Using Machine Learning for formulating new wall functions for Large Eddy Simulation: A Second Attempt

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  - Through as much data as possible at ML?
- In my case, input and output are numerical values. Regression methods should then be used [2]; I use support vector regression (SVR) methods available in Python.

Training: I need a target database

$$\frac{\partial \bar{v}_i}{\partial x_i} = 0$$

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial \bar{v}_i}{\partial x_j}\right]$$

- Fully-developed Channel flow
- IDDES. $96 \times 96 \times 96$, Reynolds number is 5 200
- Database: average in x and z

$$\bar{U}_{1^{st}}(x,z) = \frac{1}{\Delta X \Delta Z}\int_{x,z}^{x+\Delta X, z+\Delta Z} \bar{u}\, dxdz$$

$$\bar{u}_\tau(x,z) = \frac{1}{\Delta X \Delta Z}\int_{x,z}^{x+\Delta X, z+\Delta Z} u_\tau\, dxdz$$

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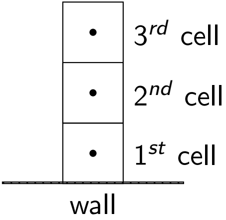

1^{st} cell	$\langle \Delta y^+ \rangle$
Location 1	12
Location 2	31
Location 3	49
Location 4	66
Location 5	76
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Location 7	135
Location 8	155
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300 independent instantaneous samples of $\bar{U}$ stored at all 3×9 cells

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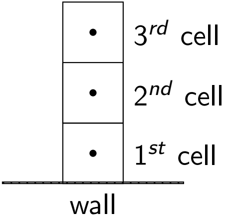

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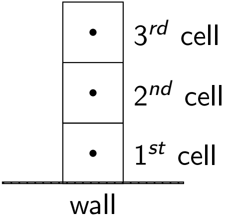

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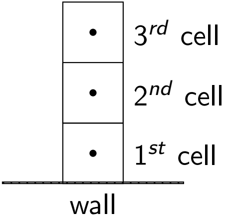

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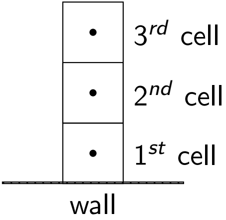
- $T = \nu/\bar{U}^2$



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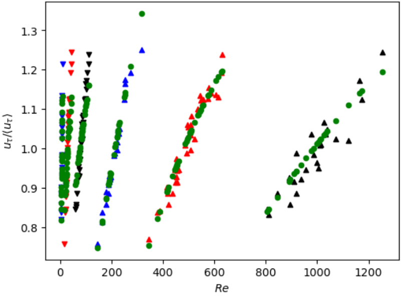
Predicted output using ML: 1st attempt



(A) ▲: IDDES, Location 1; ▲: IDDES, Location 2; ▲: IDDES, Location 3; ▼: IDDES, Location 4; ▼: IDDES, Location 5; ▼: IDDES, Location 6. ○: svr.

- Output on y axis

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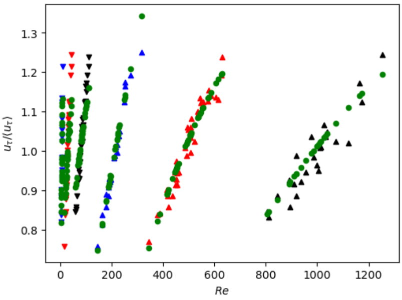

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- Output on y axis
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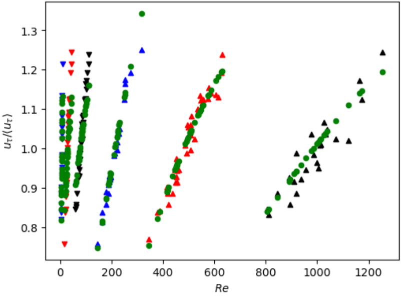

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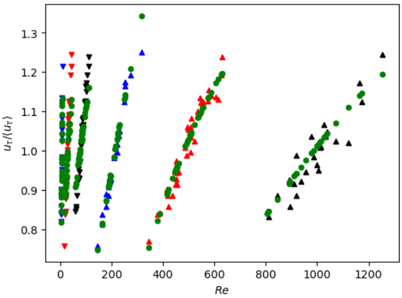

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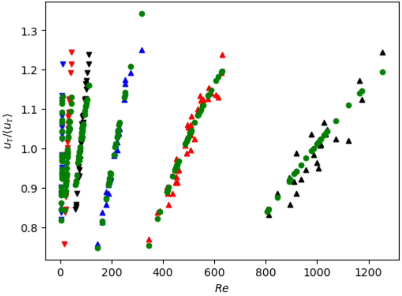

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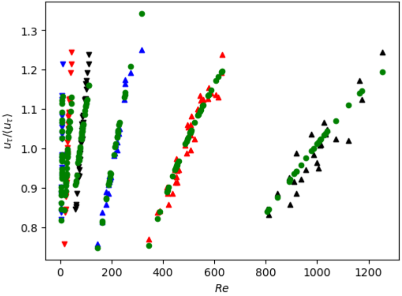

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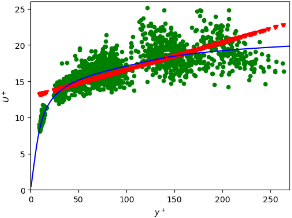

—: $\langle\bar{u}\rangle$, IDDES; ▼: `svrLINEAR`: ●: IDDES, test data. 9% normalized error.

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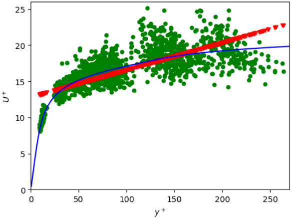

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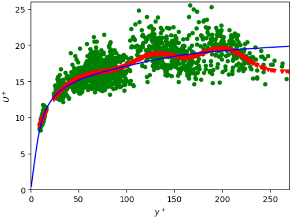

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Standard wall functions

- The machine-learning wall functions will be compared to wall functions based on Reichardt's law

$$\frac{\bar{u}_P}{u_\tau} \equiv U^+ = \frac{1}{\kappa}\ln(1-0.4y^+) + 7.8\left[1-\exp\left(-y^+/11\right) - (y^+/11)\exp\left(-y^+/\right.\right.$$

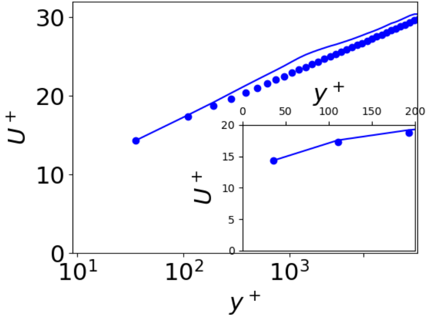
- The friction velocity is then obtained by solving the implicit equation

$$u_\tau - \bar{u}_P\left(\ln(1-0.4y^+)/\kappa + 7.8\left[1-\exp\left(-y^+/11\right) - (y^+/11)\exp\left(-y^+/3\right)\right]\right)^{-1} = 0$$

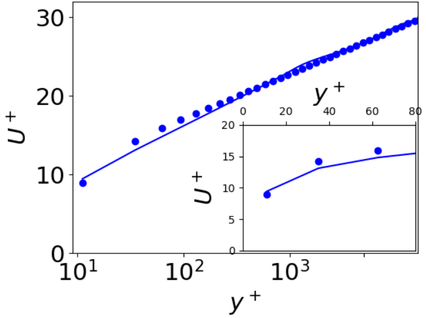
using the Newton-Raphson method `scipy.optimize.newton` in Python.

- $\bar{u}_P$ denotes the wall-parallel velocity in the first, second or third wall-adjacent cell.

Results, channel flow, ML, $Re_\tau = 16\,000$



(A) $N_y = 66$, stretching 11%.



(B) $N_y = 68$, stretching 14.7%.

Figure: Channel flow. svr. $Re_\tau = 16\,000$. Velocity. •: Reichardt's law.

Reichardt's wall function, $Re_\tau = 16\,000$

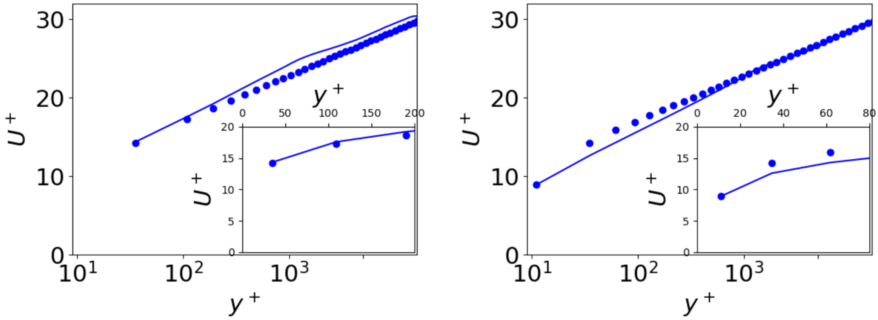


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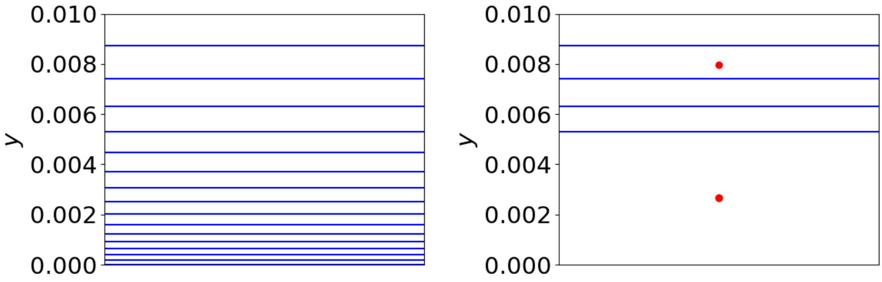
Figure: Channel flow. Reichardt's wall function. $Re_\tau = 16\,000$. Velocity.
•: Reichardt's law.

New Grid Strategy

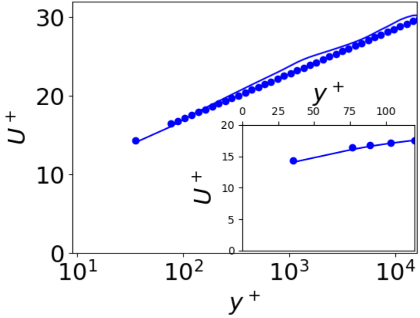


(A) Low-Re number IDDES grid. (B) Wall function grid. New grid strategy.

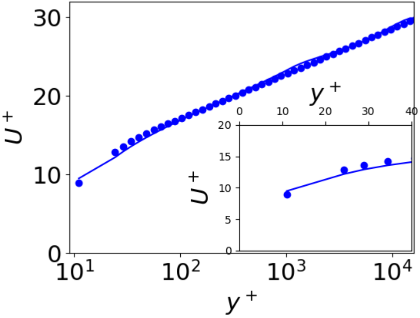
Figure: Different grids. ——— : grid lines.

- This strategy was used in [1] for channel flow and impinging jets (RANS)

Channel flow, ML, $Re_\tau = 16\,000$, new grid



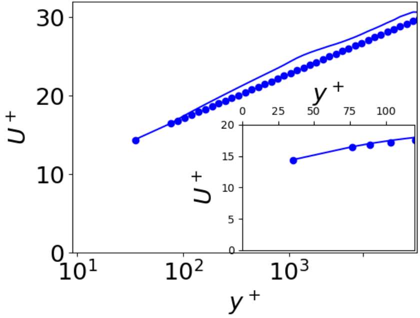
(A) $N_y = 78$.



(B) $N_y = 92$.

FIGURE: Channel flow. $Re_\tau = 16\,000$. Velocity. svr. •: Reichardt's law.

Reichardt's wall function, $Re_\tau = 16\,000$



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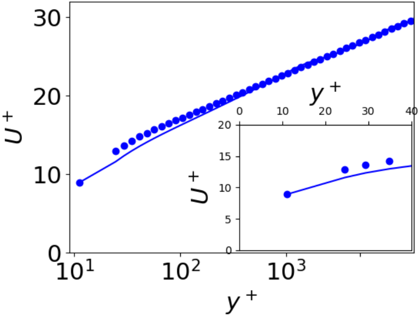

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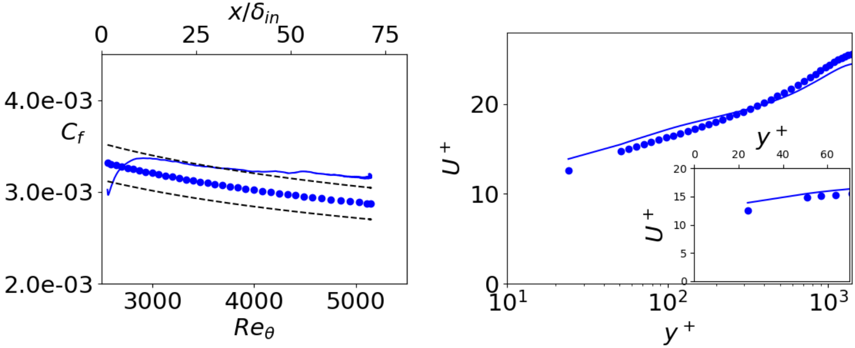
Figure: Channel flow. $Re_\tau = 16\,000$. Velocity. Reichardt's wall function.
●: Reichardt's law.

Developing Boundary Layer Flow

- $Re_\theta = U_{free}\theta/\nu = 2\,550$ at the inlet.
- Domain $(96 \times 7 \times 5)\delta_{in}$.
- Grid $(550 \times 82 \times 64)$.

Developing Boundary Layer Flow

- u_τ computed using 3^{rd} cell



(A) Skin friction.

(B) Velocity at $Re_\theta = 4\,000$. Markers: DNS [5]

Figure: Boundary layer flow. $Re_\theta = 2\,500$ at inlet. `svr`. $N_y = 82$

Developing Boundary Layer Flow, $2\Delta x$, $2\Delta z$

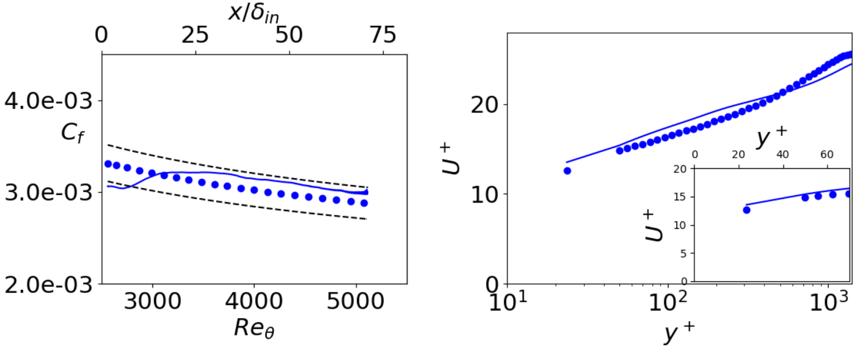


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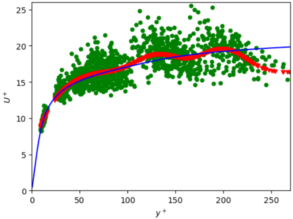
Figure: Boundary layer flow. svr. $N_y = 82$, $N_k = 32$, $\Delta x_{in} = 2\Delta x_{in,base}$

Conclusions

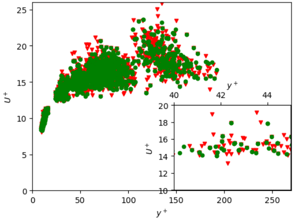
- Machine Learning (`svr`) wall functions have been developed
- Good results for channel flow placing the wall-adjacent cell at different locations
- Good results for developing boundary layer flow

Attempt 3? I

- Instantaneous data are used for training `svr`
- `svr` finds the time-averaged regression line (shown by ▼ in Fig. A)
- If I want *instantaneous* u_τ, I could find nearest neighbour (shown by ● in Fig. B)



(A) ——— : $\langle \bar{u} \rangle$, IDDES; ▼: `svr`: ●: IDDES, target data. 9% normalized error.



(B) Nearest neighbor using Python's `scipy.spatial.KDTree` ▼: KDTree; ●: IDDES, target data; 0.7% normalized error.

Attempt 3? II

- I have trained `svr` using instantaneous data but the model gives time-averaged regression line
- Maybe better to train `svr` using time-averaged data.
  - In this case many time-averaged $\langle \bar{u} \rangle$ (and $\langle \bar{p} \rangle$) profiles can be used for training
  - Maybe `svm` (Support Vector Machines) and/or Neural Network could be used to find the most appropriate profile

References I

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