Detached Eddy Simulations: Analysis of a limit on the dissipation term for reducing spectral energy transfer at cut-off

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(ID)DES: PHYSICAL MEANING OF ψ

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\begin{aligned}
C^k &= P^k + D^k - \psi\varepsilon && k - \text{equation} \\
C^\varepsilon &= C_{\varepsilon 1}\frac{\varepsilon}{k}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon^2}{k} && \varepsilon - \text{equation} \\
\psi &= \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)
\end{aligned}
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Partition of Turbulent Kinetic Energy

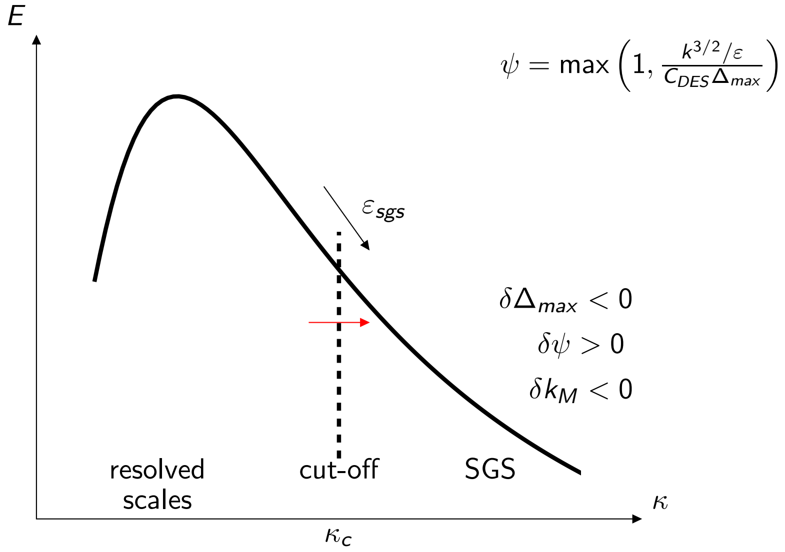

Figure: Energy spectrum.

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- i.e. limit $\frac{\partial \kappa_c}{\partial x_j}$
- by limiting $\psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$
- i.e. limiting the dissipation term, $\psi\varepsilon$, in the k equation

Perturbation Analysis

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- Along mean streamlines, k_M and ε_M are assumed to be in equilibrium. We get
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- 5 Eqns, 7 Unknowns: $\delta\varepsilon_M$, δP^k, $\delta\psi$, δk_M, $\delta\varepsilon_M$, $\underline{\delta D^k, \delta D^\varepsilon}$

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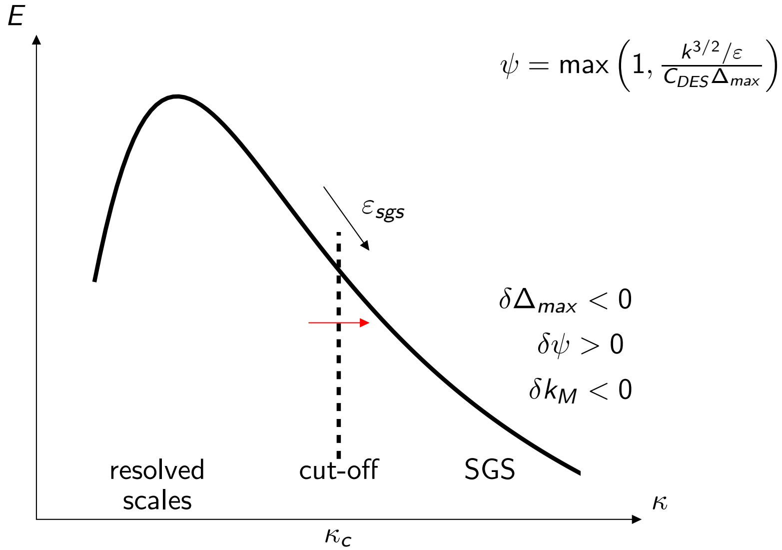

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- Same limit: $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1}$

The Turbulence Model

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- The AKN low-Reynolds number of Abe *et al.* [1] is used as underlying RANS model

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\frac{\partial k}{\partial t} + \frac{\partial \bar{u}_j k}{\partial x_j} &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \psi\varepsilon \\
\frac{\partial \varepsilon}{\partial t} + \frac{\partial \bar{u}_j \varepsilon}{\partial x_j} &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} f_1 P_k \frac{\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \\
C_{\varepsilon 1} &= 1.5, \quad C_{\varepsilon 2} = 1.9, \quad C_\mu = 0.09 \\
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- together with IDDES of Shur *et. al* [10]
- With $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1}$ is called IDDES-PC (Partition Control)

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Channel flow, periodic b.c.

- $Re_\tau = u_\tau h/\nu = 5\,200$
- The mesh has $32 \times 96 \times 32$ $(x,\ y,\ z)$ cells $\Rightarrow$ $(\Delta x^+, \Delta z^+) = (800, 400)$.

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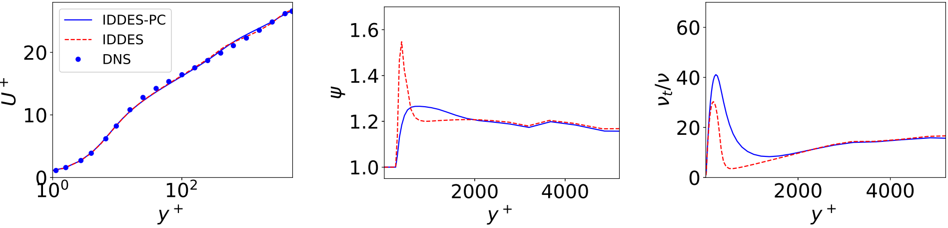

(A) Mean velocity. (B) ψ (C) Turbulent viscosity

Figure: ▬▬ : IDDES-PC model; ‒ ‒ : IDDES; Markers: DNS [8]

Channel flow inlet-outlet

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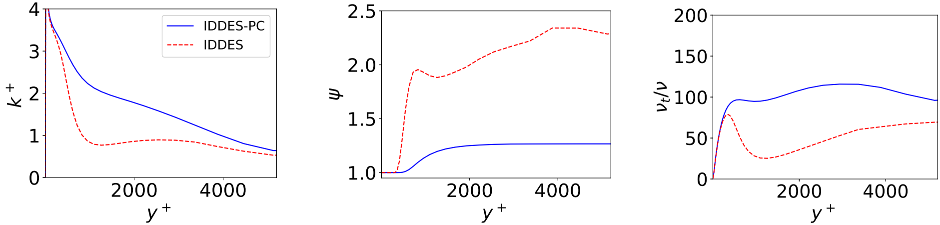

(A) Modeled k. (B) ψ. (C) Turbulent viscosity.

Figure: Profiles at $x = \delta$, i.e. one half-channel width.

HUMP FLOW

- The Reynolds number is $Re_c = 936\,000$
- The spanwise extent is $z_{max} = 0.2$.
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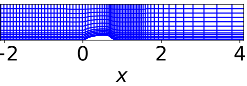
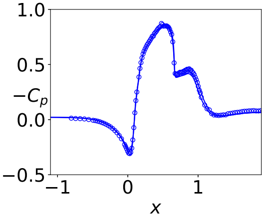
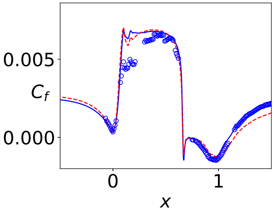


(A) Grid.



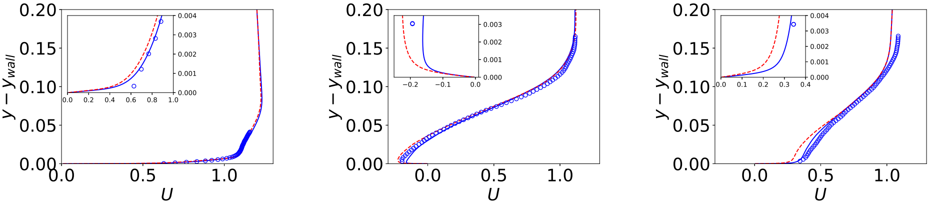
(B) Pressure coefficient.



(C) Skin friction

FIGURE: —— : IDDES-PC model; - - . IDDES. Markers: experiments [7, 6]

HUMP FLOW. VELOCITIES.



(A) $x = 0.65$. (B) $x = 1.0$. (C) $x = 1.30$.

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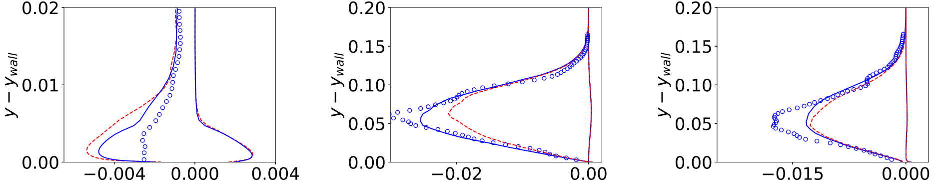
hump flow. Shear stresses.



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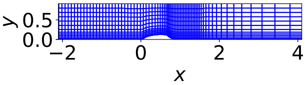

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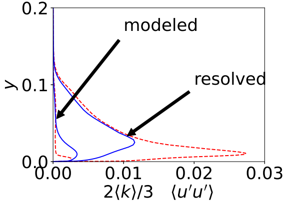
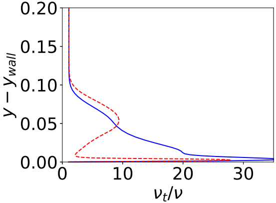


(B) Streamwise fluctuations.



(C) Turbulent viscosity.

FIGURE: $x = 0$. —— : IDDES-PC model; – – . IDDES.

Conclusions

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- Good results are obtained (better than standard IDDES for the hump flow)

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