A New Approach to Treat the RANS-LES interface in PANS [1]
Lars Davidson

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PANS Low Reynolds Number Model [4]

$$\frac{\partial k}{\partial t} + \frac{\partial (kU_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{ku}}\right)\frac{\partial k}{\partial x_j}\right] + (P - \varepsilon)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial (\varepsilon U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} P \frac{\varepsilon}{k} - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

$$\nu_t = C_\mu f_\mu \frac{k^2}{\varepsilon}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

- LRN Damping functions, f_2, f_μ as in [4]
- $f_\varepsilon = 1.0$
- LES region: $f_k = 0.4$
- RANS region: $f_k = 1.0$

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▶Zonal RANS-LES: f_k has a large gradient at the RANS-LES interface

PANS: DERIVATION

- The PANS k equation is derived by multiplying the RANS k equation by f_k. The left hand reads

$$f_k \frac{Dk_{tot}}{Dt} \tag{1}$$

where $D/Dt = \partial/\partial t + \bar{u}_i \partial/\partial x_i$, $k_{tot} = k + k_{res}$ (modeled plus resolved).

- If it is assumed that f_k constant, Eq. 1 can be re-written as

$$f_k \frac{Dk_{tot}}{Dt} = \frac{Df_k k_{tot}}{Dt} = \frac{Dk}{Dt}, \quad f_k = \frac{k}{k_{tot}} \tag{2}$$

- If f_k is not constant, Eq. 2 must be written as (Girimaji & Wallin [2])

$$f_k \frac{Dk_{tot}}{Dt} = \frac{Df_k k_{tot}}{Dt} - k_{tot}\frac{Df_k}{Dt} = \frac{Dk}{Dt} - \boxed{k_{tot}\frac{Df_k}{Dt}}$$

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- This work presents models for the boxed term at RANS-LES interfaces, i.e.
  - horizontal RANS-LES interface in boundary layer (channel flow)
  - vertical RANS-LES interface in embedded LES (channel flow)

Interface Model 1

- In [2], $k_{tot} Df_k/Dt$ is represented by introducing an additional turbulent viscosity, ν_{tr}, in the momentum equation

$$\frac{\partial}{\partial x_j}\left(\nu_{tr}\bar{s}_{ij}\right), \quad \bar{s}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$$

where

$$\nu_{tr} = \frac{P_{k_{tr}}}{|\bar{s}|^2}, \quad P_{k_{tr}} = \nu_{tr}|\bar{s}|^2 = k_{tot}\frac{Df_k}{Dt} = \frac{k}{f_k}\frac{Df_k}{Dt}$$

- The object of $P_{k_{tr}}$ is to decrease ν_t and facilitate growth of resolved turbulence on the LES side of an interface
- Hence, only $\nu_{tr} < 0$ is used which corresponds to $Df_k/Dt < 0$ (from RANS to LES).
- $\nu_t + \nu_{tr} > 0$ in the momentum equation (but not in the k equation)

Interface Model 2

- This model is identical to Model 1 except that k/f_k is replaced by k_{tot} i.e.

$$\underbrace{P_{k_{tr}} = \frac{k}{f_k}\frac{Df_k}{Dt}}_{\text{Model 1}} \qquad \underbrace{P_{k_{tr}} = k_{tot}\frac{Df_k}{Dt}}_{\text{Model 2}}$$

$$k_{tot} = k + \frac{1}{2}\langle \bar{u}_i' \bar{u}_i' \rangle_{r.a}$$

where subscript $r.a.$ denotes running average.

- In PANS, f_k is defined as $f_k = k/k_{tot}$
- In post-processing it is usually found that $f_k > k/k_{tot}$ (approx. a factor 4 larger) $\Rightarrow |P_{k_{tr}}|_{model\ 2} > |P_{k_{tr}}|_{model\ 1}$

Interface Model 3

- In Models 1 & 2, $\nu_{tr} = P_{k_{tr}}/|\bar{s}|^2$ which may cause numerical problems.
- Model 3 does not involve ν_{tr}. The original term $k_{tot} Df_k/Dt$ is used in the k equation
- Adding the term

$$-0.5\bar{u}'_i \frac{Df_k}{Dt} - \frac{k\bar{u}'_i}{\langle \bar{u}'_m \bar{u}'_m \rangle} \frac{Df_k}{Dt}$$

in the momenum equation corresponds to the time-averaged term

$$-\left\langle \frac{\bar{u}'_i \bar{u}'_i}{2} \frac{Df_k}{Dt} \right\rangle - \frac{k \langle \bar{u}'_i \bar{u}'_i \rangle}{\langle \bar{u}'_m \bar{u}'_m \rangle} \frac{Df_k}{Dt} = -\langle k_{tot} \rangle \frac{Df_k}{Dt}$$

in the k_{res} equation.

- However, this term causes numerical instability. Hence it is not used.
- Only the term in the k equation, $k_{tot} Df_k/Dt < 0$, is used.

Interface Models: summary

- Models 1 & 2: additional turbulent viscosity, $\nu_{tr} = P_{k_{tr}}/|\bar{s}|^2 < 0$, in P_k and momentum equations
  - Limit in momentum equations: $\nu_t + \nu_{tr} > 0$
  - Model 1: $P_{k_{tr}} = \frac{k}{f_k}\frac{Df_k}{Dt}$
  - Model 2: $P_{k_{tr}} = \langle k_{tot}\rangle_{r.a}\frac{Df_k}{Dt}$
- Model 3: additional production term, $P_{k_{tr}}$, in k equation without use of ν_{tr}

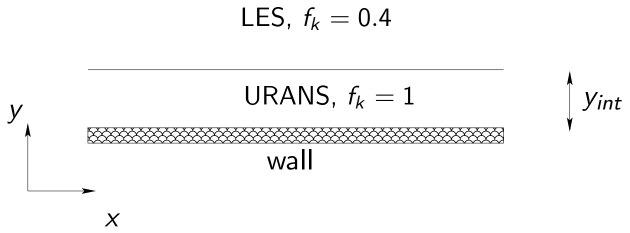
$$P_{k_{tr}} = \langle k_{tot}\rangle_{r.a}\frac{Df_k}{Dt} < 0$$

- Models 1-3 correspond to the non-commutivity in DES beteen filtering and spatial derivative at RANS-LES interfaces (Hamba [3])

$$\overline{\frac{\partial f}{\partial x_i}} = \frac{\partial \bar{f}}{\partial x_i} - \frac{\partial \Delta}{\partial x_i}\frac{\partial \bar{f}}{\partial \Delta}$$

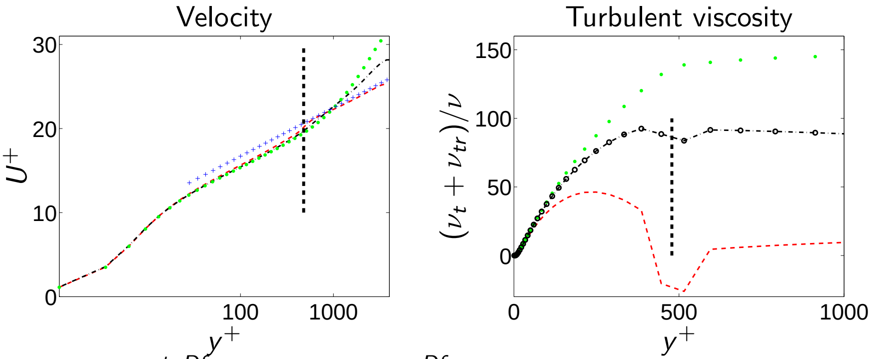
Fully Developed Channel Flow

- The URANS and the LES regions.



- $Re_\tau = u_\tau \delta/\nu = 2\,000$, $Re = 4\,000$ and $Re = 8\,000$
- $x_{max} = 3.2$, $y_{max} = 2$ and $z_{max} = 1.6$.
- 32×32 cells in the $x - z$ plane
- $N_y = 80$ cells ($Re_\tau = 2\,000$ and $4\,000$) or $N_y = 96$ ($Re_\tau = 8\,000$)

FULLY DEVELOPED CHANNEL FLOW: RESULTS

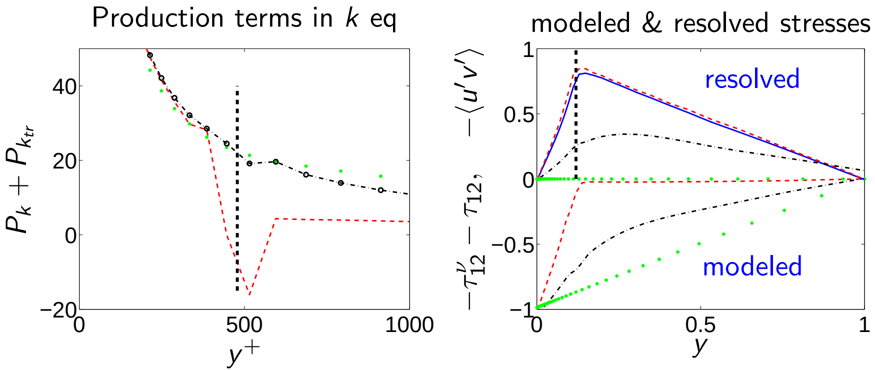


- - -: Model $\frac{k}{f_k}\frac{Df_k}{Dt}$ - -: Model $k_{tot}\frac{Df_k}{Dt}$. ···: no interface model.

$+$: $U^+ = \ln(y^+)/0.4 + 5.2$

•: location of the computational cell centers.

Fully Developed Channel Flow: Results



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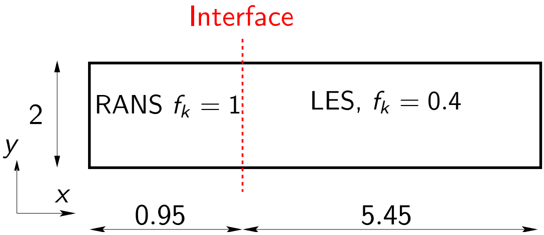
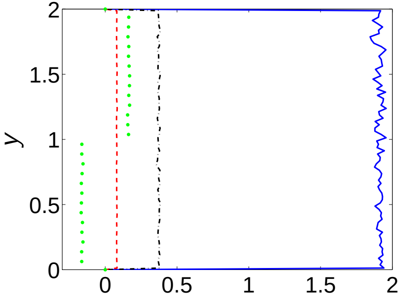
Embedded Channel Flow

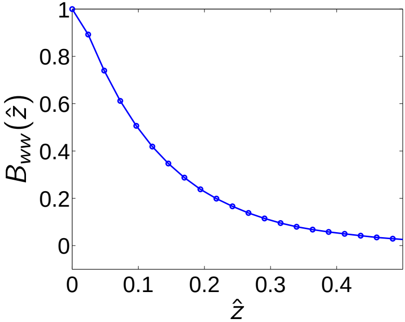


- $Re_\tau = u_\tau \delta/\nu = 950$
- The domain size is $6.4 \times 2 \times 1.6$ (x, y, z)
- $128 \times 80 \times 64$ cells.

Synthetic Fluctuations at the Interface



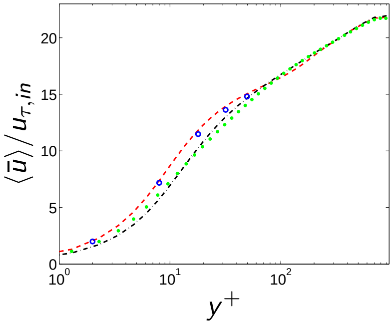
—: u^2_{rms}/u^2_τ
– –: v^2_{rms}/u^2_τ
– · –: w^2_{rms}/u^2_τ
···: $\langle u'v'\rangle/u^2_\tau$



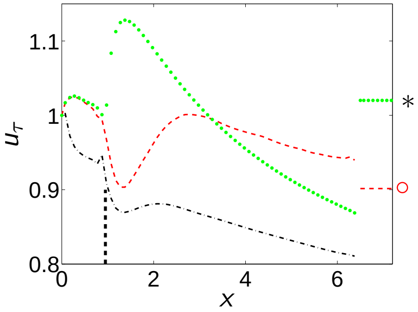
Two-point correlation of synthetic inlet fluctuations

Velocity and Skin Friction

Velocity at $x = 5.5$



Friction velocity



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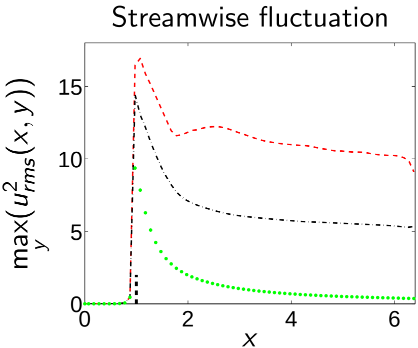
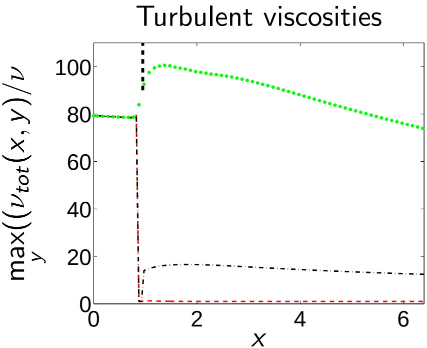
- - -: Model $k_{tot}\frac{Df_k}{Dt}$
- · · ·: no interface model
- ○: $U^+ = \ln(y^+)/0.4 + 5.2$

- $*$: u_τ, RANS when $x \to \infty$
- ○: u_τ, PANS when $x \to \infty$

Resolved and Modeled Turbulence





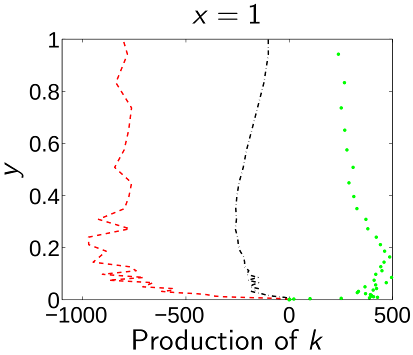
- - - : Model $\frac{k}{f_k}\frac{Df_k}{Dt}$

- - : Model $k_{tot}\frac{Df_k}{Dt}$

· · · : no interface model

$$\nu_{tot} = \nu + \nu_t + \nu_{tr}$$

Production at Interface



- - -: $P_k + P_{k_{tr}}$, Model $\frac{k}{f_k}\frac{Df_k}{Dt}$.

– –: $P_k + P_{k_{tr}}$, Model $k_{tot}\frac{Df_k}{Dt}$.

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Conclusions

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  - $k_{tot}\frac{Df_k}{Dt}$ added no $\nu_{t_{tr}}$ to the k eq (Model 3)

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- Model 2 works very well
- Model 3 gives identical results to Model 2 (not shown in this presentation)

Three-Day CFD Course at Chalmers

- Unsteady Simulations for Industrial Flows: LES, DES, hybrid LES-RANS and URANS
- 29-31 October 2014 at Chalmers, Gothenburg, Sweden
- Max 16 participants
- 50% lectures and 50% workshops in front of a PC
- Registration deadline: 10 October 2014
- For info, see http://www.tfd.chalmers.se/˜lada/cfdkurs/cfdkurs.html

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