# LARGE EDDY SIMULATION OF HEAT TRANSFER IN BOUNDARY LAYER AND BACKSTEP FLOW USING PANS [4]
# LARS DAVIDSON

Lars Davidson, www.tfd.chalmers.se/˜lada

# PANS Low Reynolds Number Model [7]

$$\frac{\partial k_u}{\partial t} + \frac{\partial (k_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{ku}}\right)\frac{\partial k_u}{\partial x_j}\right] + (P_u - \varepsilon_u)$$

$$\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial (\varepsilon_u U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon_u}{\partial x_j}\right] + C_{\varepsilon 1} P_u \frac{\varepsilon_u}{k_u} - C^*_{\varepsilon 2}\frac{\varepsilon_u^2}{k_u}$$

$$\nu_u = C_\mu f_\mu \frac{k_u^2}{\varepsilon_u}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

$C_{\varepsilon 1}$, $C_{\varepsilon 2}$, $\sigma_k$, $\sigma_\varepsilon$ and $C_\mu$ same values as [1]. $f_\varepsilon = 1$. $f_2$ and $f_\mu$ read

$$f_2 = \left[1 - \exp\left(-\frac{y^*}{3.1}\right)\right]^2 \left\{1 - 0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}$$

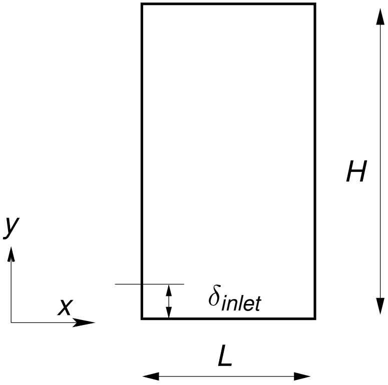
$$f_\mu = \left[1 - \exp\left(-\frac{y^*}{14}\right)\right]^2 \left\{1 + \frac{5}{R_t^{3/4}}\exp\left[-\left(\frac{R_t}{200}\right)^2\right]\right\}$$

- Baseline model: $f_k = 0.4$.

# Numerical Method

- Incompressible finite volume method
- Pressure-velocity coupling treated with fractional step
- Differencing scheme for momentum eqns:
  - 95% $2^{nd}$ order central and 5% $2^{nd}$ order upwind differencing scheme (baseline) OR
  - 100% $2^{nd}$ order central differencing
- Hybrid $1^{st}$ order upwind/$2^{nd}$ order central scheme $k$ & $\varepsilon$ eqns.
- $2^{nd}$-order Crank-Nicholson for time discretization

# BOUNDARY LAYER FLOW: DOMAIN




- Inlet: $\delta_{inlet} = 1$ (covered by 45 cells), $Re_\theta = 3\,600$, $U_{in} = \rho = 1$. Stretching 1.12 up to $y/\delta \simeq 1$.
- Domain: $L/\delta_{in} = 3.2$, $H/\delta_{in} = 15.6$, $Z_{max} = 1.5\delta_{in}$
- Resolution: $\Delta z^+_{in} \simeq 27$, $\Delta x^+_{in} \simeq 54$
- Grid: $66 \times 96 \times 64$ $(x, y, z)$

# Anisotropic Synthetic Fluctuations: I [3, 2, 5]

- Prescribe an homogeneous Reynolds tensor, $\overline{u_i u_j}$ (here from DNS)
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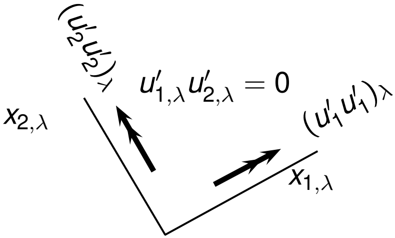
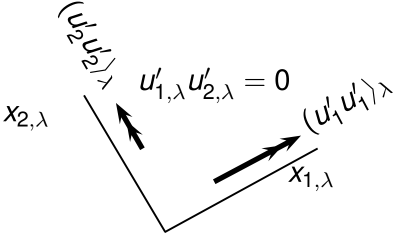


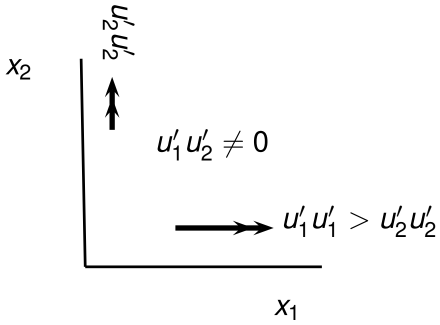

- Prescribe an homogeneous Reynolds tensor, $\overline{u_i u_j}$ (here from DNS)
- isotropic fluctuations in principal directions, $(u'_1 u'_1)_\lambda = (u'_2 u'_2)_\lambda$, $u'_{1,\lambda} u'_{2,\lambda} = 0$
-

# ANISOTROPIC SYNTHETIC FLUCTUATIONS: I [3, 2, 5]




- Prescribe an homogeneous Reynolds tensor, $\overline{u_iu_j}$ (here from DNS)
- isotropic fluctuations in principal directions, $(u'_1u'_1)_\lambda = (u'_2u'_2)_\lambda$, $u'_{1,\lambda}u'_{2,\lambda} = 0$
- re-scale the normal components, $(u'_1u'_1)_\lambda > (u'_2u'_2)_\lambda$, using the eigenvalues $u'_{1,\lambda}u'_{2,\lambda} = 0$

# Anisotropic Synthetic Fluctuations: II

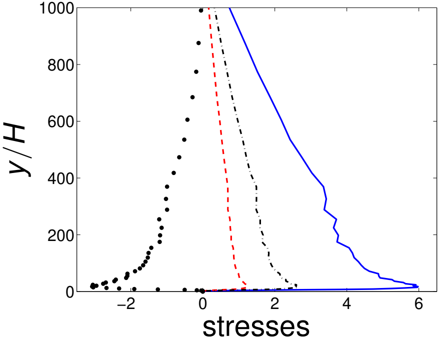


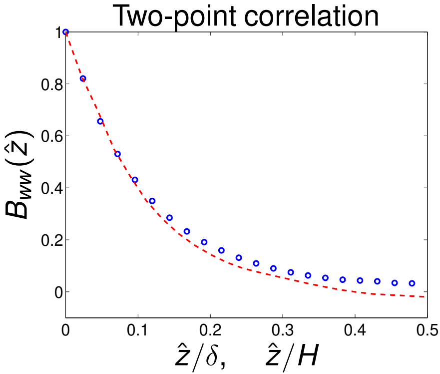

- Transform from $(x_{1,\lambda}, x_{2,\lambda})$ to $(x_1, x_2)$
- $\frac{u'^2_1}{u'^2_2} = 23$, $\frac{u'^2_1}{u'^2_3} = 5$ from $(u'_1 u'_1)_{peak}$ in DNS channel flow, $Re_\tau = 500$

# INLET CONDITIONS FOR $k_u$ AND $\varepsilon_u$ AS IN [6]

- A pre-cursor RANS simulation using the AKN model (i.e. PANS with $f_k = 1$) is carried out. At $Re_\theta = 3\,600$, $U_{RANS}$, $V_{RANS}$, $k_{RANS}$ are taken.
- $\bar{u}_{in} = U_{RANS} + u'_{synt}$, $\bar{v}_{in} = V_{RANS} + v'_{synt}$, $\bar{w}_{in} = w'_{synt}$
- Anisotropic synthetic fluctuations are used. The fluctuations are scaled with $k_u/k_{u,max}$.
- $k_{u,in} = f_k k_{RANS}$, $\varepsilon_{u,in} = C_\mu^{3/4} k_{u,in}^{3/2}/\ell_{sgs}$, $\ell_{sgs} = C_s\Delta$, $\Delta = V^{1/3}$, $C_s = 0.05$

# INLET TURB. FLUCTUATION, TWO-POINT CORRELATIONS

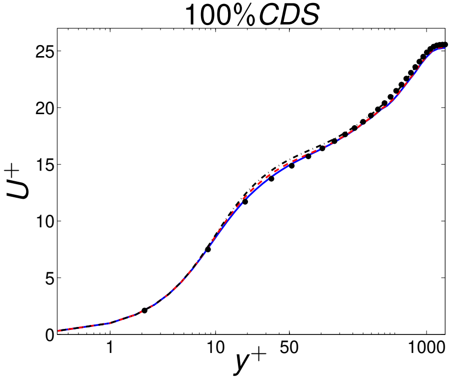


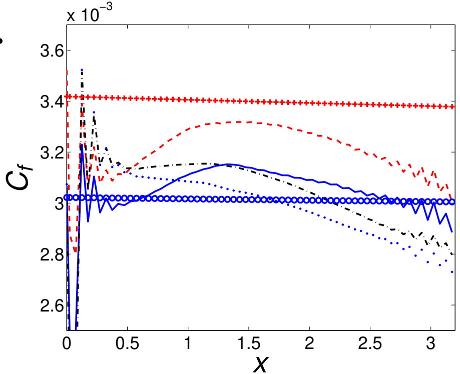




—— : $u^+_{rms}$, - - - : $v^+_{rms}$, - · - : $w^+_{rms}$
· · · · : $\langle u'v' \rangle^+$

∘: inlet; - - - : $x = 3\delta_{in}$

# BOUNDARY LAYER: VELOCITY AND SKIN FRICTION

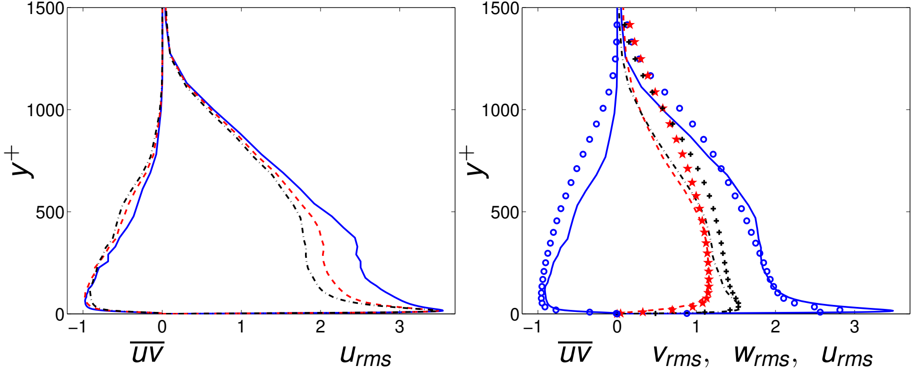



——— : $x = \delta_{in}$; - - - : $x = 2\delta_{in}$; - . - : $x = 3\delta_{in}$; ■: DNS [8]

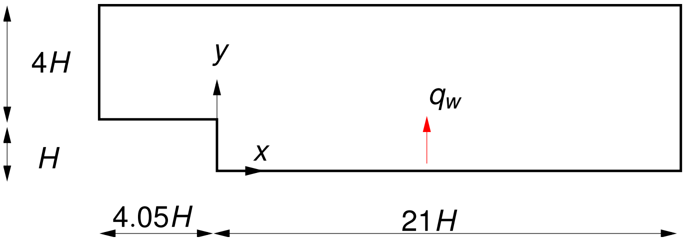
——— : 100% CDS; - - - : 100% CDS, $U_{in}$ from AKN; - . - : 25% larger inlet fluct.; ····: 25% larger inlet fluct., $C_s = 0.07$; markers: $0.37\,(log_{10}Re_x)^{-2.584}$ (+: AKN; ∘: DNS);

# Reynolds Stresses



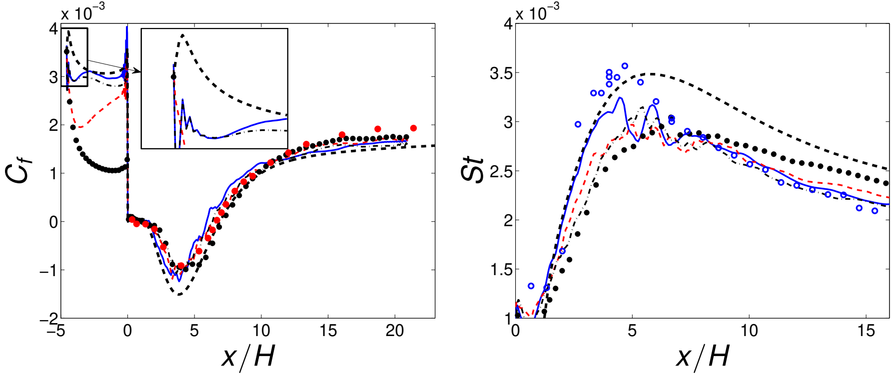

——— : $x = \delta_{in}$; - - - : $x = 2\delta_{in}$; - · - · : $x = 3\delta_{in}$; Markers: DNS [8]
$x = 3\delta_{in}$.

# Backward Facing Step: Domain



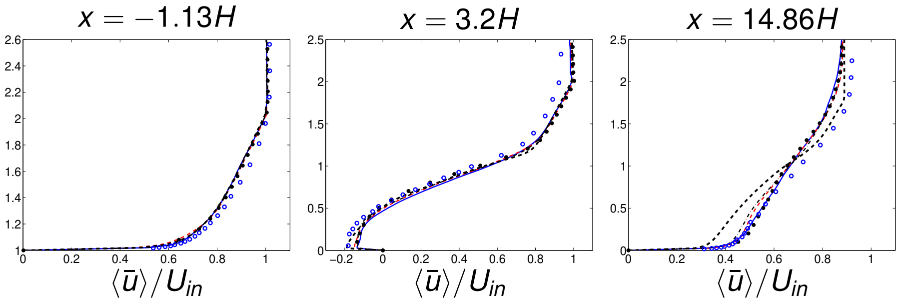

- $Re_H = 28\,000$ Experiments by Vogel & Eaton [9]
- Mean inlet profiles from RANS (same as in boundary layer)
- Grid: $336 \times 120$ in $x \times y$ plane. $Z_{max} = 1.6H$, $N_k = 64$, $\Delta z^+_{in} = 31$.
- Anisotropic synthetic fluctuations, $u'$, $v'$, $w'$ (same as for boundary layer flow); no fluctuations for $t'$
- Constant heat flux, $q_w$, on lower wall.

# Backstep flow. Skin friction and Stanton number



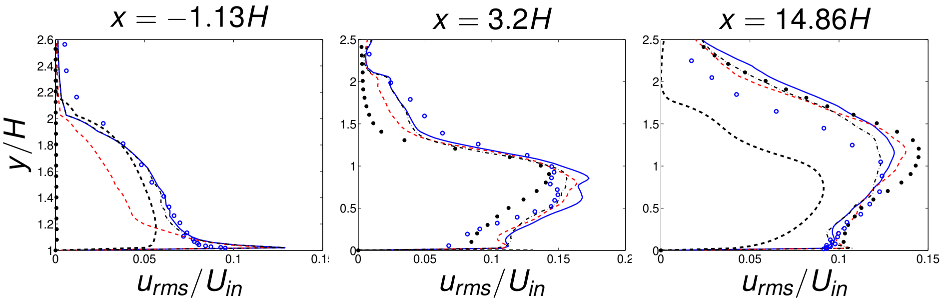

—— : PANS; - - - : PANS, 50% smaller inlet fluctuations; - · - : WALE; •: PANS, no inlet fluctuations; - - - : 2D RANS; ∘,•: experiments [9].

# BACKSTEP FLOW: VELOCITIES.



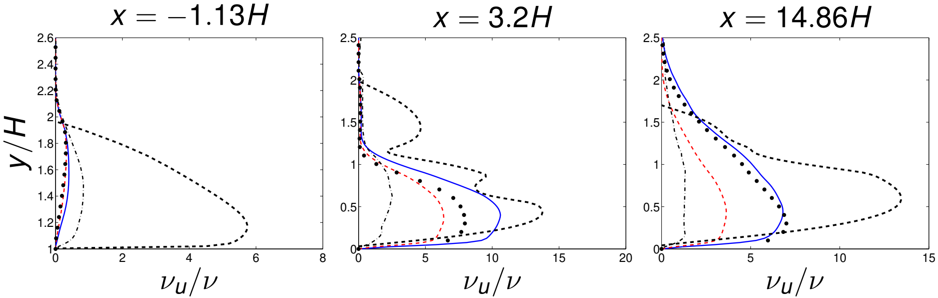

—— : PANS; - - - : PANS, 50% smaller inlet fluctuations; - - - : WALE;
•: PANS, no inlet fluctuations; - - - : 2D RANS; ○: experiments [9].

# BACKSTEP FLOW: RESOLVED STREAMWISE STRESS.




—— : PANS; - - - : PANS, 50% smaller inlet fluctuations; - · - : WALE;
•: PANS, no inlet fluctuations; - - - : 2D RANS; ○: experiments [9].

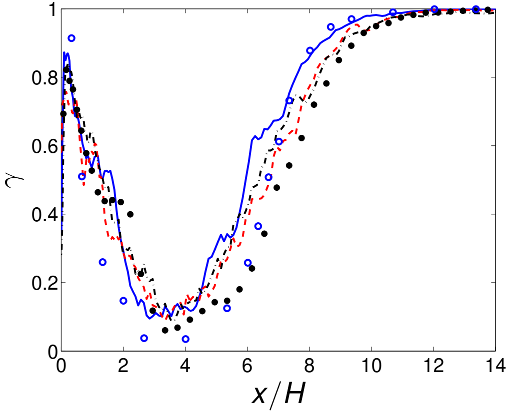
# Backstep flow: Turbulent viscosities.




—— : PANS; - - - : PANS, 50% smaller inlet fluctuations; - · - : WALE;
•: PANS, no inlet fluctuations; - - - : 2D RANS/10;

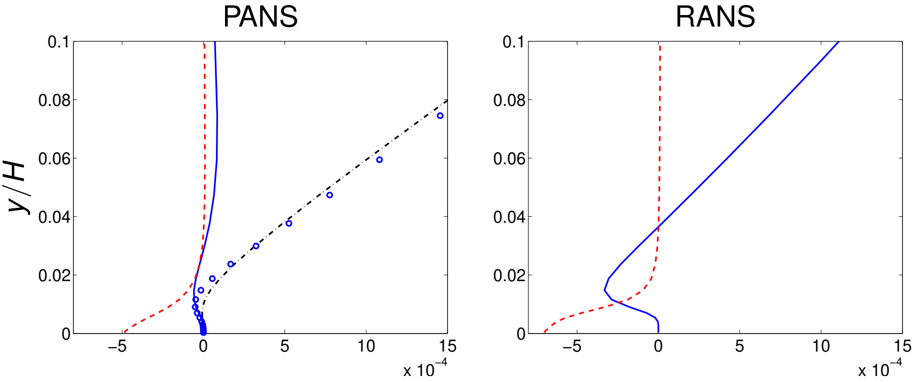
# Forward/Backward Flow

- Fraction of time, $\gamma$, when the flow along the bottom wall is in the downstream direction.



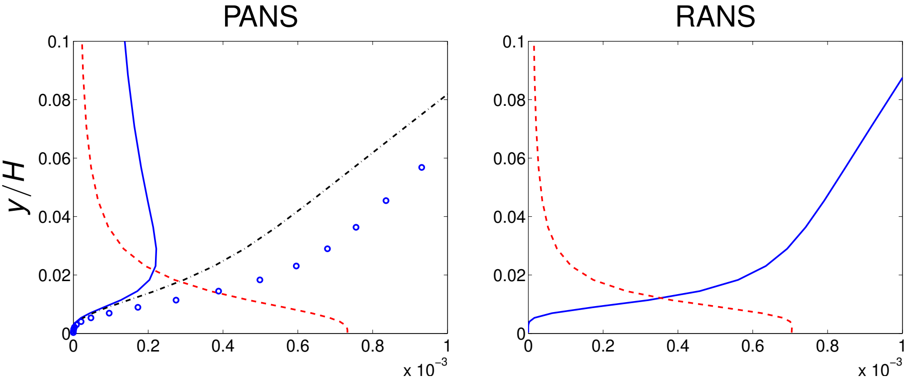

—— : PANS; - - - : PANS, 50% smaller inlet fluctuations; - · - : WALE;
•: PANS, no inlet fluctuations; ∘: experiments [9].

# SHEAR STRESSES. $x = 3.2H$



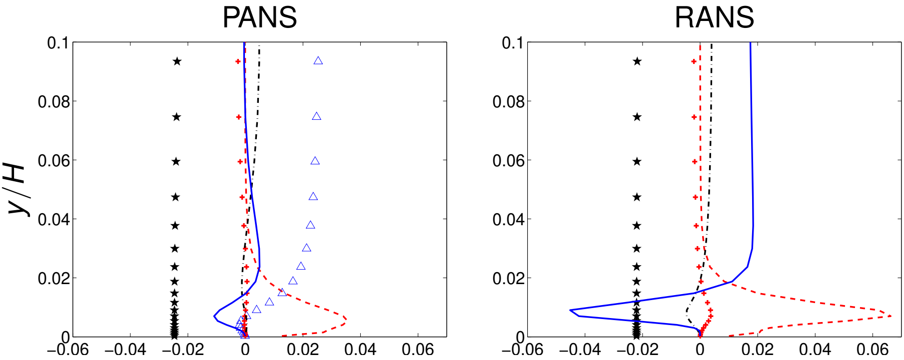

—— : $2\langle\nu_t \bar{s}_{12}\rangle$; - - - : $\nu \dfrac{\partial\langle\bar{u}\rangle}{\partial y}$; - · - : $-\langle\overline{uv}\rangle$; ○: $2\langle\nu_t \bar{s}_{12}\rangle - \langle\overline{uv}\rangle$.

# SHEAR STRESSES. $x = 14.86$



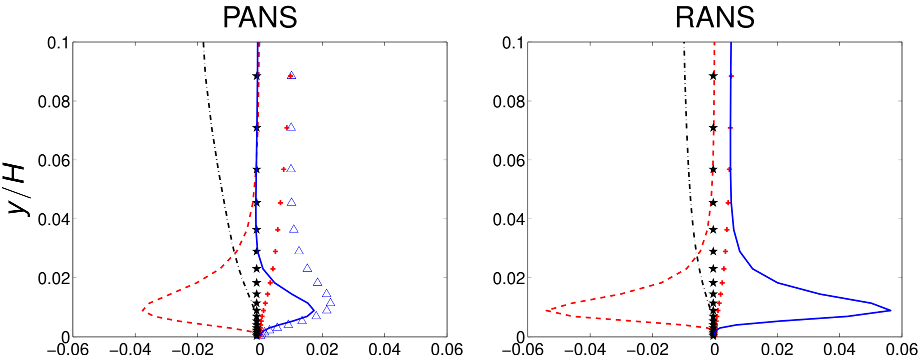

$—$ : $2\langle \nu_t \bar{s}_{12} \rangle$; $- - -$ : $\nu \frac{\partial \langle \bar{u} \rangle}{\partial y}$; $-\cdot-$ : $-\langle \overline{uv} \rangle$; $\circ$: $2\langle \nu_t \bar{s}_{12} \rangle - \langle \overline{uv} \rangle$.

# Terms in the $\langle \bar{u} \rangle$ Equation. $x = 3.2H$



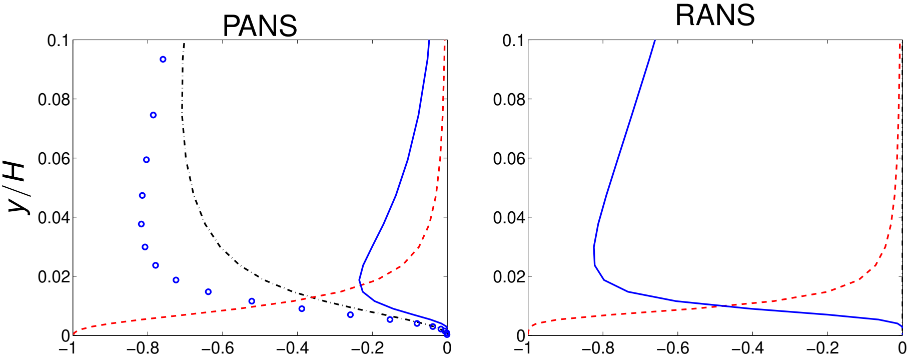

$—$: $\frac{\partial}{\partial y}(2\langle \nu_t \bar{s}_{12} \rangle)$; $---$: $\nu \frac{\partial^2 \langle \bar{u} \rangle}{\partial y^2}$; $-\cdot-$: $-\frac{\partial \langle \bar{u} \rangle \langle \bar{u} \rangle}{\partial x}$; $+$: $-\frac{\partial \langle \bar{u} \rangle \langle \bar{v} \rangle}{\partial y}$; $\star$: $-\frac{\partial \langle \bar{p} \rangle}{\partial x}$, $\triangle$: $-\frac{\partial \langle \overline{uv} \rangle}{\partial y}$.

# Terms in the $\langle \bar{u} \rangle$ Equation. $x = 14.86H$



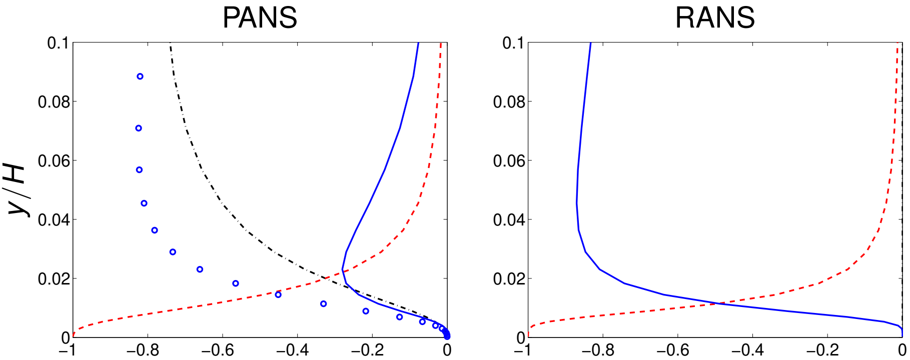

$-$ : $\frac{\partial}{\partial y}\left(2\langle \nu_t \bar{s}_{12} \rangle\right)$; $--$ : $\nu \frac{\partial^2 \langle \bar{u} \rangle}{\partial y^2}$; $-\cdot-$ : $-\frac{\partial \langle \bar{u} \rangle \langle \bar{u} \rangle}{\partial x}$; $+$: $-\frac{\partial \langle \bar{u} \rangle \langle \bar{v} \rangle}{\partial y}$; $\star$: $-\frac{\partial \langle \bar{p} \rangle}{\partial x}$, $\triangle$: $-\frac{\partial \langle \overline{uv} \rangle}{\partial y}$.

# HEAT FLUXES. $x = 3.2H$



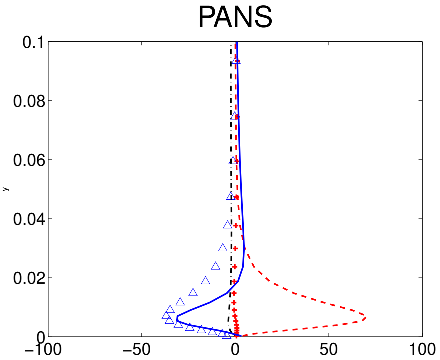

$$— : \left\langle \frac{\nu_t}{\sigma_t}\frac{\partial \bar{t}}{\partial y} \right\rangle ; \; - - - : \frac{\nu}{\sigma_\ell}\frac{\partial \langle \bar{t} \rangle}{\partial y} ; \; -\cdot-\cdot- : -\langle \overline{v' t'} \rangle . \; \circ : \left\langle \frac{\nu_t}{\sigma_t}\frac{\partial \bar{t}}{\partial y} \right\rangle - \langle \overline{v' t'} \rangle .$$

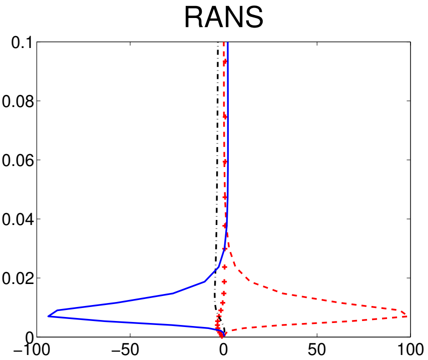
# HEAT FLUXES. $x = 14.86H$




$—: \left\langle \frac{\nu_t}{\sigma_t} \frac{\partial \bar{t}}{\partial y} \right\rangle$; $---: \frac{\nu}{\sigma_\ell} \frac{\partial \langle \bar{t} \rangle}{\partial y}$; $-\cdot-: -\langle \overline{v' t'} \rangle$. $\circ$: $\left\langle \frac{\nu_t}{\sigma_t} \frac{\partial \bar{t}}{\partial y} \right\rangle - \langle \overline{v' t'} \rangle$.

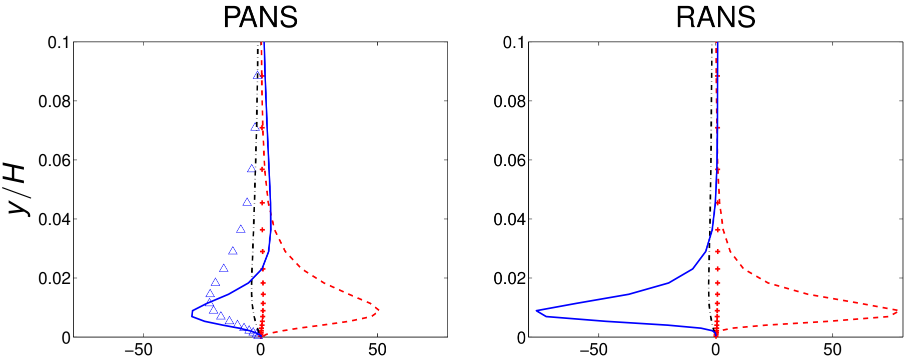
# Terms in the $\langle \bar{t} \rangle$ equation. $x = 3.2H$




—— : $\frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_t}\frac{\partial \langle \bar{t} \rangle}{\partial y}\right)$; - - - : $\frac{\nu}{\sigma_\ell}\frac{\partial^2 \langle \bar{t} \rangle}{\partial y^2}$; - · - : $-\frac{\partial \langle \bar{u} \rangle \langle \bar{t} \rangle}{\partial x}$; +: $-\frac{\partial \langle \bar{v} \rangle \langle \bar{t} \rangle}{\partial y}$; △: $-\frac{\partial \langle \overline{v' t'} \rangle}{\partial y}$.

# Terms in the $\langle \bar{t} \rangle$ Equation. $x = 14.86H$




$$— : \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_t}\frac{\partial \langle \bar{t} \rangle}{\partial y}\right); \; - - - : \frac{\nu}{\sigma_\ell}\frac{\partial^2 \langle \bar{t} \rangle}{\partial y^2}; \; -\cdot - : -\frac{\partial \langle \bar{u} \rangle \langle \bar{t} \rangle}{\partial x}; \; + : -\frac{\partial \langle \bar{v} \rangle \langle \bar{t} \rangle}{\partial y}; \; \triangle : -\frac{\partial \langle \overline{v' t'} \rangle}{\partial y}.$$

# CONCLUDING REMARKS

- Developing boundary layer
  - Synthetic fluctuations give fully developed conditions after a couple of boundary layer thicknesses
  - 5% upwinding dampens resolved fluctuations; can be compensated by 25% larger inlet fluctuations
- Backstep flow
  - Very good agreement with experiments
  - 2D RANS predicts turbulent diffusion surprisingly well
  - Synthetic inlet fluctuations give an improved Stanton number; otherwise small effect in the reciculation region
  - LRN PANS and WALE equally good
  - 5% upwinding has a negligible effect in the recirculation region

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