This is an appendix in the lecture notes of the course *MTF270 Turbulence modeling* which can be downloaded here
`http://www.tfd.chalmers.se/~lada/turbulent_flow/lecture_notes.html`

J MTF270: Computation of wavenumber vector and angles

For each mode n, create random angles φ^n, α^n and θ^n (see Figs. J.1 and 22.1) and random phase ψ^n. The probability distributions are given in Table J.1. They are chosen so as to give a uniform distribution over a spherical shell of the direction of the wavenumber vector, see Fig. J.1.

J.1 The wavenumber vector, κ_j^n

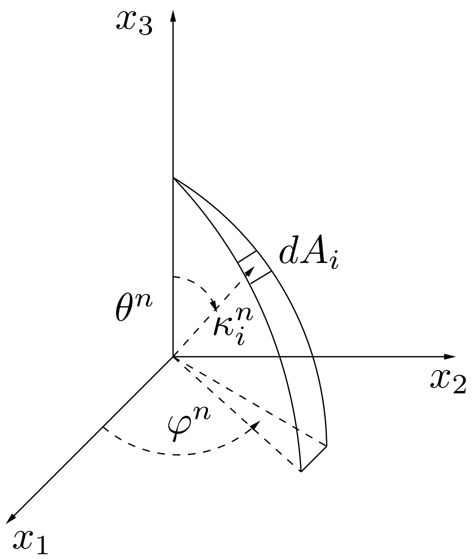

Figure J.1: The probability of a randomly selected direction of a wave in wave-space is the same for all dA_i on the shell of a sphere.

Compute the wavenumber vector, κ_j^n, using the angles in Section J according to Fig. J.1, i.e.

$$\begin{aligned}\kappa_1^n &= \sin(\theta^n)\cos(\varphi^n)\\ \kappa_2^n &= \sin(\theta^n)\sin(\varphi^n)\\ \kappa_3^n &= \cos(\theta^n)\end{aligned} \tag{J.1}$$

$p(\varphi^n) = 1/(2\pi)$	$0 \le \varphi^n \le 2\pi$
$p(\psi^n) = 1/(2\pi)$	$0 \le \psi^n \le 2\pi$
$p(\theta^n) = 1/2\sin(\theta)$	$0 \le \theta^n \le \pi$
$p(\alpha^n) = 1/(2\pi)$	$0 \le \alpha^n \le 2\pi$

Table J.1: Probability distributions of the random variables.

$\boldsymbol{\kappa_i^n}$	$\boldsymbol{\sigma_i^n}$	$\boldsymbol{\alpha^n}$
$(1,0,0)$	$(0,0,-1)$	0
$(1,0,0)$	$(0,1,0)$	90
$(0,1,0)$	$(0,0,-1)$	0
$(0,1,0)$	$(-1,0,0)$	90
$(0,0,1)$	$(0,1,0)$	0
$(0,0,1)$	$(-1,0,0)$	90

Table J.2: Examples of value of κ_i^n, σ_i^n and α^n from Eqs. J.1 and J.3.

J.2 Unit vector σ_i^n

Continuity requires that the unit vector, σ_i^n, and κ_j^n are orthogonal. This can be seen by taking the divergence of Eq. 22.1 which gives

$$\nabla \cdot \mathbf{v}' = 2\sum_{n=1}^{N} \hat{u}^n \cos(\boldsymbol{\kappa}^n \cdot \mathbf{x} + \psi^n)\boldsymbol{\sigma}^n \cdot \boldsymbol{\kappa}^n \tag{J.2}$$

i.e. $\sigma_i^n \kappa_i^n = 0$ (superscript n denotes Fourier mode n). Hence, σ_i^n will lie in a plane normal to the vector κ_i^n, see Fig. 22.1. This gives

$$\begin{aligned}
\sigma_1^n &= \cos(\varphi^n)\cos(\theta^n)\cos(\alpha^n) - \sin(\varphi^n)\sin(\alpha^n)\\
\sigma_2^n &= \sin(\varphi^n)\cos(\theta^n)\cos(\alpha^n) + \cos(\varphi^n)\sin(\alpha^n)\\
\sigma_3^n &= -\sin(\theta^n)\cos(\alpha^n)
\end{aligned} \tag{J.3}$$

The direction of σ_i^n in this plane (the $\xi_1^n - \xi_2^n$ plane) is randomly chosen through α^n. Table J.2 gives the direction of the two vectors in the case that κ_i is along one coordinate direction and $\alpha = 0$ and $\alpha = 90^o$.