EMBEDDED LES USING PANS [2, 3]

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PANS Low Reynolds Number Model [4]

$$\frac{\partial k_u}{\partial t} + \frac{\partial (k_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{ku}}\right)\frac{\partial k_u}{\partial x_j}\right] + (P_u - \varepsilon_u)$$

$$\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial (\varepsilon_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon_u}{\partial x_j}\right] + C_{\varepsilon 1} P_u \frac{\varepsilon_u}{k_u} - C^*_{\varepsilon 2}\frac{\varepsilon_u^2}{k_u}$$

$$\nu_u = C_\mu f_\mu \frac{k_u^2}{\varepsilon_u}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

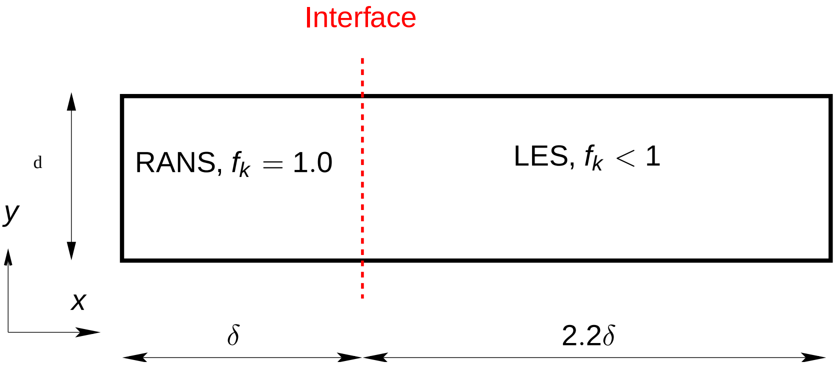
$C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_k, σ_ε and C_μ same values as [1]. $f_\varepsilon = 1$. f_2 and f_μ read

$$f_2 = \left[1 - \exp\left(-\frac{y^*}{3.1}\right)\right]^2 \left\{1 - 0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}$$

$$f_\mu = \left[1 - \exp\left(-\frac{y^*}{14}\right)\right]^2 \left\{1 + \frac{5}{R_t^{3/4}}\exp\left[-\left(\frac{R_t}{200}\right)^2\right]\right\}$$

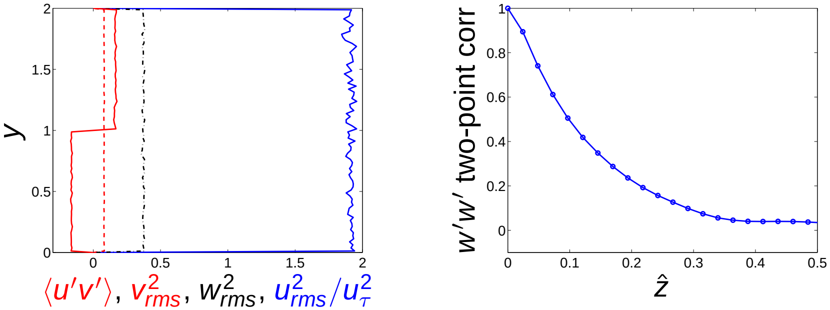
- Baseline model: $f_k = 0.4$. Range of $0.2 < f_k < 0.6$ is evaluated

Channel flow: Domain



- Interface: Synthetic turbulent fluctuations are introduced as additional convective fluxes in the momentum equations and the continuity equation
- $f_k = 0.4$ is the baseline value for LES [4]

Inlet fluctuations



- Anisotropic synthetic fluctuations, u', v', w',
- Integral length scale $\mathcal{L} \simeq 0.13$ (see 2-p point correlation)
- Asymmetric time filter $(\mathcal{U}')^m = a(\mathcal{U}')^{m-1} + b(u')^m$ with $a = 0.954, b = (1-a^2)^{1/2}$ gives a time integral scale $\mathcal{T} = 0.015$ ($\Delta t = 0.00063$)

INTERFACE CONDITIONS FOR k_u AND ε_u

- For k_u & ε_u we prescribe "inlet" boundary conditions at the interface.
- First, the usual convective and diffusive fluxes at the interface are set to zero
- Next, new convective fluxes are added. Which "inlet" values should be used at the interface?
    - $k_{u,int} = f_k k_{RANS}(x = 0.5\delta)$, $\varepsilon_{u,int} = C_\mu^{3/4} k_{u,int}^{3/2} / \ell_{sgs}$, $\ell_{sgs} = C_s \Delta$, $\Delta = V^{1/3}$
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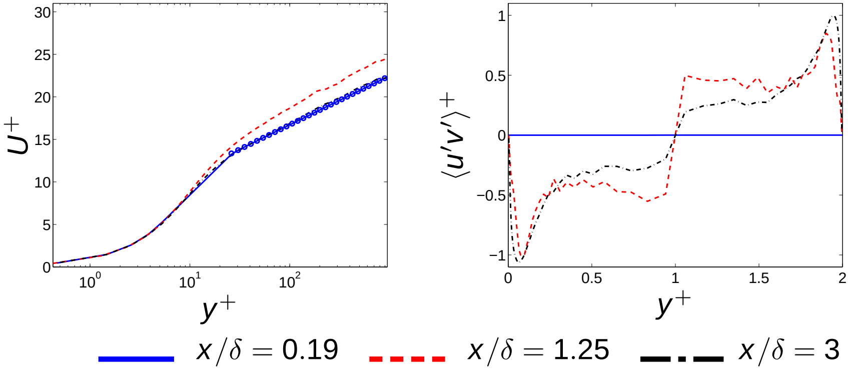
  - Baseline $C_s = 0.07$; different C_s values are tested

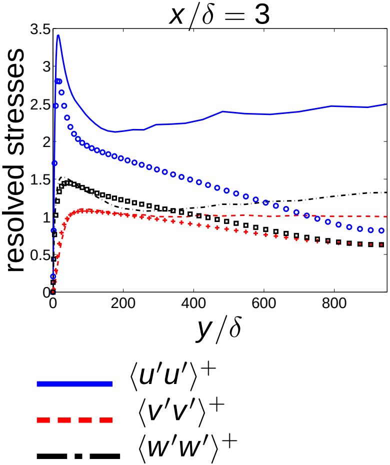
NUMERICAL METHOD

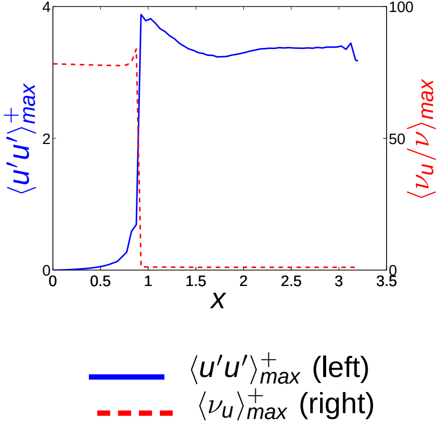
- Incompressible finite volume method
- Pressure-velocity coupling treated with fractional step
- 2^{nd} order central differencing scheme for momentum eqns in LES region.
- 2^{nd} order upwind differencing scheme (van Leer) for momentum eqns in RANS region.
- Hybrid 1^{st} order upwind/2^{nd} order central scheme k & ε eqns.
- 2^{nd}-order Crank-Nicholson for time discretization

Channel Flow: Velocity and Shear Stresses



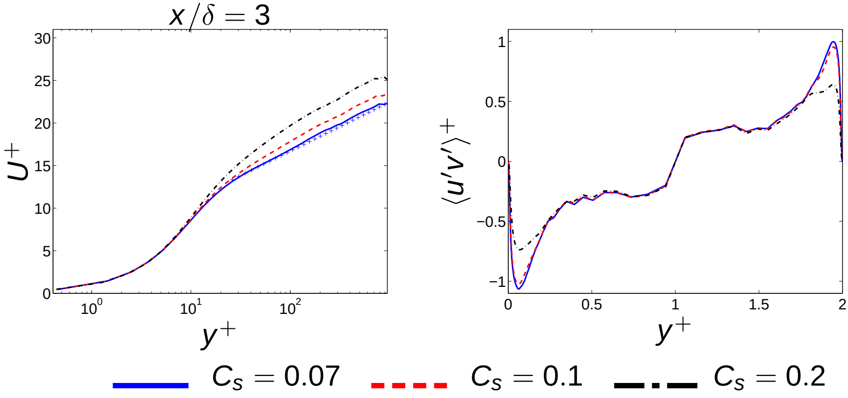
CHANNEL FLOW: STRESSES AND PEAK VALUES VS. x



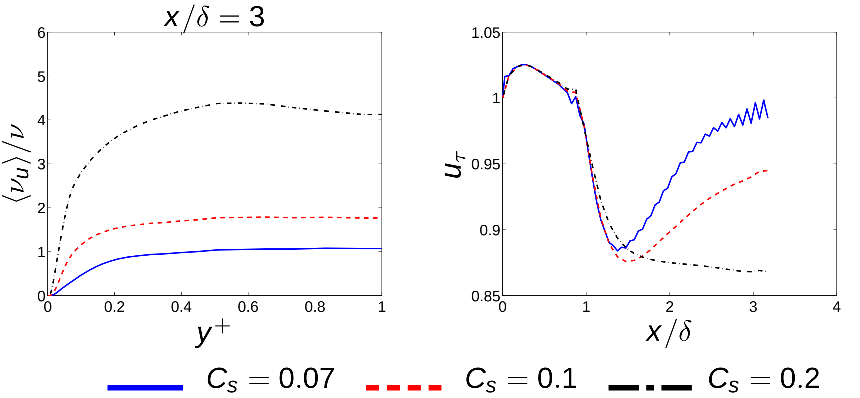


CHANNEL FLOW: DIFFERENT C_s VALUE FOR $\varepsilon_{interface}$

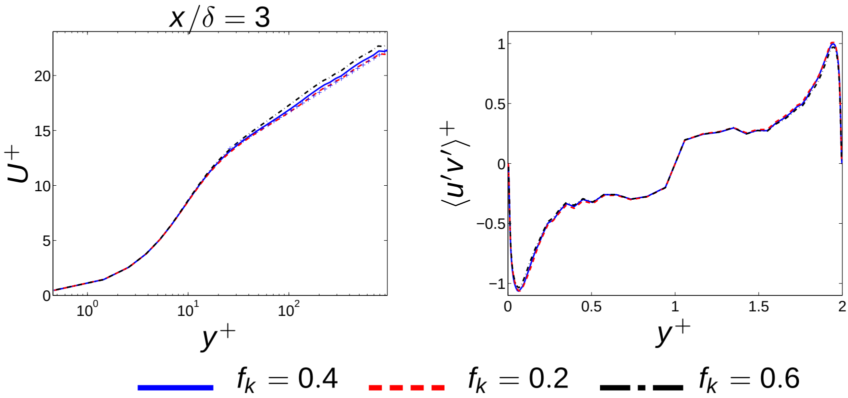
- $k_{u,int} = f_k k_{RANS}$
- $\varepsilon_{u,int} = C_\mu^{3/4} k_{u,int}^{3/2} / \ell_{sgs}$, $\ell_{sgs} = C_s \Delta$



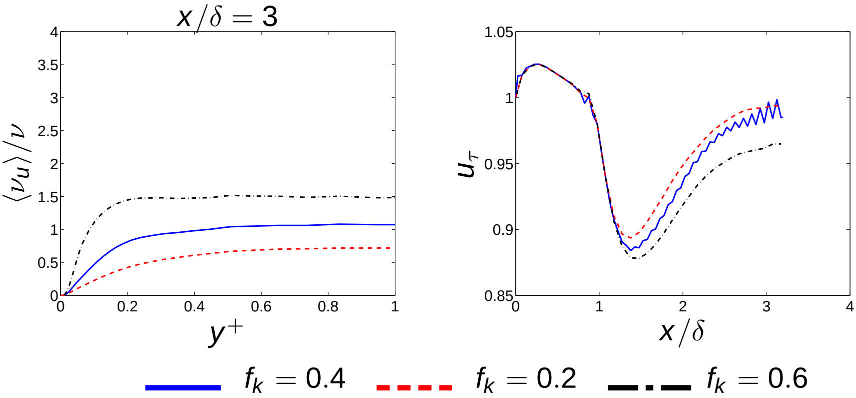
Channel Flow: different C_s value for $\varepsilon_{interface}$



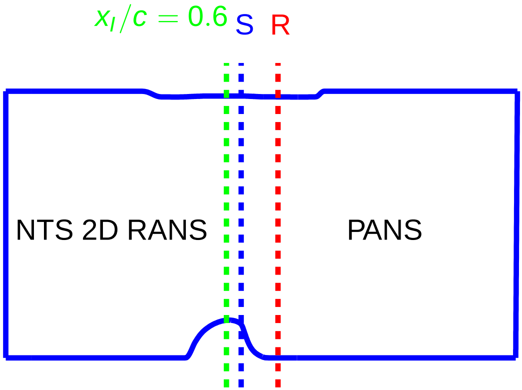
Channel Flow: different f_k values



Channel Flow: different f_k values

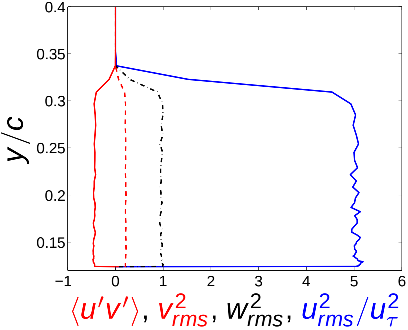


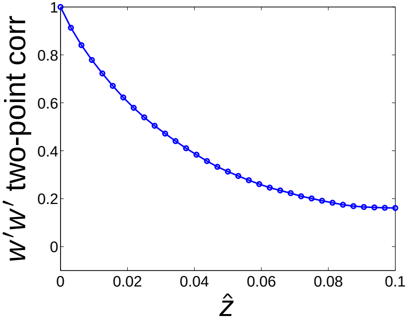
Hump Flow



- Inlet, Separation $x_S/c = 0.65$; reattachment $x_R/c = 1.1$
- $Re_c = 936\,000 \frac{U_{ij}c}{\nu}$ $(U_{in} = c = \rho = 1,\ \nu = 1/Re_c$
- $H/c = 0.91,\ h/c = 0.128,\ x/c = [0.6, 4.2]$
- Mesh: $312 \times 120 \times 64$, $Z_{max} = 0.2c$ (baseline)

BASELINE INLET FLUCTUATIONS



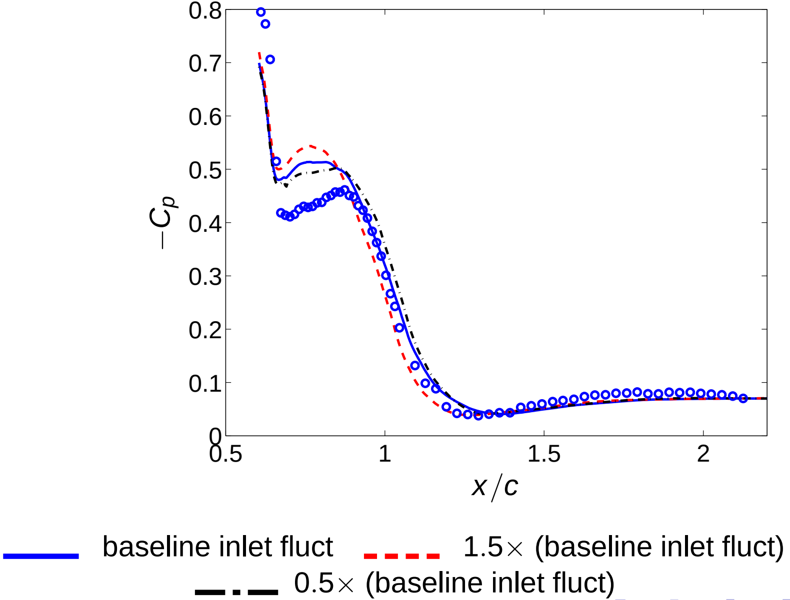


- Integral length scale $\mathcal{L} \simeq 0.04$ (see 2-p point correlation)
- Asymmetric time filter $(\mathcal{U}')^m = a(\mathcal{U}')^{m-1} + b(u')^m$ with $a = 0.954, b = (1-a^2)^{1/2}$ gives a time integral scale $\mathcal{T} = 0.038$
- $\Delta t = 0.002$. $7500 + 7500$ time steps (100 hours one core)
- Fluctuations multiplied by $f_{bl} = \max\left\{0.5\left[1 - \tanh(y - y_{bl} - y_{wall})/b\right], 0.02\right\}$, $y_{bl} = 0.2$, $b = 0.01$.

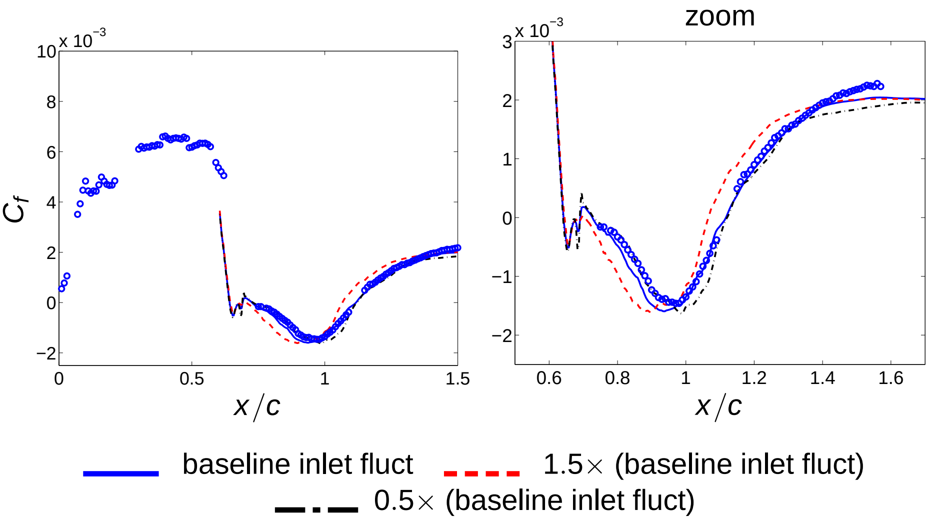
Numerical Method

- Incompressible finite volume method
- Pressure-velocity coupling treated with fractional step
- 95% 2^{nd} order central and 5% 2^{nd} order upwind differencing scheme for momentum eqns.
- Hybrid 1^{st} order upwind/2^{nd} order central scheme k & ε eqns.
- 2^{nd}-order Crank-Nicholson for time discretization

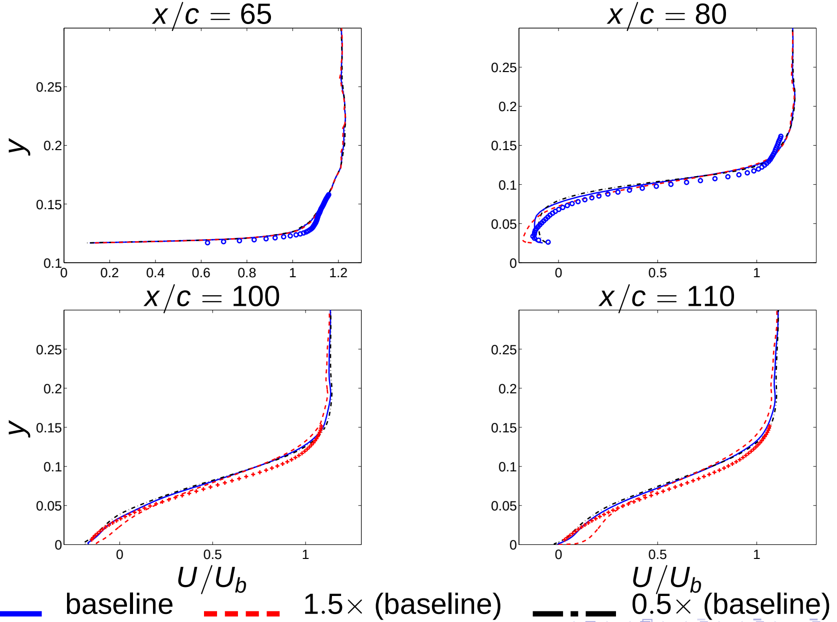
PRESSURE: AMPLITUDES OF INLET FLUCT



SKIN FRICTION: AMPLITUDES OF INLET FLUCT



Velocities: Amplitudes of Inlet Fluct



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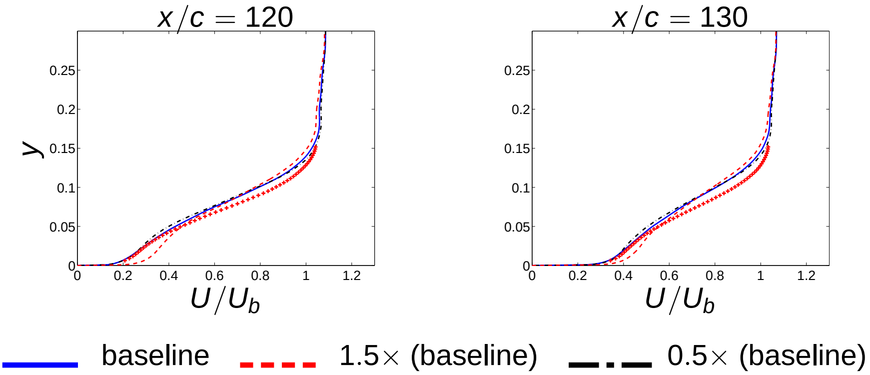
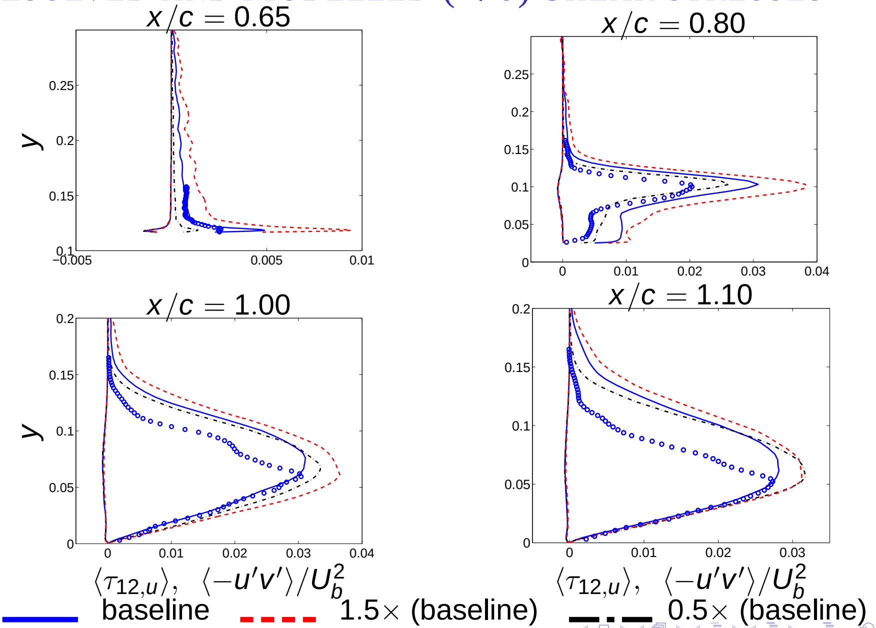
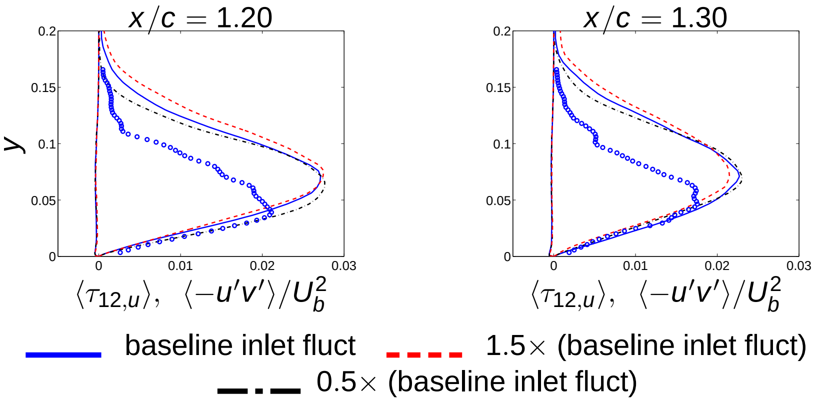


Resolved and Modelled (< 0) Shear stresses

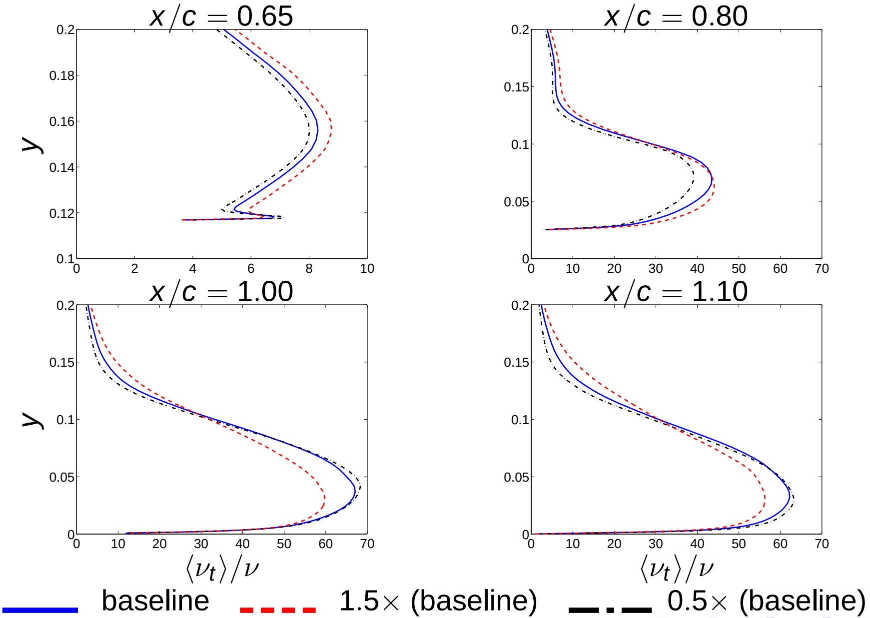


SHEAR STRESSES: AMPLITUDES OF INLET FLUCT

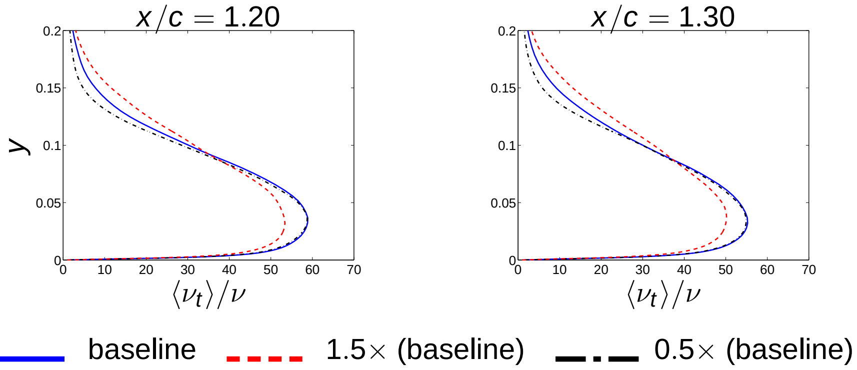
- Resolved and Modelled (< 0) Shear stresses



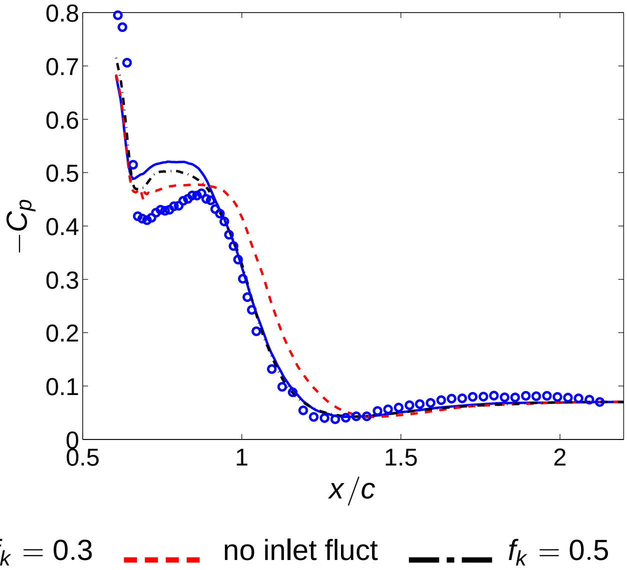
Turb Viscosity: Amplitudes Of Inlet Fluct



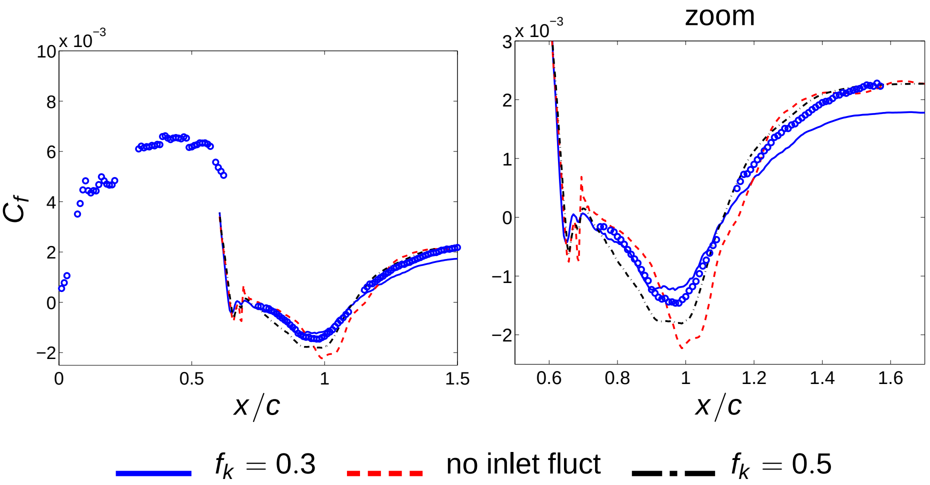
TURB VISCOSITY: AMPLITUDES OF INLET FLUCT



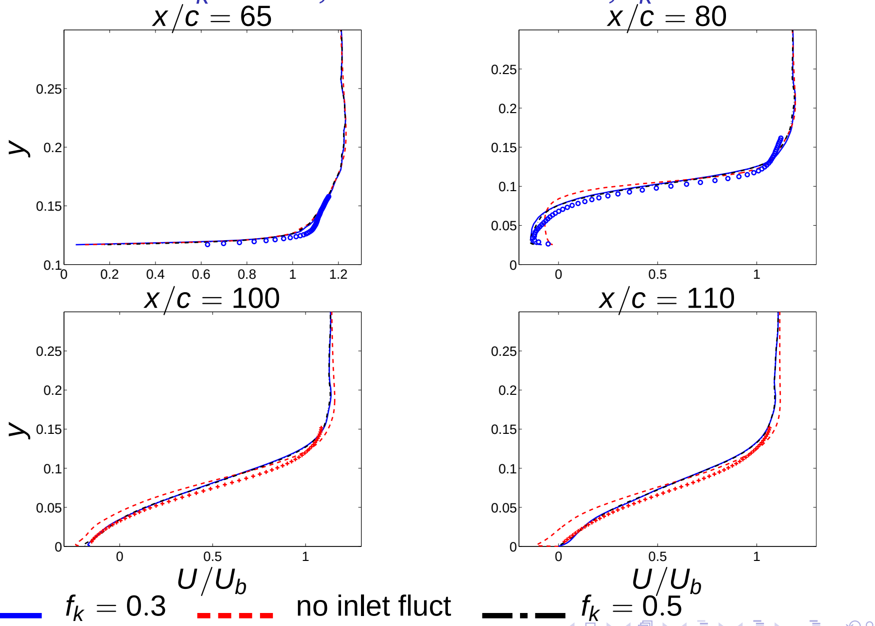
PRESSURE: $f_k = 0.3$; NO INLET FLUCT; $f_k = 0.5$



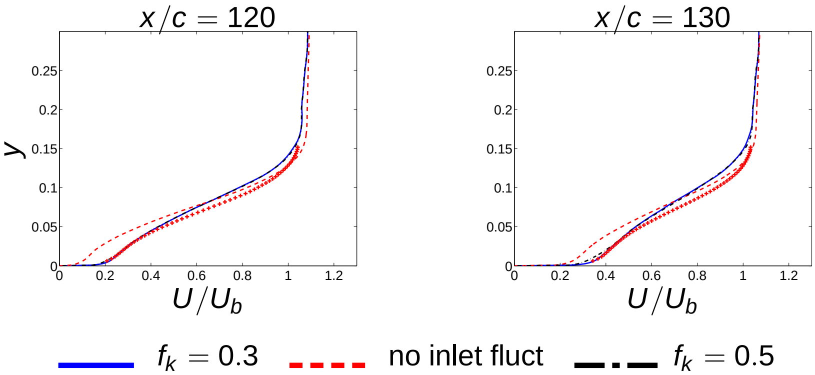
Skin Friction: $f_k = 0.3$; no inlet fluct; $f_k = 0.5$



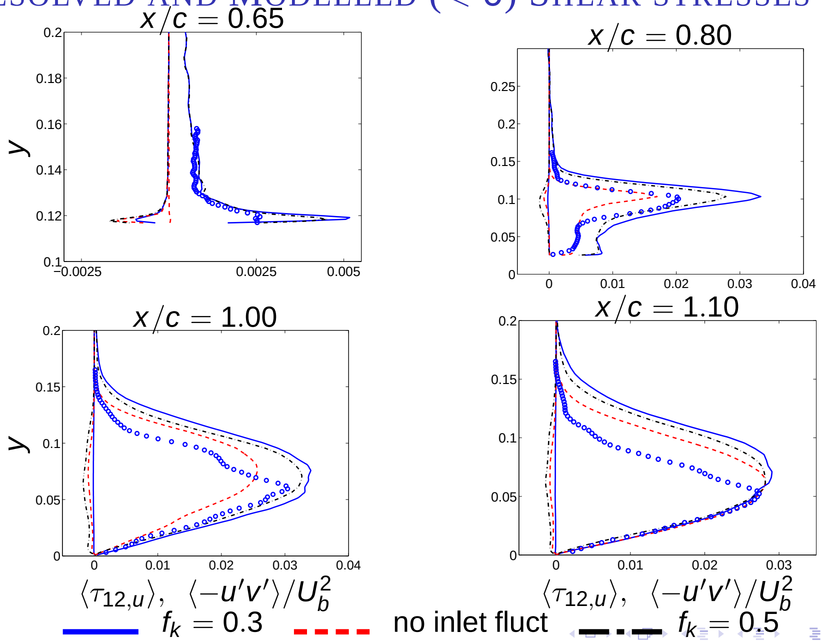
VELOCITIES: $f_k = 0.3$; NO INLET FLUCT; $f_k = 0.5$



VELOCITIES: $f_k = 0.3$; NO INLET FLUCT; $f_k = 0.5$



Resolved and Modelled (< 0) Shear stresses



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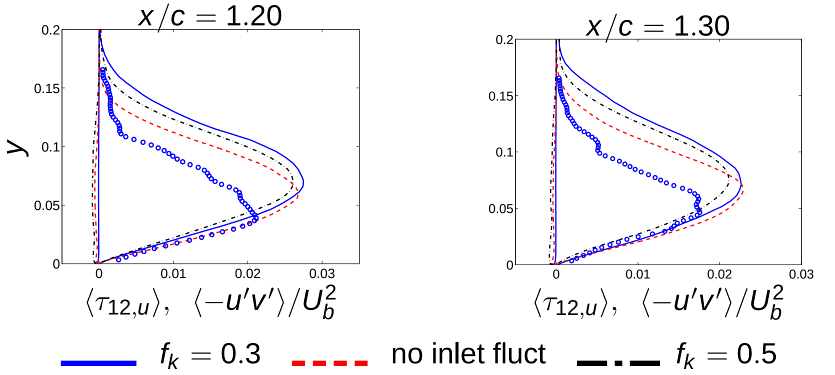
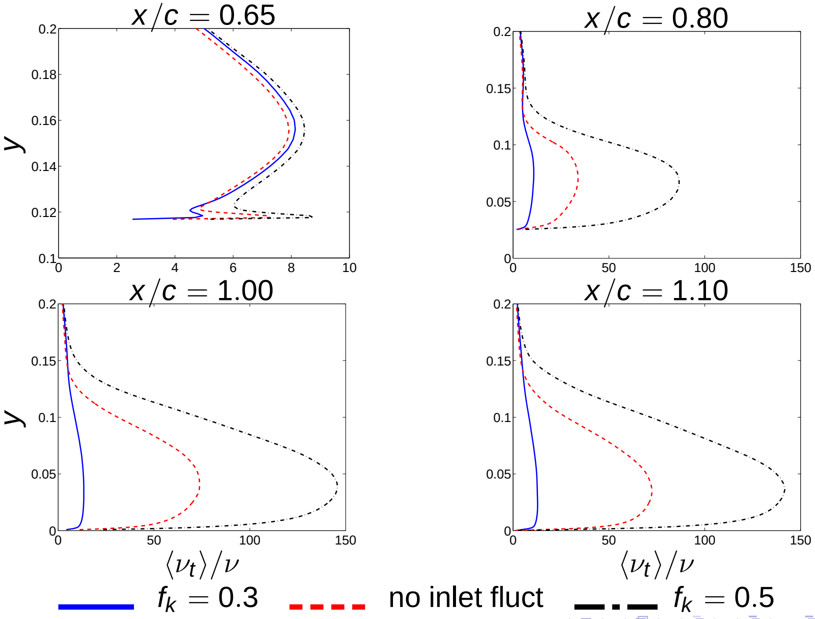
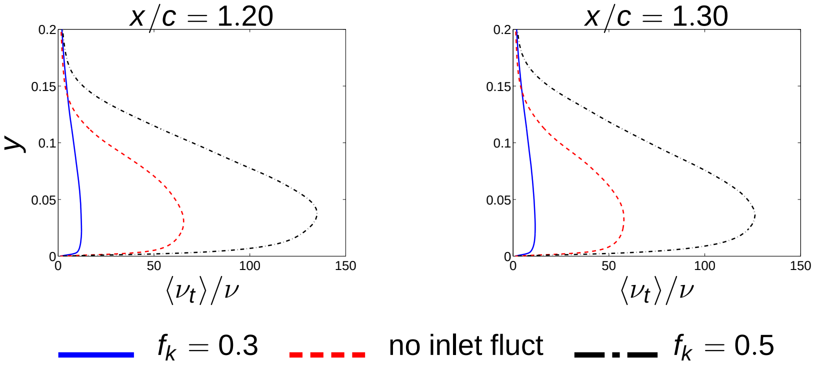


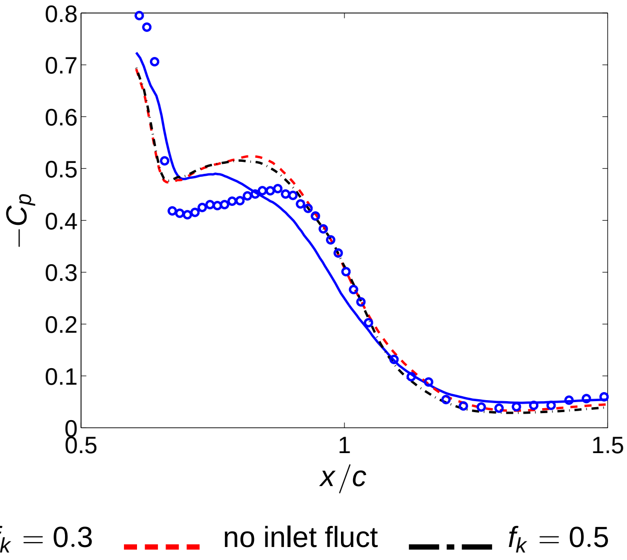
TURB VISCOSITY: $f_k = 0.3$; NO INLET FLUCT; $f_k = 0.5$



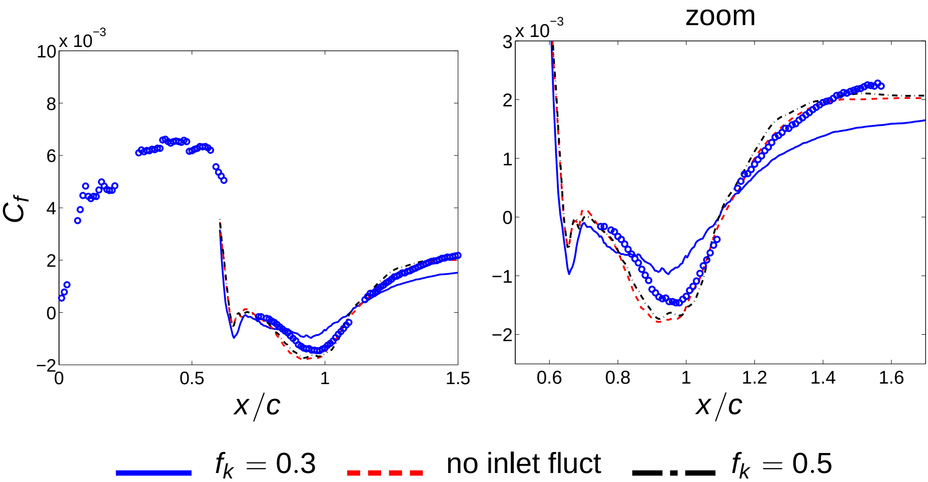
TURB VISCOSITY: $f_k = 0.3$; NO INLET FLUCT; $f_k = 0.5$



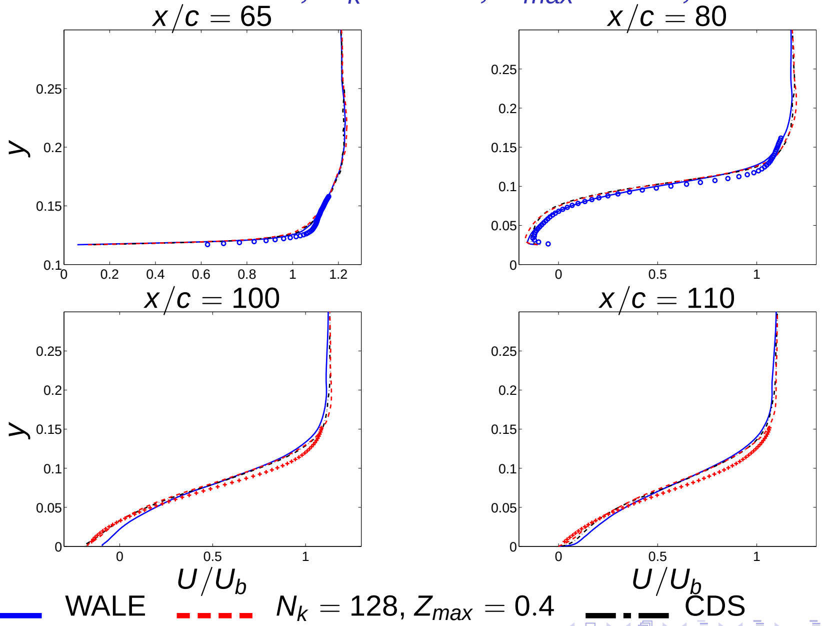
PRESSURE: WALE, $N_k = 128$, $Z_{max} = 0.4$, CDS



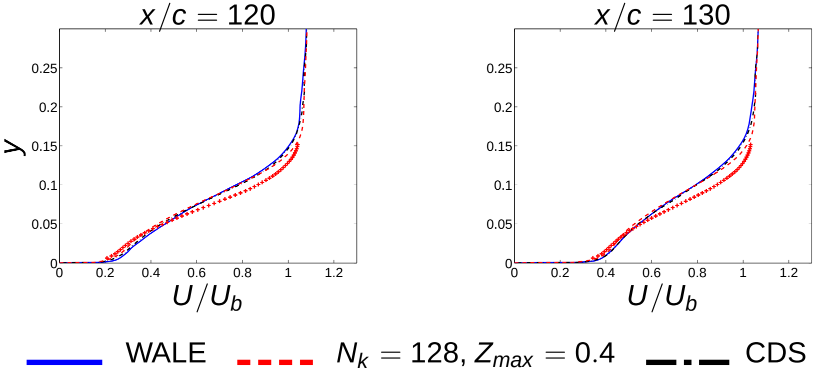
Skin Friction: WALE, $N_k = 128$, $Z_{max} = 0.4$, CDS



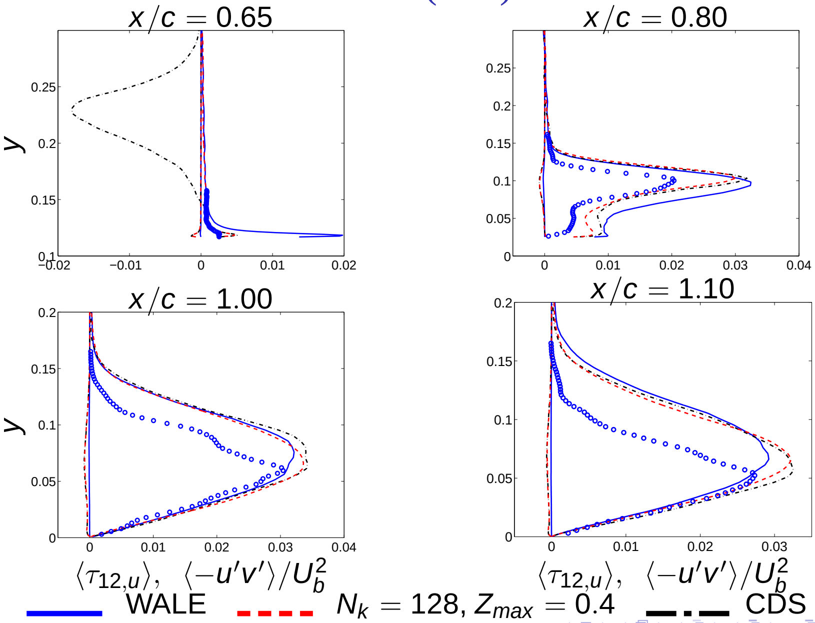
VELOCITIES: WALE, $N_k = 128$, $Z_{max} = 0.4$, CDS



Velocities: WALE, $N_k = 128$, $Z_{max} = 0.4$, CDS

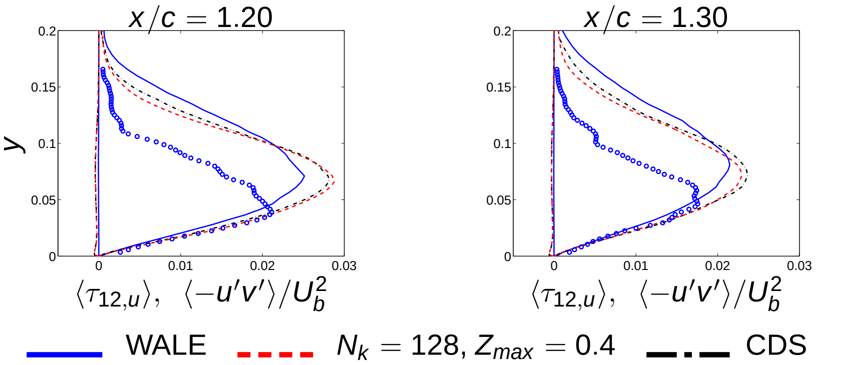


RESOLVED AND MODELLED (< 0) SHEAR STRESSES

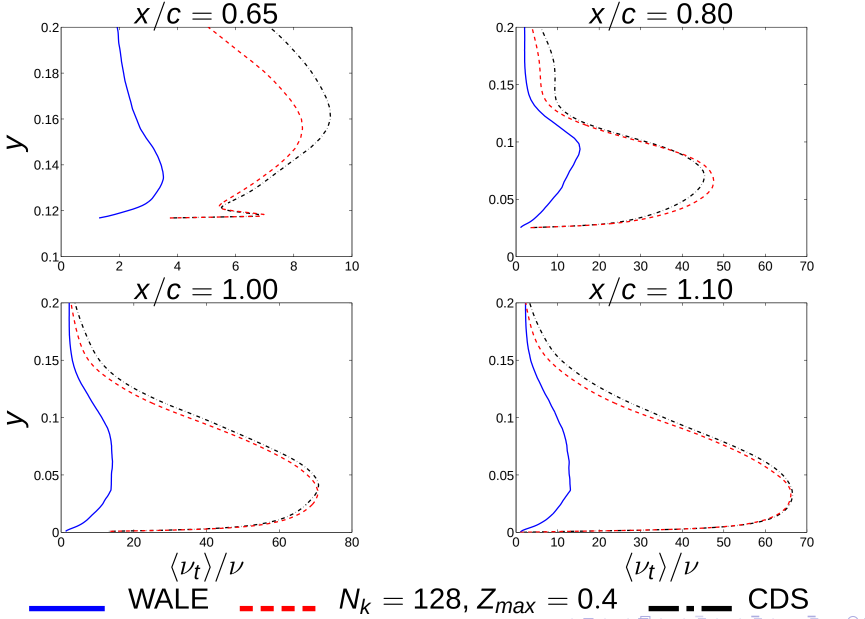


Shear stresses: WALE, $N_k = 128$, $Z_{max} = 0.4$, CDS

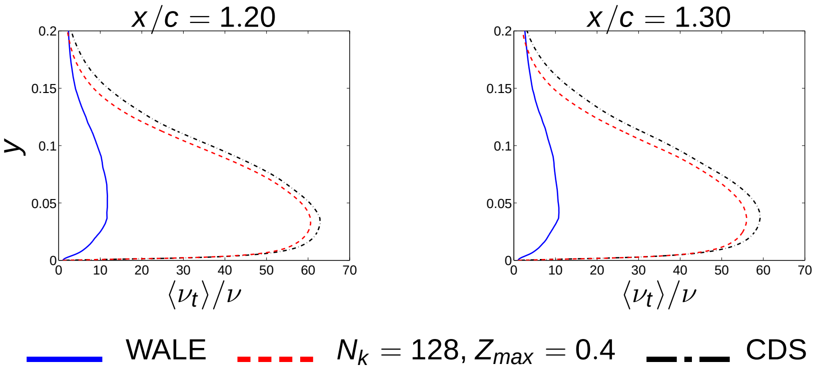
- Resolved and Modelled (< 0) Shear stresses



TURB VISCOSITY: WALE, $N_k = 128$, $Z_{max} = 0.4$, CDS



TURB VISCOSITY: WALE, $N_k = 128$, $Z_{max} = 0.4$, CDS



Concluding Remarks

- LRN PANS has been shown to work well as an embedded LES method
- Channel flow: At two δ downstream the interface, the resolved turbulence in good agreement with DNS data and the wall friction velocity has reached 99% of its fully developed value.
- Channel flow: The treatment of the modelled k_u and ε_u across the interface is important.
- LRN PANS predicts the hump flow well. LRN PANS better than WALE.
- Hump flow: CDS gives unphysical fluctuations near the inlet. The because larger the smaller the inlet fluctuations

[1] ABE, K., KONDOH, T., AND NAGANO, Y.
A new turbulence model for predicting fluid flow and heat transfer in separating and reattaching flows - 1. Flow field calculations.
Int. J. Heat Mass Transfer 37 (1994), 139–151.

[2] DAVIDSON, L., AND PENG, S.-H.
Emdedded LES with PANS.
In *6th AIAA Theoretical Fluid Mechanics Conference, AIAA paper 2011-3108* (27-30 June, Honolulu, Hawaii, 2011).

[3] DAVIDSON, L., AND PENG, S.-H.
Embedded LES using PANS applied to flow in a channel and over a hump (submitted).
AIAA Journal (2012).

[4] MA, J., PENG, S.-H., DAVIDSON, L., AND WANG, F.
A low Reynolds number variant of Partially-Averaged Navier-Stokes model for turbulence.
International Journal of Heat and Fluid Flow 32 (2011), 652–669. 10.1016/j.ijheatfluidflow.2011.02.001.