Large Eddy Simulations in Industry

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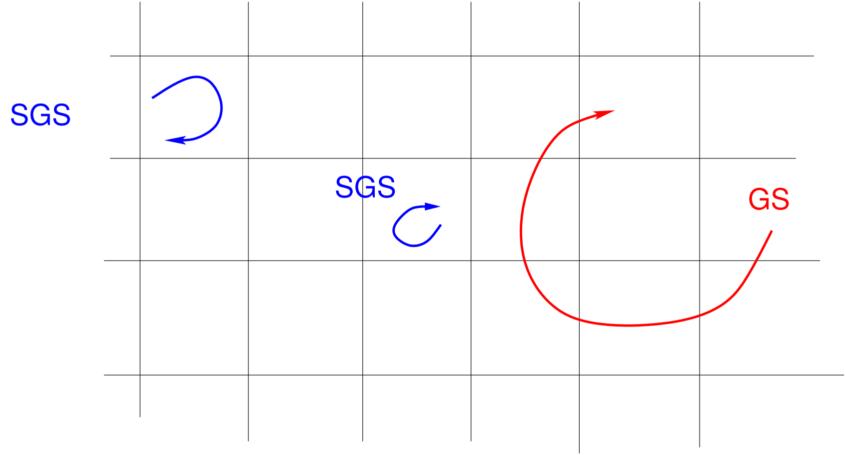
Three-Day CFD Course at Chalmers

▶ This lecture is a condensed version of the course

- Unsteady Simulations for Industrial Flows: LES, DES, hybrid LES-RANS and URANS, 11-13 November 2019, http://www.cfd-sweden.se/
- Turbulence modeling, 8 weeks MSc course, see http://www.tfd.chalmers.se/˜lada/comp_turb_model/
- Course literature: eBook [3].

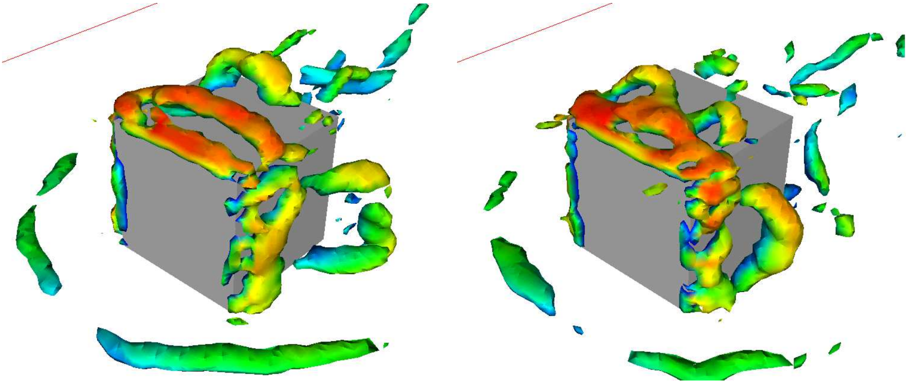
LARGE EDDY SIMULATIONS



- In LES, large (Grid) Scales (GS) are resolved and the small (Sub-Grid) Scales (SGS) are modelled.
- LES is suitable for bluff body flows where the flow is governed by large turbulent scales

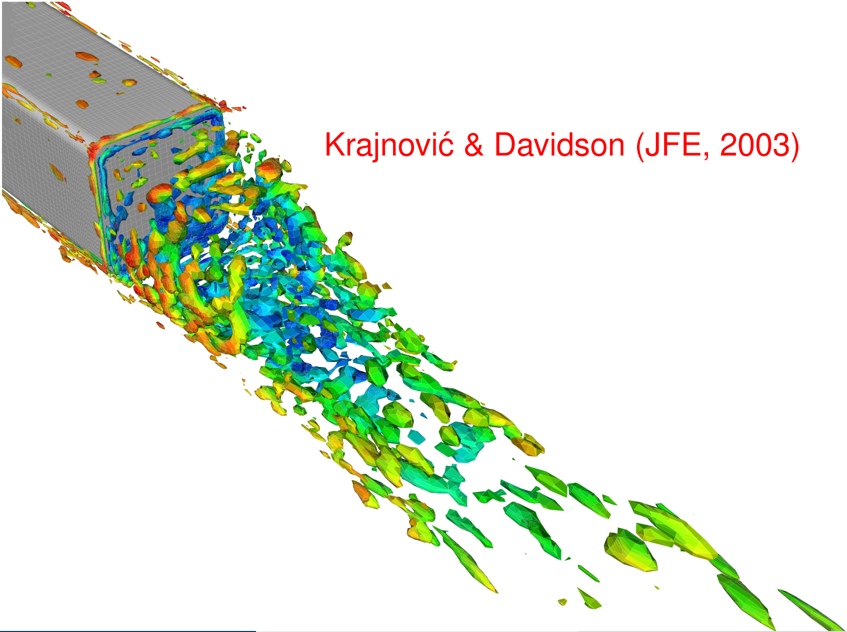
BLUFF-BODY FLOW: SURFACE-MOUNTED CUBE[5]

Krajnović & Davidson (AIAA J., 2002)



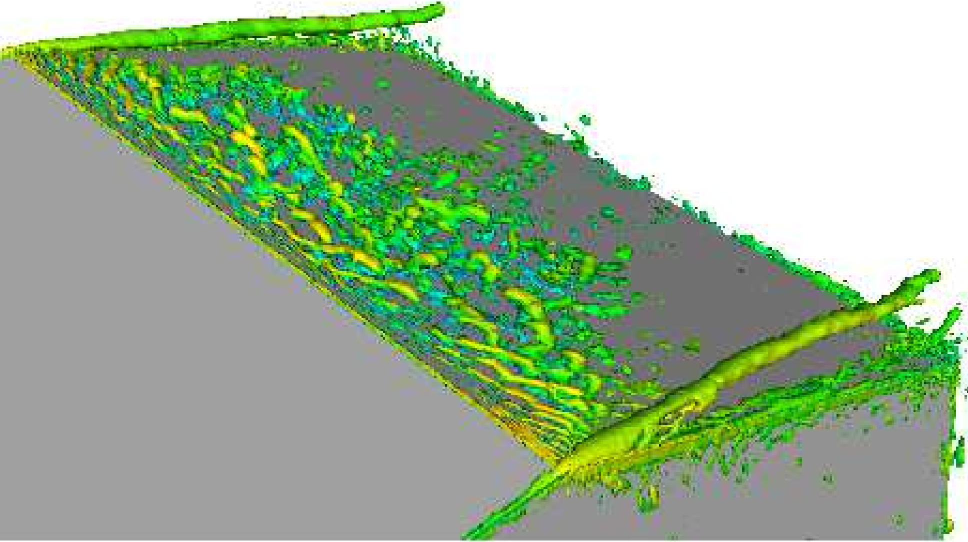
Snapshots of large turbulent scales illustrated by $Q = -\frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_i}$

BLUFF-BODY FLOW: FLOW AROUND A BUS[6]

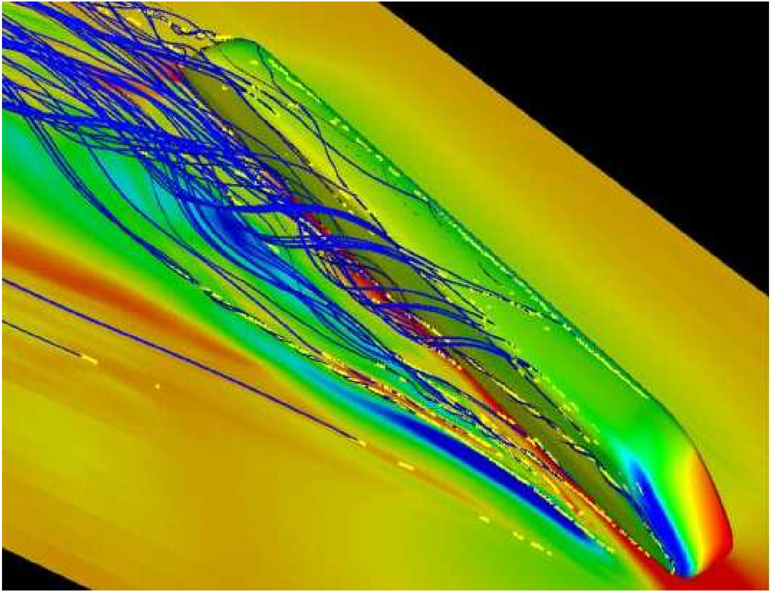


Krajnović & Davidson (JFE, 2003)

BLUFF-BODY FLOW: FLOW AROUND A CAR[7]

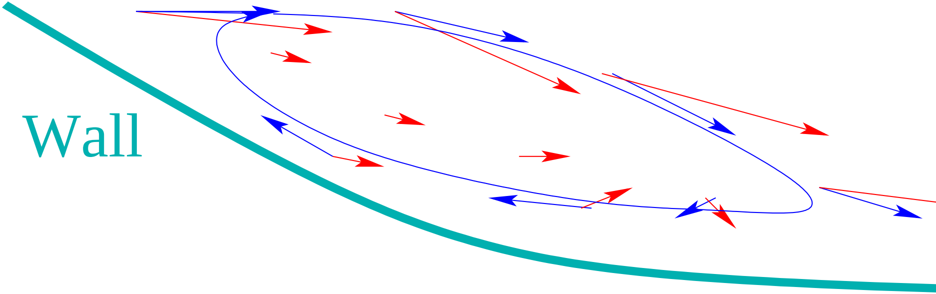


BLUFF-BODY FLOW: FLOW AROUND A TRAIN[4]



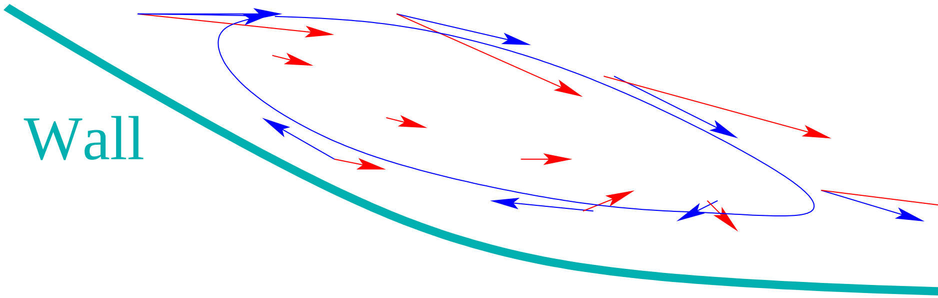
Hemida & Krajnović, 2006

SEPARATING FLOWS



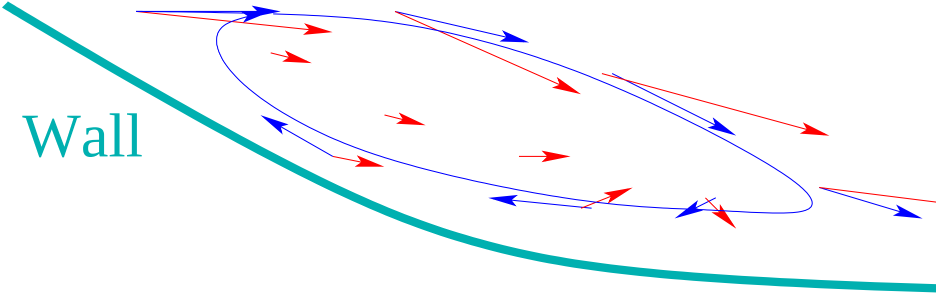
- TIME-AVERAGED flow and INSTANTANEOUS flow

Separating Flows



- TIME-AVERAGED flow and INSTANTANEOUS flow
- In average there is backflow (negative velocities). Instantaneous, the negative velocities are often positive.

SEPARATING FLOWS



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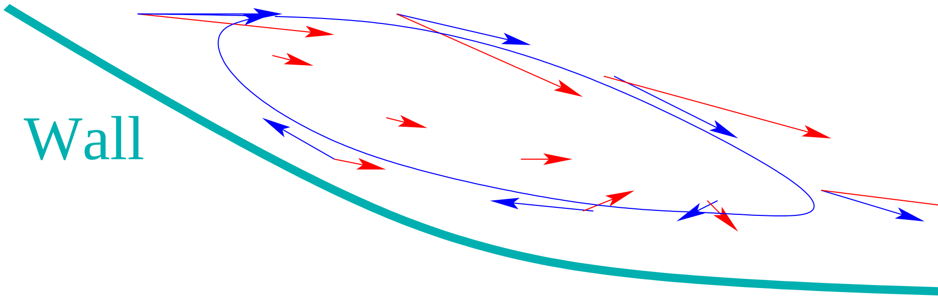

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- Is it reasonable to require a turbulence model to fix this?

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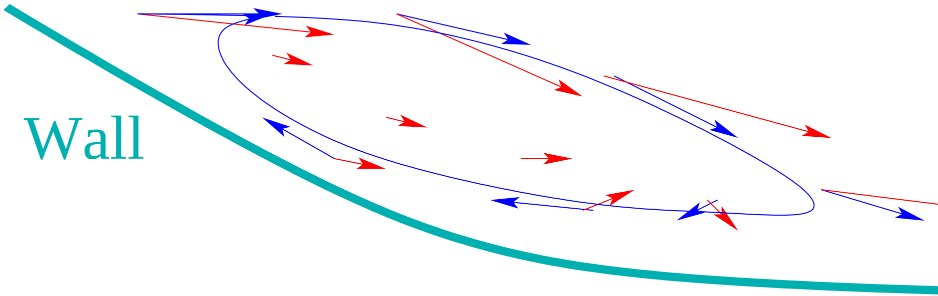

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- How easy is it to model fluctuations that are as large as the mean flow?
- Is it reasonable to require a turbulence model to fix this?
- Isn't it better to RESOLVE the large fluctuations?

TIME AVERAGING AND FILTERING

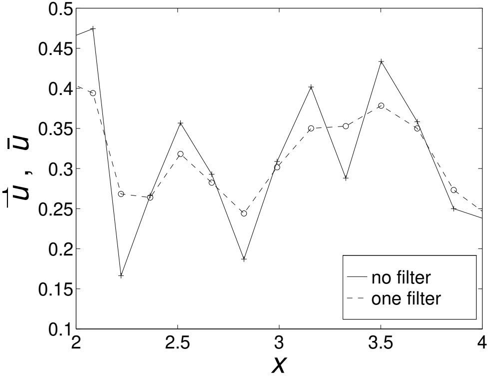
RANS: time average. This is called Reynolds time averaging:

$$\langle \Phi \rangle = \frac{1}{2T}\int_{-T}^{T} \Phi(t)dt, \; \Phi = \langle \Phi \rangle + \Phi'$$

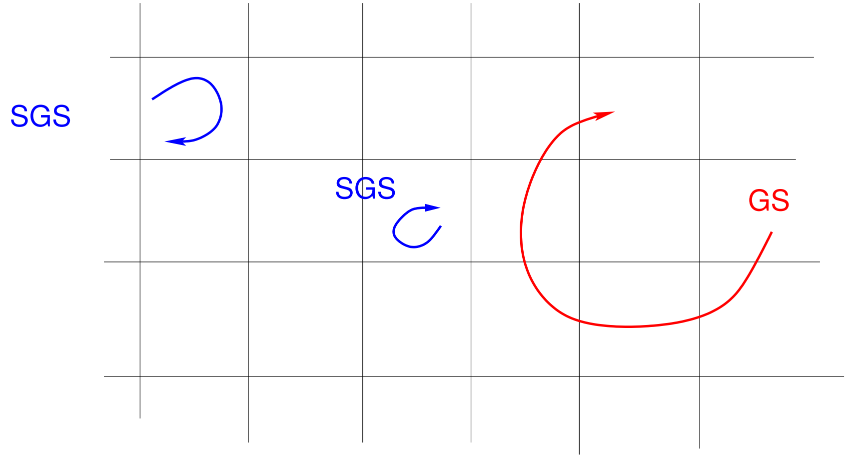
In LES we filter (volume average) the equations. In 1D we get:

$$\bar{\Phi}(x,t) = \frac{1}{\Delta x}\int_{x-0.5\Delta x}^{x+0.5\Delta x} \Phi(\xi, t)d\xi \quad (1$$

$$\Phi = \bar{\Phi} + \Phi''$$



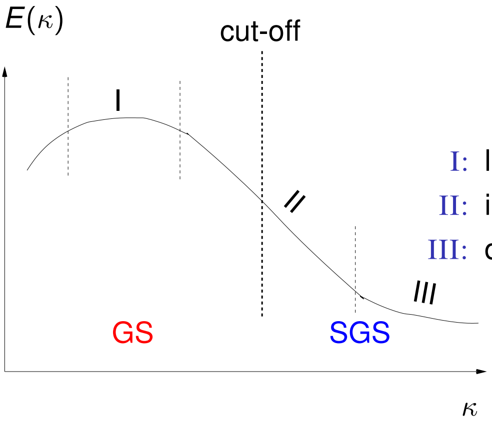
LARGE EDDY SIMULATIONS



- Large scales (GS) are resolved; small scales (SGS) are modelled.

ENERGY SPECTRUM

The limit (cut-off) between GS and SGS is supposed to take place in the inertial subrange (II)



I: large scales

II: inertial subrange, $-5/3$-range

III: dissipation subrange

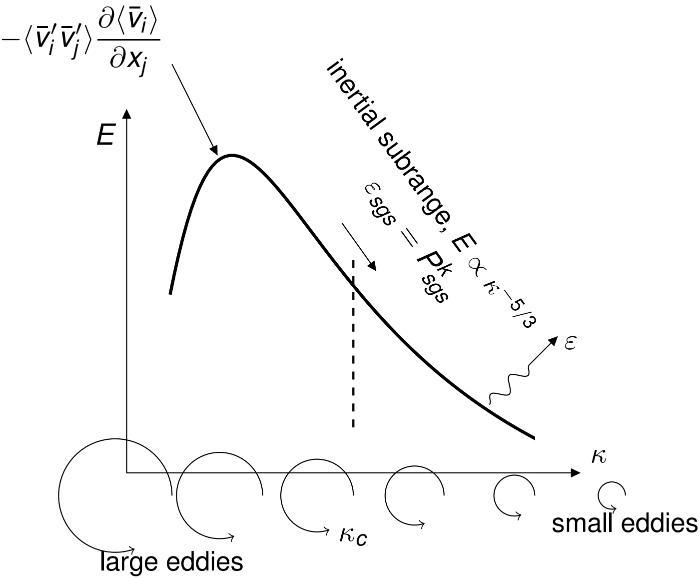
Subgrid model

- We need a subgrid model for the SGS turbulent scales
- The simplest model is the Smagorinsky model [9]:

$$\nu_{sgs} = (C_S \Delta)^2 \sqrt{2\bar{s}_{ij}\bar{s}_{ij}} \equiv (C_S\Delta)^2 |\bar{s}|$$
$$\bar{s}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right), \quad \Delta = (\Delta V_{IJK})^{1/3} \tag{2}$$

- In RANS we always use two-equation models (or more). But not in LES? Why?
  - In LES, less turbulence is modeled.
  - However, on coarse meshes, it may indeed be better to use one-equation (DES) or two-equation models (PANS)

Energy path



$-\langle \bar{v}'_i \bar{v}'_j \rangle \frac{\partial \langle \bar{v}_i \rangle}{\partial x_j}$
E
inertial subrange, $E \propto \kappa^{-5/3}$
$\varepsilon_{sgs} = P^k_{sgs}$
ε
κ
κ_c
large eddies
small eddies

LES vs. RANS

LES can handle many flows which RANS (Reynolds Averaged Navier Stokes) cannot; the reason is that in LES large, turbulent scales are resolved. Examples are:

- *o* Flows with large separation
- *o* Bluff-body flows (e.g. flow around a car); the wake often includes large, unsteady, turbulent structures
- *o* Transition

- In RANS all turbulent scales are modelled ⇒ inaccurate

- In LES only small, isotropic turbulent scales are modelled ⇒ accurate

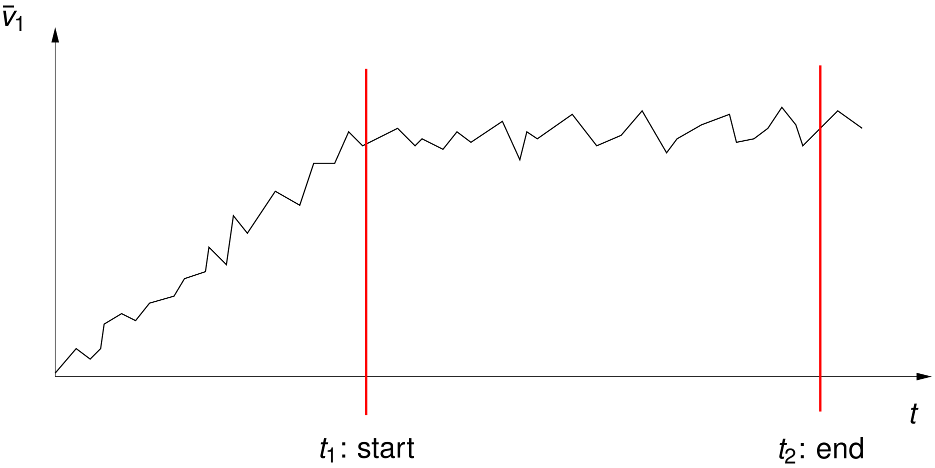
LES is *very* much more expensive than RANS.

Finite Volume RANS and LES Codes.

	RANS	LES
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	central differencing
Time discretization	1st order	2nd order (e.g. C-N)
Turbulence model	$\geq$ two-equations	zero- or one-eq

Time averaging in LES

- t_1: Start time averaging
- t_2: Stop time averaging



NEAR-WALL RESOLUTION

- Biggest problem with LES: near walls, it requires very fine mesh in all directions, not only in the near-wall direction.

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- The reason: violent violent low-speed outward ejections and high-speed in-rushes must be resolved (often called streaks).

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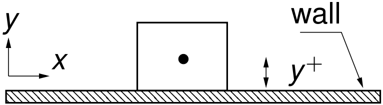
- In the presentation we use Hybrid LES-RANS for which the grid requirements are much smaller than for LES

Near-Wall Resolution Cont'd

- In RANS when using wall-functions, $30 < y^+ < 100$ for the wall-adjacent cells



Near-Wall Resolution Cont'd

- In RANS when using wall-functions, $30 < y^+ < 100$ for the wall-adjacent cells
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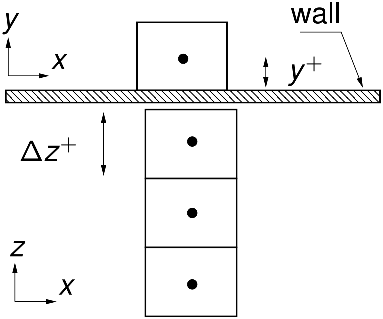
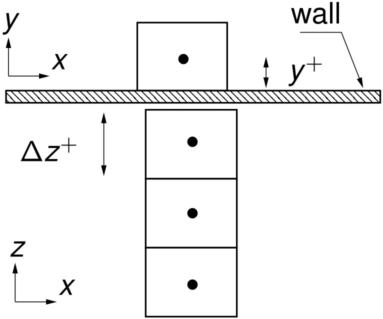


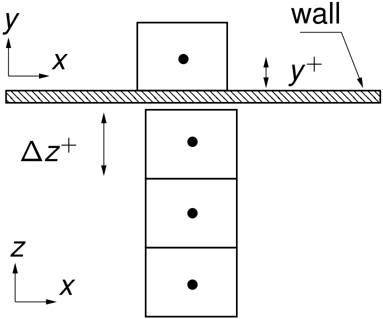
NEAR-WALL RESOLUTION CONT'D

- In RANS when using wall-functions, $30 < y^+ < 100$ for the wall-adjacent cells
- In LES, $\Delta z^+ \simeq 30$ EVERYWHERE

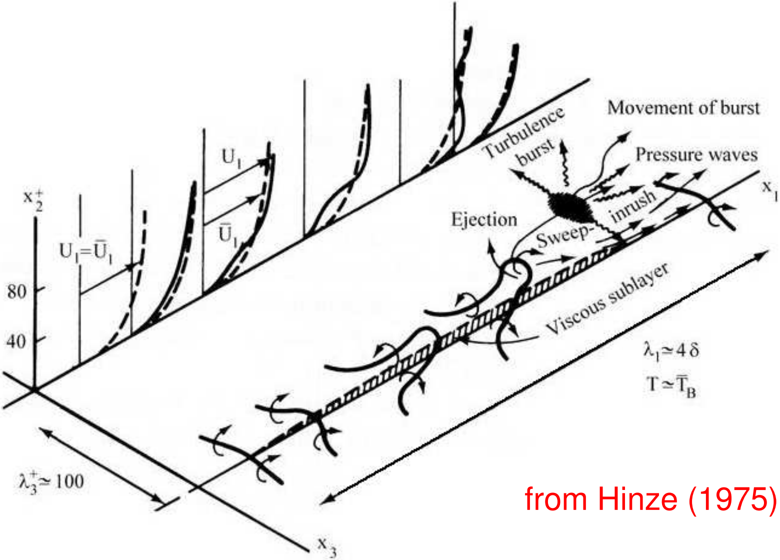


Near-Wall Resolution Cont'd

- In RANS when using wall-functions, $30 < y^+ < 100$ for the wall-adjacent cells
- In LES, $\Delta z^+ \simeq 30$ EVERYWHERE
- AND $\Delta x^+ \simeq 100$, $\Delta y^+_{min} \simeq 1$

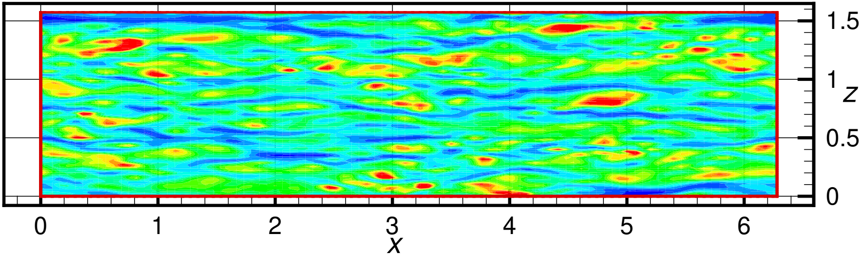


NEAR-WALL TREATMENT



from Hinze (1975)

Near-Wall Treatment



- Fluctuating streamwise velocity at $y^+ = 5$. DNS of channel flow.
- We find that the structures in the spanwise direction are very small which requires a very fine mesh in z direction.

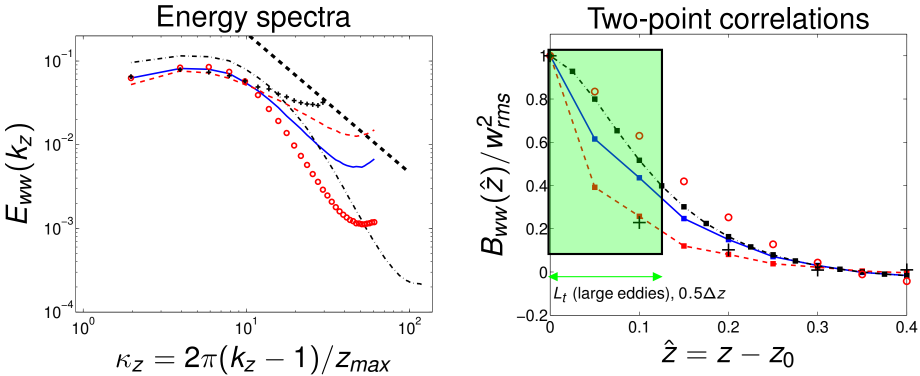
RESOLUTION

- For the near-wall region, we know how fine the mesh should be in terms of viscous units (see Slide 17)
- An appropriate resolution for the fully turbulent part of the boundary layer is $\delta/\Delta x \simeq 10 - 20$ and $\delta/\Delta z \simeq 20 - 40$
- This may be relevant also for jets and shear layers

How to estimate resolution in general? [1, 2]

- Energy spectra (both in spanwise direction and time)
- Two-point correlations
- Ratio of SGS turbulent kinetic energy $\langle k_{sgs} \rangle$ to resolved $0.5\langle u'u' + v'v' + w'w' \rangle$
- Ratio of SGS shear stress $\langle \tau_{sgs,12} \rangle$ to resolved $\langle u'v' \rangle$
- Ratio of SGS viscosity, $\langle \nu_{sgs} \rangle$ to molecular, ν

CHANNEL FLOW, $Re_\tau = 4000$, $y^+ = 440$



—— : $(\Delta x, \Delta z)$ - - - : $0.5\Delta x$ -·-· : $0.5\Delta z$ ∘ $2\Delta x$; $+$: $2\Delta z$

The $(\Delta x, \Delta z)$ mesh is $(\delta/\Delta x, \delta/\Delta z) = (10, 20)$

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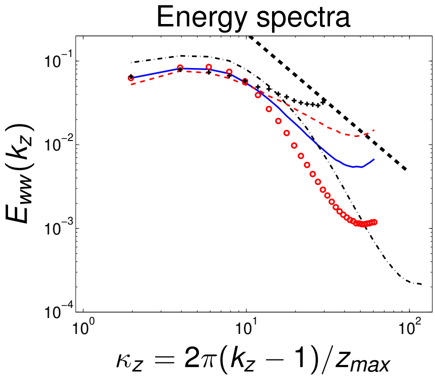

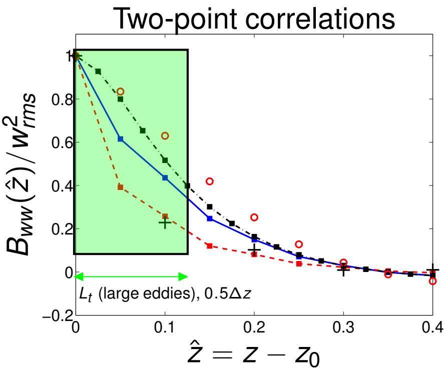

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- Shows that $2\Delta z$ and $2\Delta x$ are too coarse.

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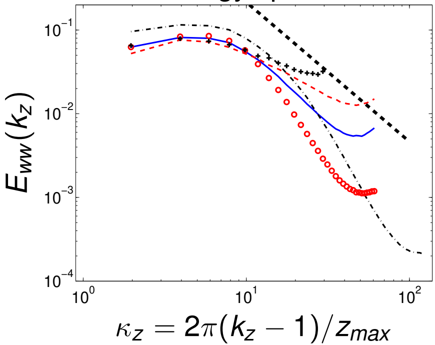
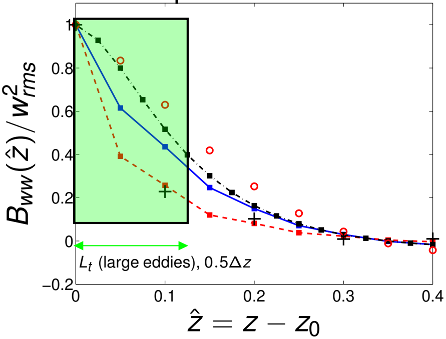
Energy spectra



Two-point correlations



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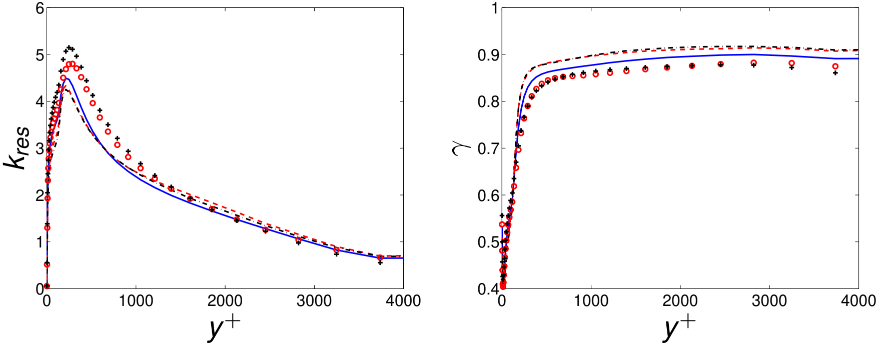
- Two-point correlation is better
- Shows that $2\Delta z$ and $2\Delta x$ are too coarse.
- integral lenghscale,
  $L_t = \int_0^{z_{maz}} B_{ww}(\hat{z}) d\hat{z}$

Channel flow, $Re_\tau = 4000$, $y^+ = 440$

$k_{res} = (u'^2 + v'^2 + w'^2)/2$ $\quad\quad$ $\gamma = \frac{k_{res}}{<k_T> + k_{res}}$



- Pope [8] suggests $\gamma > 0.8$ indicates well resolved flow

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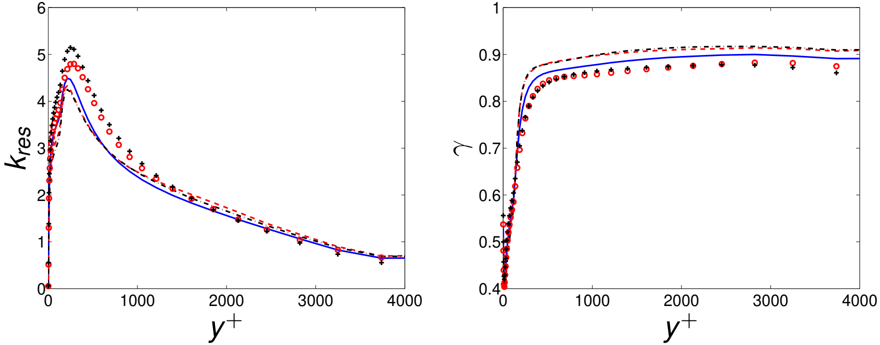

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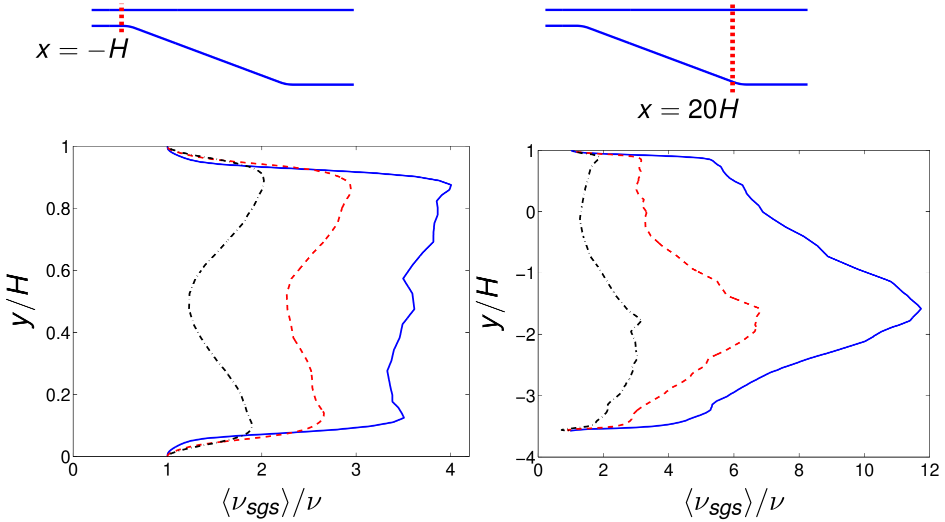
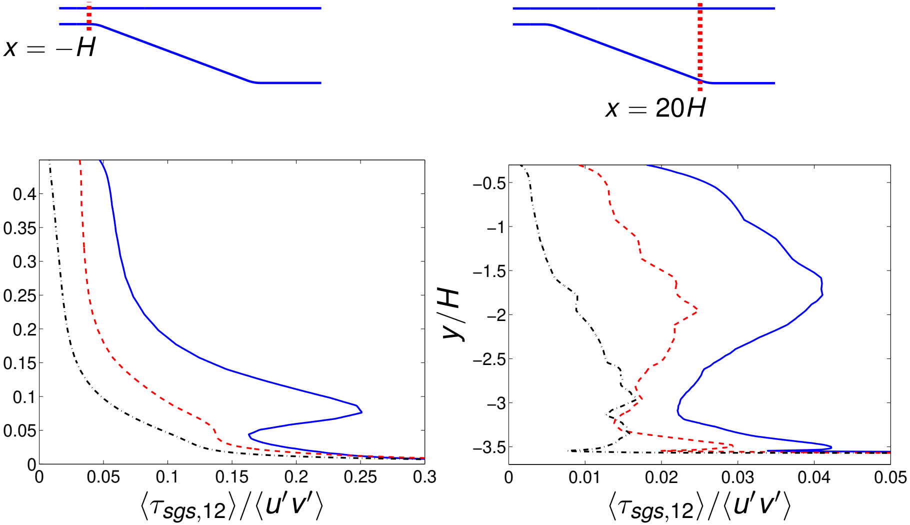
- Pope criterion does not work here

SGS vs. Molecular Viscosity [2]



— $N_z = 32$; - - - $N_z = 64$; - · - $N_z = 128$.

SGS vs. Resolved Shear Stresses



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DES: Detached-Eddy Simulations

▶DES: Use RANS near walls and LES away from walls

- RANS: high turbulent viscosity
- LES: low turbulent viscosity

▶The S-A one-equation model (RANS) reads

$$\frac{d\rho\tilde{\nu}_t}{dt} = \frac{\partial}{\partial x_j}\left(\frac{\mu+\mu_t}{\sigma_{\tilde{\nu}_t}}\frac{\partial\tilde{\nu}_t}{\partial x_j}\right) + \text{cr. term} + P - \boxed{C_{w1}\rho f_w\left(\frac{\tilde{\nu}_t}{d}\right)^2}, \quad d = x_n$$

▶Replace d with $\tilde{d}$:

$$\left(\frac{\tilde{\nu}_t}{d}\right)^2 \Rightarrow \left(\frac{\tilde{\nu}_t}{\tilde{d}}\right)^2, \quad \tilde{d} = \min\{C_{DES}\Delta, d\}, \quad \Delta = \max\{\Delta x_1, \Delta x_3, \Delta x_3\}$$

▶This is the DES S-A one-equation model

DES based on Two-equation DES models

- RANS: high turbulent viscosity
- LES: low turbulent viscosity

▶RANS $k - \varepsilon$. The k equation reads

$$\frac{\partial k}{\partial t} + \bar{v}_j \frac{\partial k}{\partial x_j} = P^k + \frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left((\nu + \frac{\nu_t}{\sigma_k})\frac{\partial k}{\partial x_j}\right) - F_{DES}\varepsilon, \quad F_{DES} = 1$$

- DES:

$$F_{DES} = \max\left(1, \frac{L_t}{C_{DES}\Delta}\right) = \max\left(1, \frac{k^{3/2}}{\varepsilon C_{DES}\Delta}\right)$$

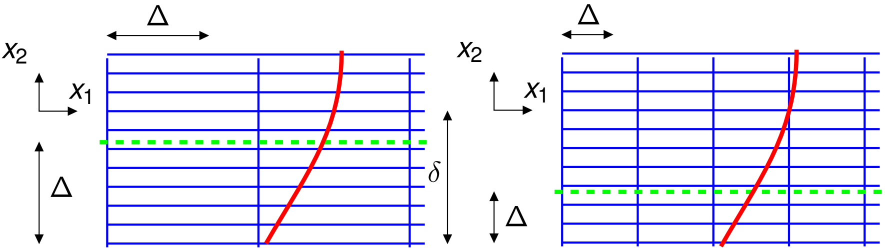
▶In LES region, $F_{DES} > 1$ which decreases k and $\nu_t = C_\mu k^2/\varepsilon$.

DDES: DELAYED DES

- F_{DES} may switch to LES because Δx_1 is too small (but not sufficiently small)
- Hence boundary layer is treated in LES mode with too a coarse mesh $\Rightarrow$ poorly resolved LES $\Rightarrow$ inaccurate predictions.

▶The solution is DDES (Delayed DES)

DDES: Delayed DES cont'd



: grid; : U; - - - RANS-LES interface. $\Delta = C_{DES} \max(\Delta x_1, \Delta x_2, \Delta x_3)$

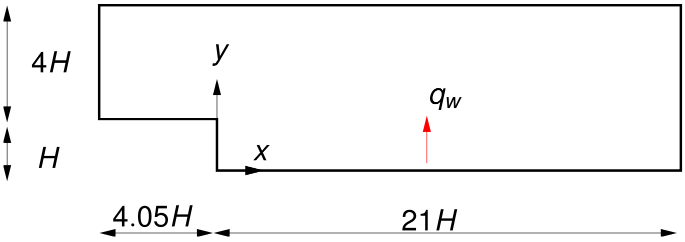
- Good DES mesh since entire b.l. modeled by RANS.
- poor DES grid since the outer part of the b.l. is in LES mode

▶In DDES, F_{DES} is computed as ($C_{DES} = 0.67$)

$$F_{DES} = \max\left\{\frac{L_t}{C_{DES}\Delta}(1 - F_1), 1\right\}$$

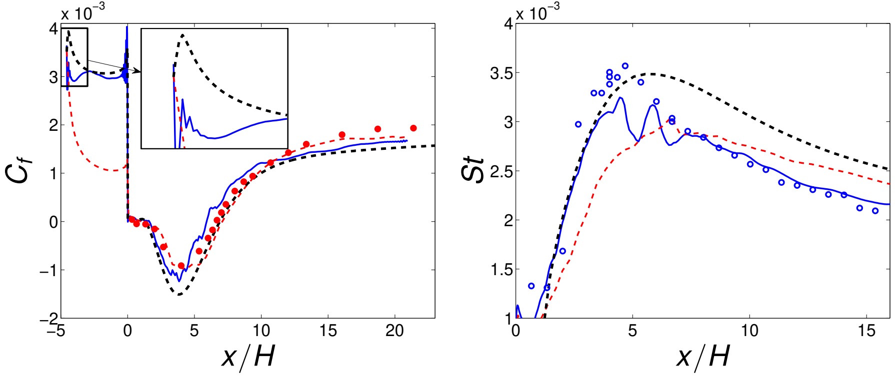
where F_1 ($F_1 = 1$ in the boundary layer) is taken from SST-$k-\omega$

Backward Facing Step: Domain



- $Re_H = 28\,000$ Experiments by Vogel & Eaton [10]
- Mean inlet profiles from RANS (same as in boundary layer)
- Grid: 336×120 in $x \times y$ plane. $Z_{max} = 1.6H$, $N_k = 64$, $\Delta z^+_{in} = 31$.
- Anisotropic synthetic fluctuations, u', v', w' (same as for boundary layer flow); no fluctuations for t'
- Constant heat flux, q_w, on lower wall.

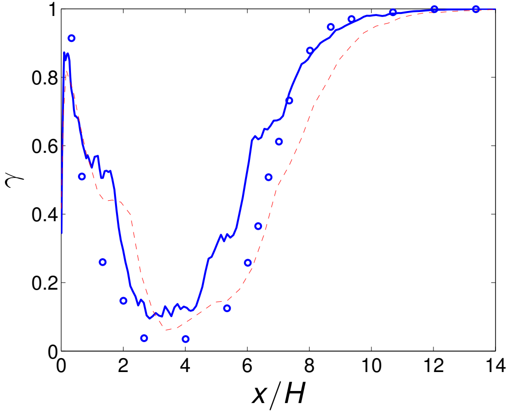
Backstep flow. Skin friction and Stanton number



——— : PANS; - - - : PANS, no inlet fluctuations; - - - : 2D RANS; expts. [10].

FORWARD/BACKWARD FLOW

- Fraction of time, γ, when the flow along the bottom wall is in the downstream direction.



—— : PANS;
- - - : PANS, no inlet fluctuations; ∘: experiments [10].

Concluding Remarks

- LES/DES is expensive and accurate
- RANS is cheap and sometimes inaccurate
- After you have made an LES/DES: try to verify if the resolution is sufficient

References I

[1] Davidson, L.
Large eddy simulations: how to evaluate resolution.
International Journal of Heat and Fluid Flow 30, 5 (2009), 1016–1025.

[2] Davidson, L.
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