meaning

$$\overline{\frac{\partial u_i k}{\partial x_i}} = \frac{\partial \bar{u}_i k}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i k}{\partial \Delta}$$

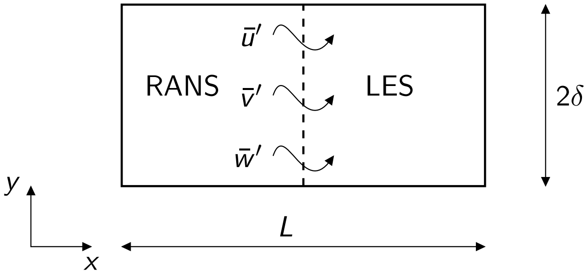
- Consider a fluid particle in a RANS region moving in the x_1 direction and passing across a RANS-LES interface.
- The filterwidth decreases across the interface, i.e. $\partial\Delta/\partial x_1 < 0$
- k decreases when going from RANS to LES $\Rightarrow$ $\partial \bar{u}_1 k/\partial\Delta = \underbrace{(k_{LES} - k_{RANS})}_{<0} / \underbrace{(\Delta_{LES} - \Delta_{RANS})}_{<0} > 0$
- $\Rightarrow$ The commutation term > 0
- $\Rightarrow$ The commutation term < 0 on the right-side of the k equation.

Commutation term: physical meaning

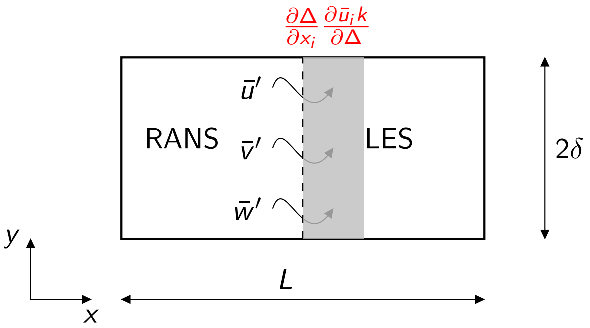
$$\overline{\frac{\partial u_i k}{\partial x_i}} = \frac{\partial \bar{u}_i k}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i k}{\partial \Delta}$$

- Consider a fluid particle in a RANS region moving in the x_1 direction and passing across a RANS-LES interface.
- The filterwidth decreases across the interface, i.e. $\partial \Delta / \partial x_1 < 0$
- k decreases when going from RANS to LES $\Rightarrow$ $\partial \bar{u}_1 k / \partial \Delta = \underbrace{(k_{LES} - k_{RANS})}_{<0} / \underbrace{(\Delta_{LES} - \Delta_{RANS})}_{<0} > 0$
- $\Rightarrow$ The commutation term > 0
- $\Rightarrow$ The commutation term < 0 on the right-side of the k equation.
- Hence, the commutation term at the RANS-LES interface reduces k.

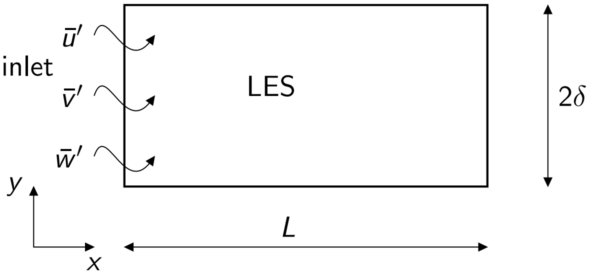
Commutation term at the RANS-LES interface



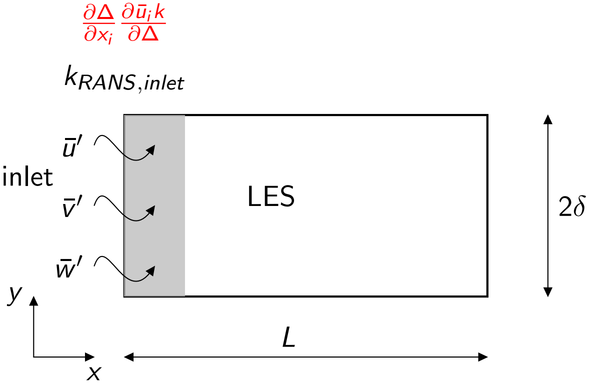
Commutation term at the RANS-LES interface



Commutation term at the LES inlet



Commutation term at the LES inlet



Commutation term in the ω equation

- Let us start by looking at the ε equation.
  - What happens with ε when a fluid particle moves from a RANS region into an LES region?
  - The answer is, nothing. The dissipation is the same in a RANS region as in an LES region.
- Transformation of the k and ε equations to an ω equation gives

$$\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{\varepsilon}{C_k k}\right) = \frac{1}{C_k k}\frac{d\varepsilon}{dt} + \frac{\varepsilon}{C_k}\frac{d(1/k)}{dt} = \frac{1}{C_k k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

Commutation term in the ω equation

- Let us start by looking at the ε equation.
  - What happens with ε when a fluid particle moves from a RANS region into an LES region?
  - The answer is, nothing. The dissipation is the same in a RANS region as in an LES region.
- Transformation of the k and ε equations to an ω equation gives

$$\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{\varepsilon}{C_k k}\right) = \frac{1}{C_k k}\frac{d\varepsilon}{dt} + \frac{\varepsilon}{C_k}\frac{d(1/k)}{dt} = \cancel{\frac{1}{C_k k}\frac{d\varepsilon}{dt}} - \frac{\omega}{k}\frac{dk}{dt}$$

- Hence, the commutation error in the ω equation is the commutation term in the k equation multiplied by $-\omega/k$ so that

$$\overline{\frac{\partial u_i \omega}{\partial x_i}} = \frac{\partial \bar{u}_i \omega}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i \omega}{\partial \Delta} = \frac{\partial \bar{u}_i \omega}{\partial x_i} + \frac{\omega}{k}\frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{u}_i k}{\partial \Delta}$$

Summary of inlet treatment of k and ω

- Prescribe RANS values of k and ω at the inlet obtained from RANS simulations

Summary of inlet treatment of k and ω

- Prescribe RANS values of k and ω at the inlet obtained from RANS simulations
- Add commutation term to the cell slice(s) near the inlet

Summary of inlet treatment of k and ω

- Prescribe RANS values of k and ω at the inlet obtained from RANS simulations
- Add commutation term to the cell slice(s) near the inlet
  - k equation: $-\dfrac{\partial \Delta}{\partial x_1}\dfrac{\partial \bar{u}_1 k}{\partial \Delta}$ (sink term)

Summary of inlet treatment of k and ω

- Prescribe RANS values of k and ω at the inlet obtained from RANS simulations
- Add commutation term to the cell slice(s) near the inlet
  - k equation: $-\frac{\partial \Delta}{\partial x_1}\frac{\partial \bar{u}_1 k}{\partial \Delta}$ (sink term)
  - ω equation: $\frac{\omega}{k}\frac{\partial \Delta}{\partial x_1}\frac{\partial \bar{u}_1 k}{\partial \Delta}$ (source term)

Summary of inlet treatment of k and ω

- Prescribe RANS values of k and ω at the inlet obtained from RANS simulations
- Add commutation term to the cell slice(s) near the inlet
  - k equation: $-\dfrac{\partial\Delta}{\partial x_1}\dfrac{\partial \bar{u}_1 k}{\partial\Delta}$ (sink term)
  - ω equation: $\dfrac{\omega}{k}\dfrac{\partial\Delta}{\partial x_1}\dfrac{\partial \bar{u}_1 k}{\partial\Delta}$ (source term)
- The present approach is similar to adding the commutation term in PANS [3]

$$f_k\frac{Dk_{tot}}{Dt} = \frac{D(f_k k_{tot})}{Dt} - k_{tot}\frac{Df_k}{Dt} = \frac{Dk}{Dt} - k_{tot}\frac{Df_k}{Dt}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{u}_i\frac{\partial}{\partial x_i}, \qquad k_{tot} = k + \frac{1}{2}\langle u_i' u_i'\rangle$$

Synthetic inlet fluctuations

1. A pre-cursor RANS simulation is made using the PDH model [8].

Synthetic inlet fluctuations

1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].

Synthetic inlet fluctuations

1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].
3. Synthetic turbulence fluctuations based on homogeneous turbulence

Synthetic inlet fluctuations

1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].
3. Synthetic turbulence fluctuations based on homogeneous turbulence
   - we can only use the Reynolds stress tensor in one point

Synthetic inlet fluctuations

1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].
3. Synthetic turbulence fluctuations based on homogeneous turbulence
   - we can only use the Reynolds stress tensor in one point

Synthetic inlet fluctuations

1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].
3. Synthetic turbulence fluctuations based on homogeneous turbulence
   - we can only use the Reynolds stress tensor in one point
   - We need to chose a relevant location for the Reynolds stress tensor
   - In boundary layer flow, the turbulent shear stress is the single most important stress component

Synthetic inlet fluctuations

1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].
3. Synthetic turbulence fluctuations based on homogeneous turbulence
   - we can only use the Reynolds stress tensor in one point
   - We need to chose a relevant location for the Reynolds stress tensor
   - In boundary layer flow, the turbulent shear stress is the single most important stress component
   - Hence, the Reynolds stress tensor is taken at the location where the magnitude of the turbulent shear stress is largest.

Synthetic inlet fluctuations

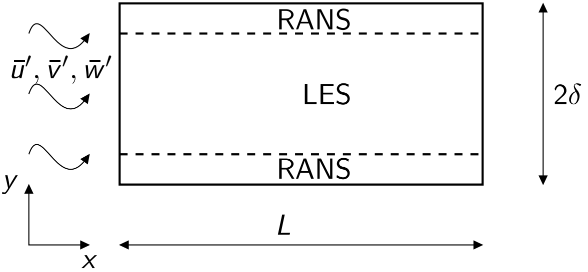
1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].
3. Synthetic turbulence fluctuations based on homogeneous turbulence
   - we can only use the Reynolds stress tensor in one point
   - We need to chose a relevant location for the Reynolds stress tensor
   - In boundary layer flow, the turbulent shear stress is the single most important stress component
   - Hence, the Reynolds stress tensor is taken at the location where the magnitude of the turbulent shear stress is largest.
4. Finally, the synthetic fluctuations are scaled with $\left(|\overline{u'v'}|/|\overline{u'v'}|_{max}\right)^{1/2}_{RANS}$ which is taken from the RANS simulation.

Synthetic inlet fluctuations

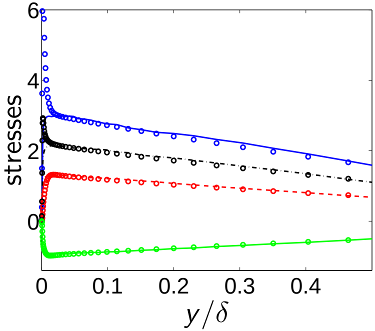
1. A pre-cursor RANS simulation is made using the PDH model [8].
2. The Reynolds stress tensor is computed using the EARSM model [10].
3. Synthetic turbulence fluctuations based on homogeneous turbulence
   - we can only use the Reynolds stress tensor in one point
   - We need to chose a relevant location for the Reynolds stress tensor
   - In boundary layer flow, the turbulent shear stress is the single most important stress component
   - Hence, the Reynolds stress tensor is taken at the location where the magnitude of the turbulent shear stress is largest.
4. Finally, the synthetic fluctuations are scaled with $\left(|\overline{u'v'}|/|\overline{u'v'}|_{max}\right)^{1/2}_{RANS}$ which is taken from the RANS simulation.
5. Matlab codes can be downloaded [1] (Google "synthetic inlet fluctuations")

Channel flow

- Reynolds number is $Re_\tau = 8\,000$.
- A $256 \times 96 \times 32$ mesh is used
- $\Delta x = 0.1$, $\Delta z = 0.05$
- The mean U, k and ω taken from 1D RANS simulation using the PDH $k - \omega$ model
- The wall-parallel RANS-LES interface is prescribed at a fixed gridline at $y^+ \simeq 500$.



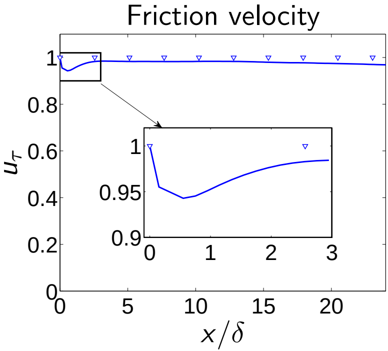
Inlet fluctuations



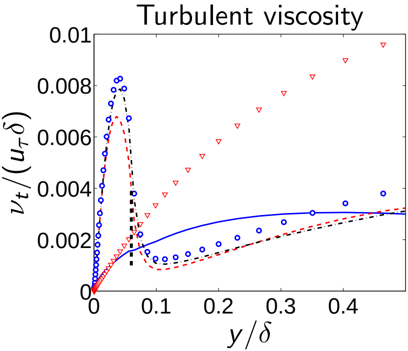
—: $\langle u'u' \rangle^+$; - -: $\langle v'v' \rangle^+$; -·-: $\langle w'w' \rangle^+$; —: $\langle u'v' \rangle^+$.

Markers: EARSM.

Results

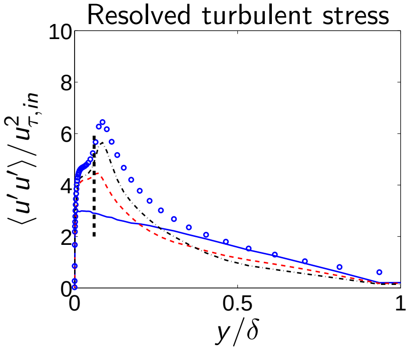


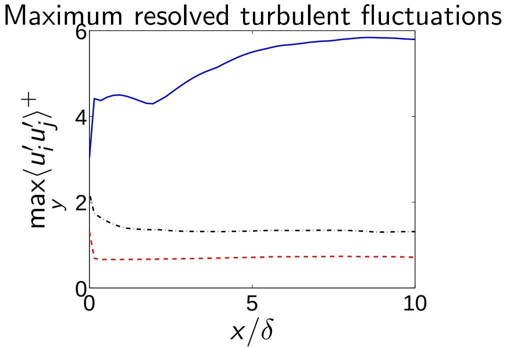
∇: $u_\tau = 1$ (target value)



—: $x = 0.05\delta$; - -: $x = 2.5\delta$; -·-: $x = 5.65\delta$; ∇: $\nu_t/(u_\tau \delta)/10$ at inlet (i.e. RANS).

Results



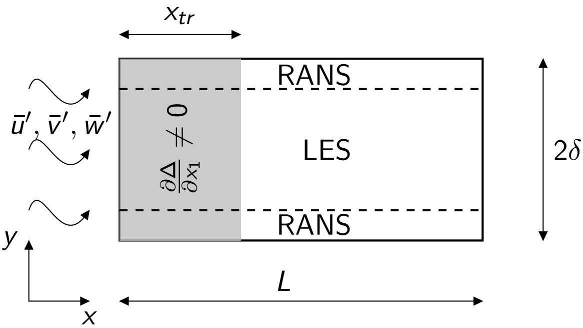


—: $x/\delta = 0.05$; - - -: $x = 2.5\delta$; -·-: $x = 5.85\delta$; ○: fully developed channel flow with Zonal hybrid RANS-LES model.

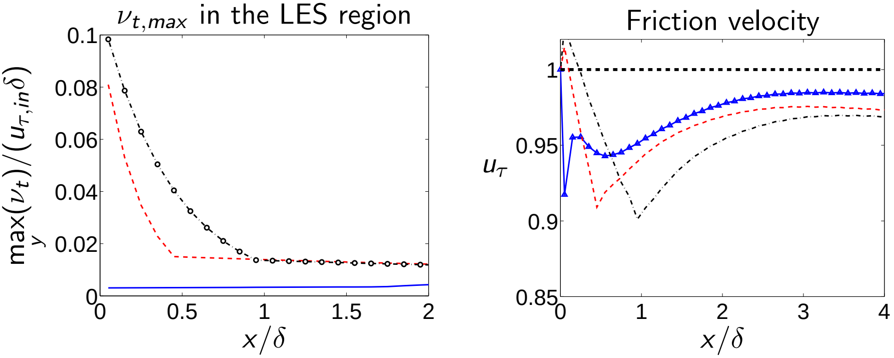
—: $\langle u'u' \rangle^+$; - - -: $\langle v'v' \rangle^+$; -·-: $\langle w'w' \rangle^+$.

Length of source region

- In how large a region, x_{tr}, should the commutation terms be added?



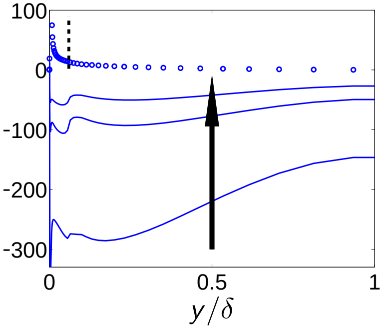
Results



—: $x_{tr}/\delta = 0.1$; - -: $x_{tr}/\delta = 0.5$; -·-: $x_{tr}/\delta = 1$

○: cell center

Source terms in k equation



$x/\delta = 0.05$. $\circ$: Production term, P^k, $x_{tr} = 0.05$.

—: commutation term. $(x_{tr}/\delta = 0.1, 0.5, 1)$

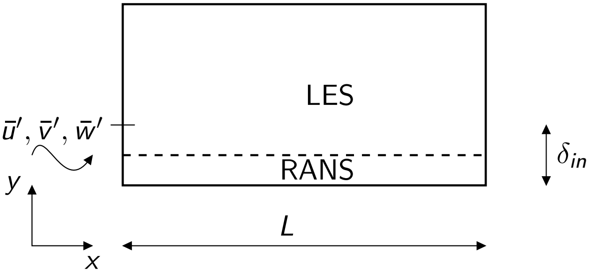
Arrow shows increasing x_{tr}

Boundary layer

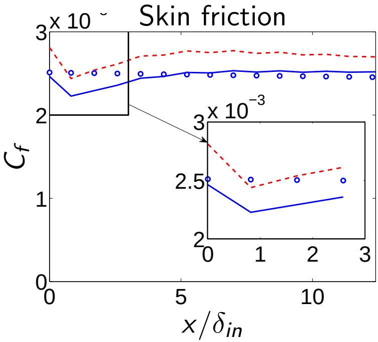
- The Reynolds number is $Re_\theta = 11\,000$ ($Re_{\tau,in} = 3\,400$).
- A $128 \times 192 \times 32$ mesh is used with $\Delta x = 0.1$, $\Delta z = 0.05$
- U_{in} as ($\kappa = 0.38$, $B = 4.1$, $\Pi = 0.5$ [6, 7])

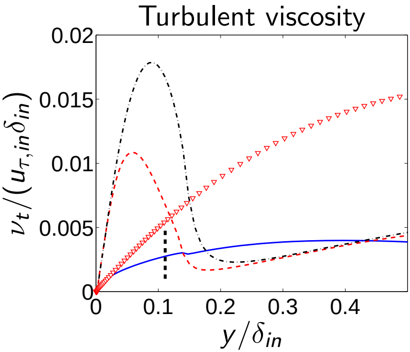
$$U_{in}^+ = \begin{cases} y^+ & y^+ \leq 5 \\ -2.23 + 4.49\ln(y^+) & 5 < y^+ < 30 \\ \dfrac{1}{\kappa}\ln(y^+) + B + \dfrac{2\Pi}{\kappa}\sin^2\left(\dfrac{\pi y}{2\delta}\right) & y^+ \geq 30 \end{cases} \quad (1)$$

- k and ω from a RANS solution



Results, boundary layer



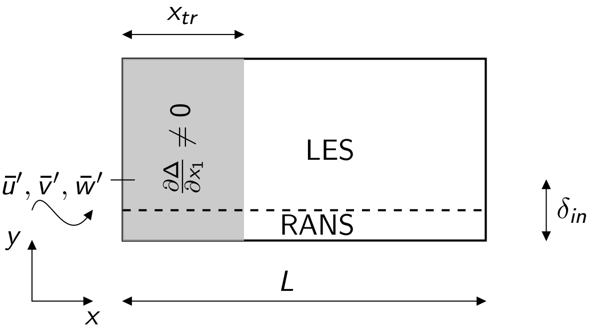


—: baseline
– –: U_{in} from RANS
○: $0.37\,(log_{10}Re_x)^{-2.584}$.

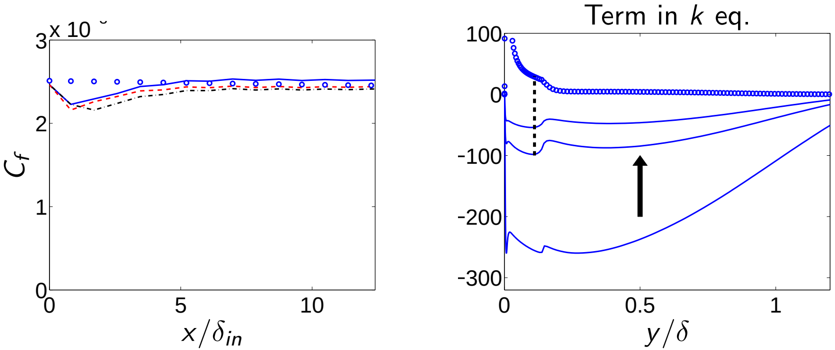
—: $x = 0.06\delta_{in}$; – –: $x = 2.35\delta_{in}$; – · –: $x = 11.9\delta_{in}$; ▽: $\nu_t/(u_{\tau,in}\delta_{in})/80$ at inlet (i.e. RANS).

Length of source region

- In how large a region, x_{tr}, should the commutation terms be added?



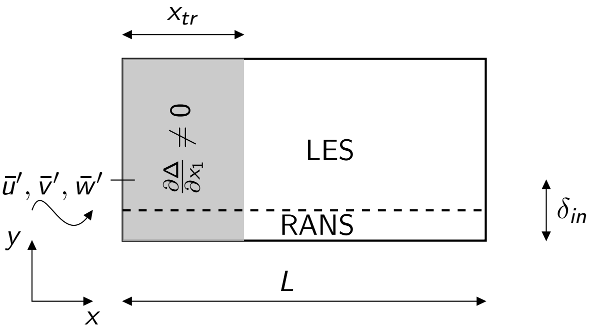
BOUNDARY LAYER: DIFFERENT x_{tr}



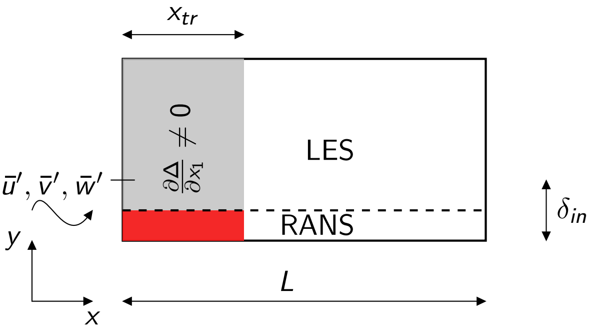
—: $x_{tr}/\delta = 0.125$; – –: $x_{tr}/\delta_{in} = 1.25$; – · –: $x_{tr}/\delta_{in} = 2.5$

$x/\delta_{in} = 0.06$. ○: Production term, $x_{tr}/\delta_{in} = 0.125$ —: commutation term. $(x_{tr}/\delta = 0.125, 1.25, 2.5)$ Arrow shows increasing x_{tr}

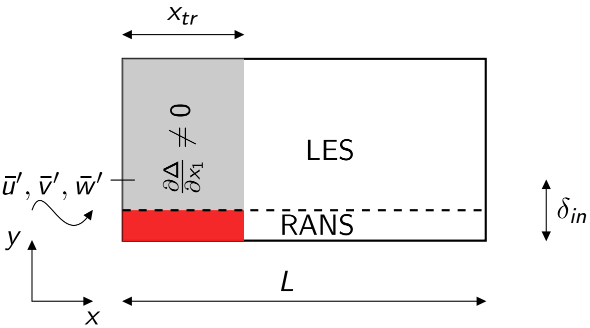
Commutation terms in the (U)RANS region?



Commutation terms in the (U)RANS region?



Commutation terms in the (U)RANS region?

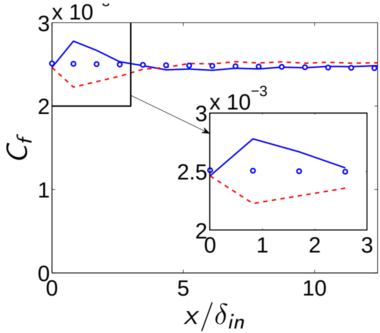


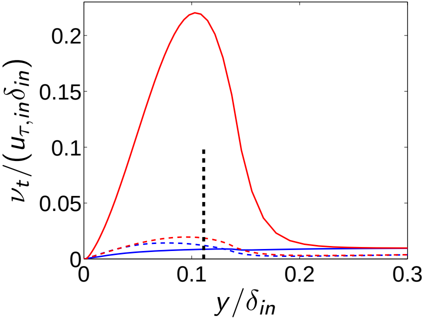
- Argument for using commutation terms in the (U)RANS region: $\nu_{t,URANS} \ll \nu_{t,RANS}$

Boundary layer: Commutation term or not?

BLUE LINES: commutation terms in the (U)RANS region

RED LINES: no commutation terms in the (U)RANS region





solid lines: $x/\delta_{in} = 0.06$

dashed lines: $x/\delta_{in} = 2.35$

Conclusions

- A novel method for prescribing inlet modelled turbulent quantities (k, ε, ω) has been presented
- It is based on the non-commutation between the divergence and the filter operators
- No tuning constants
- It is best to impose the commutation terms in one grid plane adjacent to the inlet

Three-Day CFD Course at Chalmers

- Unsteady Simulations for Industrial Flows: LES, DES, hybrid LES-RANS and URANS
- 9-11 November 2015 at Chalmers, Gothenburg, Sweden
- Max 16 participants
- 50% lectures and 50% workshops in front of a PC
- Registration deadline: 10 October 2015
- For info, see http://www.tfd.chalmers.se/~lada/cfdkurs/cfdkurs.html

References I

[1] Davidson, L.
http://www.tfd.chalmers.se/~lada/projects/inlet-boundary-conditions/pr

[2] Davidson, L.
Two-equation hybrid RANS-LES models: A novel way to treat k and ω at the inlet.
In *Turbulence, Heat and Mass Transfer, THMT-15* (Sarajevo, Bosnia and Herzegovina, 2015).

[3] Davidson, L.
Zonal PANS: evaluation of different treatments of the RANS-LES interface (to appear).
Journal of Turbulence (2015).

References II

[4] Ghosal, S., and Moin, P.
The basic equations for the large eddy simulation of turbulent flows in complex geometry.
J. Comp. Phys. 118 (1995), 24–37.

[5] Hamba, F.
Analysis of filtered Navier-Stokes equation for hybrid RANS/LES simulation.
Physics of Fluids A 23, 015108 (2011).

[6] Österlund, J.
Experimental Studies of Zero Pressure-Gradient Turbulent Boundary-Layer Flow.
PhD thesis, Department of Mechanics, Royal Institute of Technology, Stockholm, Sweden, 1999.

References III

[7] Österlund, M., Johansson, A., Nagib, H., and Hites, M.
A note on the overlap region in turbulent boundary layers.
Physics of Fluids A 12, 1 (2000), 1–4.

[8] Peng, S.-H., Davidson, L., and Holmberg, S.
A modified low-Reynolds-number $k - \omega$ model for recirculating flows.
Journal of Fluids Engineering 119 (1997), 867–875.

[9] Shur, M. L., Spalart, P. R., Strelets, M. K., and Travin, A. K.
A hybrid RANS-LES approach with delayed-DES and wall-modelled LES capabilities.
International Journal of Heat and Fluid Flow 29 (2008), 1638–1649.

References IV

[10] Wallin, S., and Johansson, A. V.
A new explicit algebraic Reynolds stress model for incompressible and compressible turbulent flows.
Journal of Fluid Mechanics 403 (2000), 89–132.