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Performance of Two-Equation Models for Numerical Simulation of Ventilation Flows

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ABSTRACT
To investigate the performance of turbulence models in the numerical simulation of recirculating ventilation flows, comparisons have been made for three types of two-equation models: the k-ε, k-ω and k-τ models. A modified k-ω model recently proposed by the authors has been introduced and implemented. All the models are applied with the wall-function method. When using the k-ω models, an extended-to-wall method is also used. Two typical recirculating flows are calculated: the separated flow behind a backward-facing step with a large expansion ratio relevant to room ventilation; and the wall-jet-induced flow in a two-dimensional ventilation enclosure. The predictions are compared with experimental data. The performance of the models is discussed. The modified k-ω model is shown to be an attractive alternative to the k-ε model.

KEY WORDS
Two-equation turbulence model, Modified k-ω model, Numerical simulation, Ventilation flow

1 INTRODUCTION
Numerical simulation techniques have become a competitive tool and been widely applied to the evaluation and prediction of thermal comfort, air quality and energy consumption in buildings, see Peng (1994). Indoor contaminant dispersion and heat transfer are always tied to air motion, created by ventilation systems and characterized usually by recirculation (either local or global). After a separation, recirculating flow often undergoes a reattachment. This process is a powerful generator of turbulence and hence mixing and losses, and thus inevitably affects the overall properties of the flow field. To achieve a detailed understanding of the complex flow features, numerical methods offer a powerful alternative to experimental methods, which are often costly and time-consuming.

When used with specific boundary conditions, the turbulence model mathematically describes the physical phenomena of fluid flow, and thus plays a key role in numerical simulations. Compared to second-order closure models and large eddy simulations, two-equation models require lower computer power and can often give reasonable prediction accuracy. On the other hand, they incorporate substantially more turbulence physics and require less *ad hoc* empiricism than the older algebraic eddy-viscosity models. The two-equation turbulence models, therefore, remain the preferred approaches in engineering applications.

The standard k-ε model (Launder and Spalding 1974), in conjunction with empirical wall functions, has been the most widely used approach to solve ventilation flow problems. In other fields, several alternative two-equation models have emerged and been applied (Wilcox 1993). They have rarely, however, been used for simulating indoor air motion and heat transfer. Their performance in predicting ventilation flows remains unclear. Investigations on various two-equation turbulence models thus have practical importance.

The k-ε model by Launder and Spalding (1974) (hereafter referred to as SKE), the k-ω model by Wilcox (1988) (referred to as SKW) and the k-τ model by Speziale et al (1992) (referred to as SKT) are representative of the frequently used and recently developed models (Wilcox 1993). In this paper, these three models were used in conjunction with the wall functions. Emphasis was placed on the k-ω model, since it appears to be a popular approach. A modified k-ω model (Peng et al 1996a) (hereafter referred to as MKW) was introduced and implemented. In addition to the wall-function method, an extended-to-wall method was used with both the SKW and MKW models. The models were applied to two typical recirculating flows relevant to room ventilation: a separated flow over a backward-facing step with a large expansion ratio, and a recirculating flow in a two-dimensional ventilation enclosure. The calculated results were compared with experimental data, and the performance of the models was discussed.

2 TWO-EQUATION TURBULENCE MODELS

Two-equation models are all based on the eddy viscosity concept, ν_t, which can be determined from the turbulent velocity scale U_t and length scale L_t, i.e. $\nu_t \sim U_t L_t$. The task with the two-equation model is to find the appropriate turbulent velocity and length scales to formulate the eddy viscosity, which is then used as a bridge to model the Reynolds shear stresses to make the equation system closed. Note that this modeling process possesses a practical weakness, that is, the eddy viscosity and diffusivity are assumed to be isotropic.

Table 1 shows the turbulence scales defined for the various models. In the table, k is the turbulent kinetic energy, ε is the dissipation rate of k, ω is the specific dissipation rate of k, τ is the turbulence dissipation time (also termed the turbulent time scale), and c_μ is a constant, different for different models. The relations between k, ε, ω and τ can readily be derived from the expressions for the eddy viscosity formulated in the different models.

Table 1 Eddy viscosity with various models

Model	k-ε	k-ω	k-τ
U_t	$k^{1/2}$	$k^{1/2}$	$k^{1/2}$
L_t	$k^{3/2}/\varepsilon$	$k^{1/2}/\omega$	$k^{1/2}\tau$
ν_t	$c_\mu k^2/\varepsilon$	$c_\mu k/\omega$	$c_\mu k\tau$

2.1 Turbulence Transport Equations

Turbulence transport equations account for the transport and history effects of turbulence. As shown in Table 1, all three types of model take k as a measure for the velocity scale of turbulent motion. The turbulent kinetic energy k is defined by $k = \frac{1}{2}\overline{u_i'u_i'}$, where u_i' (i = 1, 2, 3) are the fluctuations in the three directions x_i. The exact k-equation can be derived from the dynamic equation for the fluctuating velocity. The modeled transport equation for k thus has the same form in all two-equation models. For steady and incompressible flows, it can be written as

$$\frac{\partial(u_j k)}{\partial x_j} = P_k + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - E_k \qquad (1)$$

where E_k is the dissipation term with different expressions that depend on the second turbulence-transport equation, σ_k is a model constant, ν is the molecular viscosity, and P_k is the production term, expressed by

$$P_k = -\overline{u_i'u_j'}\frac{\partial u_i}{\partial x_j} = \nu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} \qquad (2)$$

The second turbulence-transport equation in a two-equation model, in general, is for the *dissipation* of the turbulent kinetic energy, k. If z represents ε, ω and τ, this equation can be written in a general form as

$$\frac{\partial(u_j z)}{\partial x_j} = P_z + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_z}\right)\frac{\partial z}{\partial x_j}\right] - E_z + S_z \qquad (3)$$

where P_z is the production term, E_z is the destruction term and S_z is a source term. These terms and the model constants are given in Tables 2 and 3. Note that the τ-equation is transformed from the ε-equation by means of the relation $\varepsilon \equiv \tau/k$ (Speziale et al 1993). The source term in the τ-equation is

$$S_\tau = \frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_{\tau 1}}\right)\frac{\partial k}{\partial x_j}\frac{\partial \tau}{\partial x_j} - \frac{2}{\tau}\left(\nu + \frac{\nu_t}{\sigma_{\tau 2}}\right)\frac{\partial \tau}{\partial x_j}\frac{\partial \tau}{\partial x_j} \qquad (4)$$

and the model constants are $\sigma_{\tau 1} = \sigma_{\tau 2} = 1.36$.

Table 2 Source terms in Equations (1) and (3)

Term	E_k	P_z	E_z	S_z
SKE model	$c_k\,\varepsilon$	$(c_{z1}\,\varepsilon/k)P_k$	$c_{z2}\,\varepsilon/k$	0
SKW model	$c_k\,\omega\,k$	$(c_{z1}\,\omega/k)P_k$	$c_{z2}\,\omega^2$	0
SKT model	$c_k\,k/\tau$	$(c_{z1}\,\tau/k)P_k$	c_{z2}	Eq. (4)

Table 3 Model constants

Constant	c_μ	c_k	c_{z1}	c_{z2}	σ_k	σ_z
SKE model	0.09	1.0	1.44	1.93	1.0	1.3
SKW model	1.0	0.09	0.556	0.075	2.0	2.0
SKT model	0.09	1.0	- 0.44	- 0.83	1.36	1.36

2.2 The Modified k-ω Model

When the SKW model is used to simulate recirculating ventilation flows, it is found that this model underpredicts the near-wall turbulent velocity scale, and thus underestimates the near-wall eddy viscosity. As a result, this model yields an overpredicted reattachment length, x_r, for a backward-facing step flow with a large expansion ratio. To improve the prediction accuracy, a modified form has been proposed (Peng et al 1996a).

The starting point for the modification is to enhance the near-wall turbulence energy. To achieve it, one of the best ways is to suppress the near-wall specific dissipation rate ω by improving the modeling of the ω-equation, since the greatest uncertainty usually lies in the scale-determining equation, i.e. the ω-equation. Based on the exact ω-equation, Peng et al (1996a) modeled the exact turbulent diffusion for ω in analogy to its viscous counterparts. As a result, this term was modeled with two parts: a second-order turbulent diffusion term and a first-order turbulent cross-diffusion term. Further, the near-wall asymptotical analysis showed that the molecular cross-diffusion term must be dropped from the modeled ω-equation in order not to contradict the realizability principle of turbulence modeling, see Peng et al (1996a). With Equations (1) and (3) available, the source term in Equation (3) for the MKW model (i.e. in the modified ω-equation) then becomes

$$S_\omega = c_\omega \frac{\nu_t}{k} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \qquad (5)$$

In the MKW model, E_k, P_z and E_z take the same forms as those in the SKW model. However, the model constants are revised as in Table 4.

Table 4 Model constants for the modified k-ω (MKW) model

Constant	c_μ	c_k	c_{z1}	c_{z2}	σ_z	σ_z	c_ω
MKW model	1.0	0.09	0.42	0.075	0.8	1.35	0.75

2.3 Boundary Conditions

At the inlet, the variables in different models are prescribed as follows

$$k_0 = 1.5(I_0 U_0)^2 \;;\;\; \varepsilon_0 = c_D k_0^{3/2}/l_0$$
$$or\;\; \omega_0 = \gamma k_0^{1/2}/l_0 \;\;\; or\;\; \tau_0 = c_D l_0/k_0^{1/2} \qquad (6)$$

where U_0 is the inlet velocity, I_0 is the inlet turbulence intensity, l_0 is the turbulent length scale at the inlet, usually set as a fraction of the whole inlet height, and γ and c_D are constants. When solving the backward-facing step flow, the inlet distributions are specified with the numerical solution for channel flow at the same inlet-based Reynolds number.

At the outlet, the streamwise derivatives of the variables are assumed to be zero. The velocity component normal to the outlet is specified by the global mass balance when solving for the recirculating flow in the two-dimensional ventilation enclosure.

When the wall-function method is used, for all the models, the near-wall velocity is assumed to obey the log-law, and the kinetic energy is satisfied by experimental observation, i.e. $u_\tau^2/k \approx 0.3$, where u_τ is the friction velocity. Together with the approximate expression for ε or ω or τ in the wall layer, the wall functions used for the three types of turbulence model are

$$u = \frac{u_\tau}{\kappa}\ln(Ey^+);\; k = \frac{u_\tau^2}{\sqrt{c_\mu c_k}}\;;\; \varepsilon = \frac{u_\tau^3}{\kappa y}$$
$$or\ \omega = \sqrt{\frac{c_\mu}{c_k}}\,\frac{u_\tau}{\kappa y}\ \ or\ \tau = \frac{\kappa y}{\sqrt{c_\mu c_k}\,u_\tau} \qquad (7)$$

where κ is the von Kármán constant, E = 9.0, $y^+ = u_\tau y/\nu$ and y is the distance from the wall. The constants c_μ and c_k take different values in different models, as shown in Tables 3 and 4, and $c_\mu\ c_k \equiv 0.09$.

When using the extended-to-wall method with both the SKW and MKW models, the models are directly integrated to the wall surface without using the wall functions as a bridge. Therefore, $u = v = 0$ and $k = 0$ are used on the wall surface, and ω can be specified at the near-wall first grid point with the following asymptotical solution

$$\omega \to 6\nu/(\beta y^2) \qquad as\ y \to 0 \qquad (8)$$

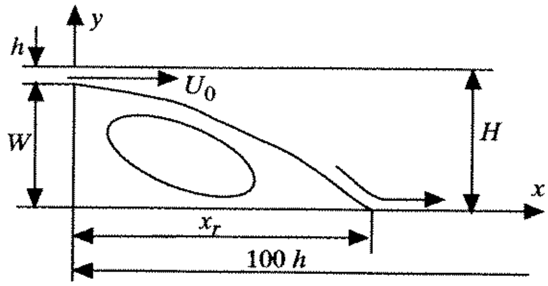
Equation (8) results from the balance between the destruction of ω and its viscous diffusion in the immediate proximity of the wall surface. A refined grid is thus required. At least one grid point should be located in the viscous sublayer.

3 APPLICATIONS AND DISCUSSION

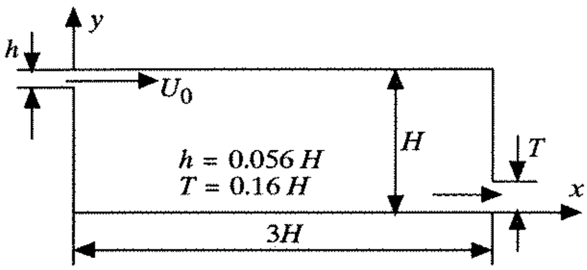
Two flows are solved here: the separated flow over a backward-facing step (Figure 1a), and the recirculating flow in a confined two-dimensional ventilation enclosure (Figure 1b). When solving the backward-facing step flow with the wall-function method, 120 x 67 cells are used; 202 x 86 cells are used with the extended-to-wall method. For the flow in the ventilation enclosure, 50 x 47 cells and 102 x 132 cells are used with the two methods. Numerical experiments are carried out to get grid-independent solutions. The calculated results for both flows are compared with Restivo's experimental data (1979) (referred to as Expt in figures). Note that the measured data used in comparison of k are for $\overline{u'^2}$.

3.1 Computed Results

Table 5 compares the reattachment lengths, x_r, for the backward-facing step flow at Re = 5050.



a Backward-facing step.



b Confined ventilation enclosure.

Figure 1 Configurations used in calculations.

Re is the inlet-based Reynolds number, i.e. Re = $U_0 h/\nu$, and W/h = 5.0. Both the SKE and MKW models give reasonable predictions; the SKT model overpredicts this quantity slightly but with acceptable accuracy. The SKW model overpredicts x_r by 10-20%, particularly when using the extended-to-wall method. The prediction is improved by the MKW model, with either the wall-function method or the extended-to-wall method.

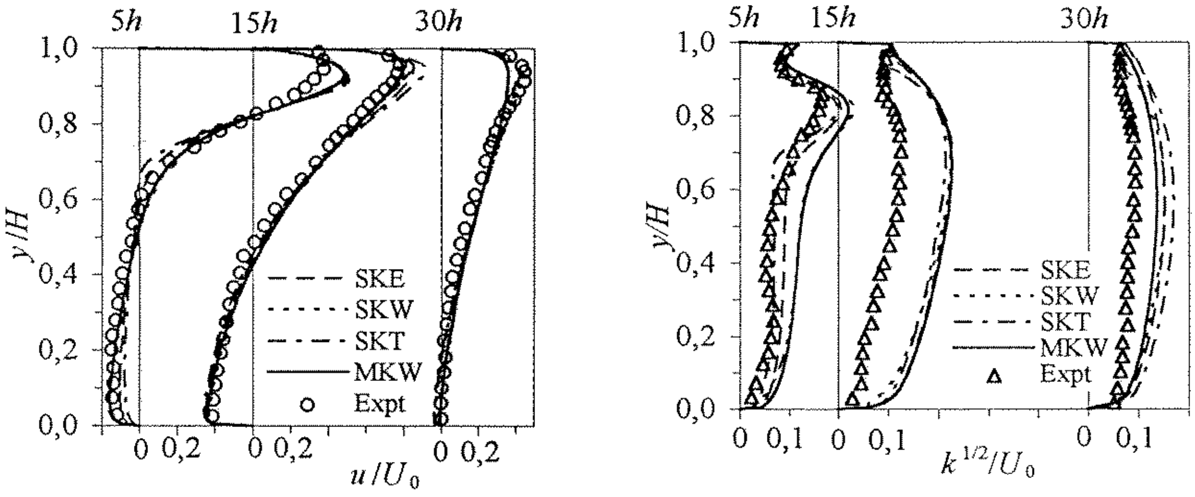
Figure 2a shows the distributions (at x = 5h, 15h, and 30h) calculated with the models when using the wall-function method. At x = 5h, both the SKT and SKE models underpredict the velocity near the lower wall, and all the models overpredict the maximum velocity in the wall-jet. In contrast to the SKW and MKW models, the SKT and SKE models also fail to reproduce the secondary bubble in the corner under the inlet. At x = 30h, the SKW model gives the best prediction for the maximum velocity in the wall-jet, but this is compensated by the underprediction of the near-floor velocity. All the models overpredict the turbulent kinetic energy. Similar results have been reported with the Launder-Sharma low-Reynolds number

Table 5 Reattachment lengths, x_r, at Re = 5050 (* With the extended-to-wall method)

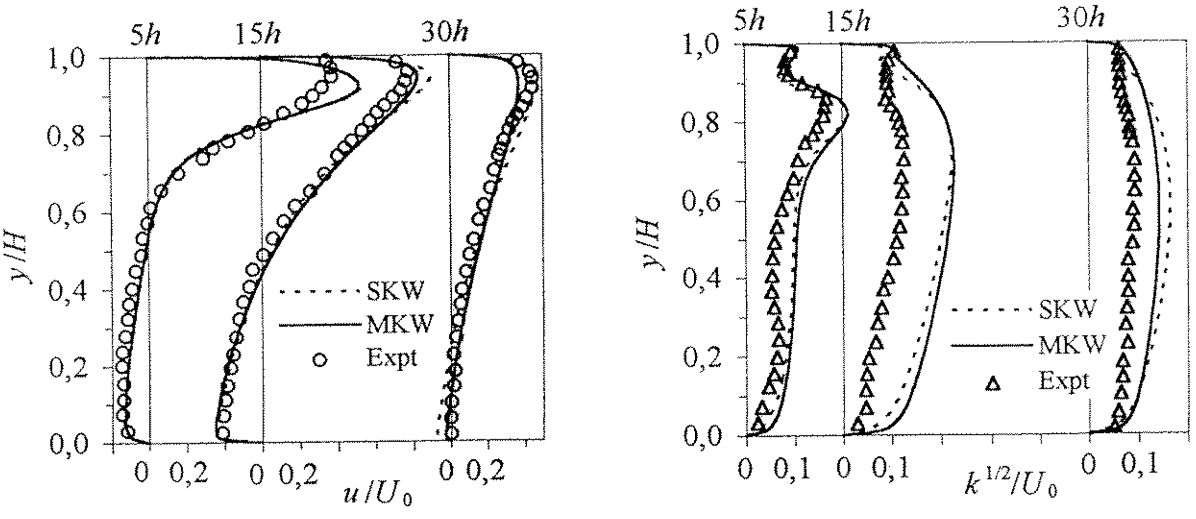
Measurement	SKE	SKW	SKW*	MKW	MKW*	SKT
6.12 W	6.16 W	6.80 W	7.40 W	6.12 W	6.32 W	6.52 W

(LRN) k-ε model (Skovgaard 1991) and other LRN models (Peng et al 1996b). Figure 2b shows the distributions calculated by the SKW and MKW models with the extended-to-wall method. The results are similar to those in Figure 2a. In the region close to the wall surface, the turbulence level has been enhanced by the MKW model compared to the prediction by the SKW model, as desired.

Figure 3 shows the results for the flow in the ventilation enclosure where the inlet-based Reynolds number, Re = U_0h/ν, is 5000. With the wall-function method (Figure 3a), the MKW model gives a higher prediction of the kinetic energy in the wall-jet ($y = H - h/2$) than the SKT and SKW models do, and performs similarly to the SKE model. The SKT model overpredicts the central-line velocity in the wall-jet, and underpredicts both the mean velocity and the turbulent energy in the outer region of the wall-jet, as shown by the distributions at sections $x = H$ and $x = 2H$. When using the extended-to-wall method with both the SKW and MKW models, the results computed with Lam-Bremhorst LRN k-ε model (hereafter referred to as LBKE) (Lam and Bremhorst 1981) is included for comparison, see Figure 3b. The MKW model shows a similar performance to the LBKE model. It predicts a higher turbulence level than the SKW model in the wall-jet and near the wall surface, as desired. Along the central line of the wall-jet, the LBKE model fails to reproduce the negative velocity close to the opposite wall. In general, the MKW model agrees better with experimental data



a Distributions computed with the wall-function method.



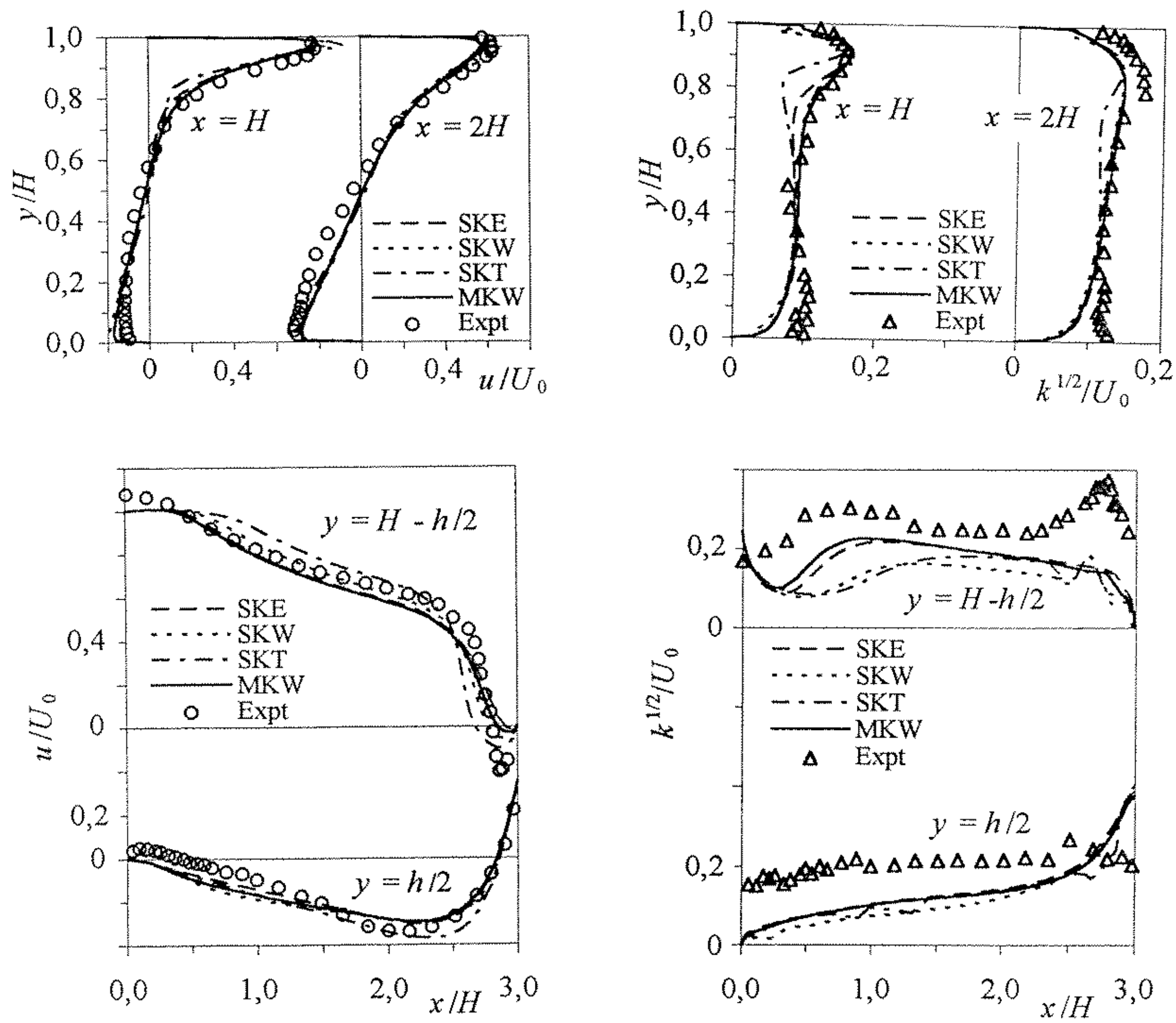
b Distributions computed with the extended-to-wall method.
Figure 2 Distributions calculated for the flow over the backward-facing step.

than the SKW model does.

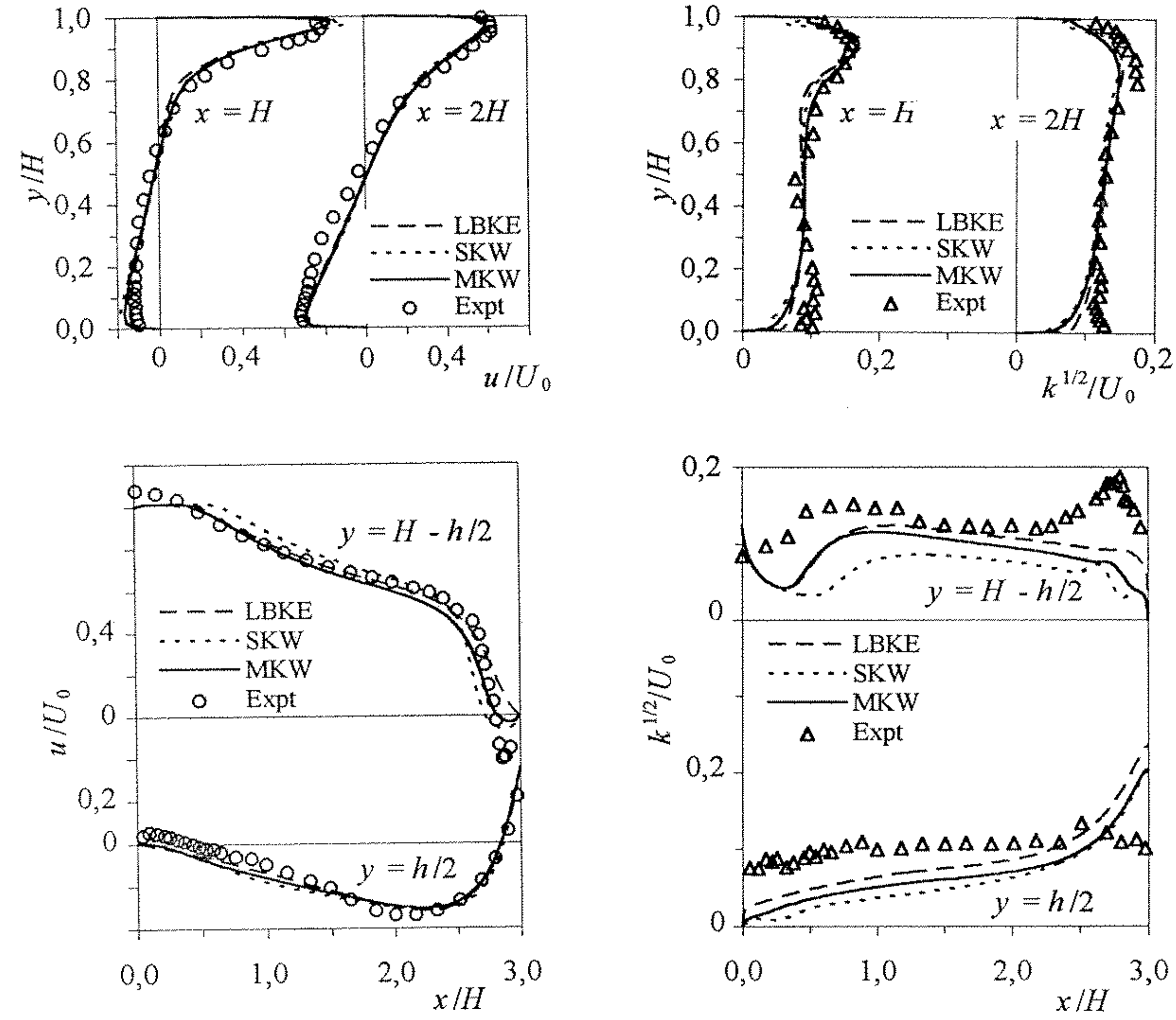
3.2 Discussion

Among the two-equation models, the main variation lies in the scale-determining equations, i.e. the equations for ε, ω and τ. By using the exact equations for k and ε, the exact equations for ω and τ can be obtained with the relations $\omega \sim \varepsilon/k$ and $\tau \sim k/\varepsilon$. The τ-equation in the SKT model can actually be derived from the modeled ε-equation. The difference in the results predicted by the SKE and SKT models is thus only due to the model constants. In general, the SKT model performs worse than the SKE model in both computational accuracy and efficiency.

The ω-equation in the SKW model excludes both the molecular and turbulent cross-diffusion terms from the direct transformation of the modeled ε-equation. These terms often have considerable impacts in regions with large gradients for both k and ω, e.g. near-wall regions (Peng et al 1996a). Usually the near-wall gradients of k and ω are of opposite sign, and the turbulent cross-diffusion term (Equation (5)) as a whole is negative. This term thus reduces the near-wall specific dissipation rate and increases the turbulence energy. When the turbulent cross-diffusion term is included and the model constants are revised as in the MKW model, the predictions are improved. The MKW model, to some extent, performs similarly to the k-ε model, with either the wall-function method or the extended-to-wall method. When the MKW model is used with the extended-to-wall method for engineering applications, two advantages exist: the wall boundary condition for ω is an exact asymptotical solution (Equation (8)), and the damping functions are excluded.



a Distributions computed with the wall-function method.



b Distributions computed with the extended-to-wall method.
Figure 3 Distributions calculated for the flow in the ventilation enclosure.

Unlike the ε-equation, the ω-equation possesses a nontrivial solution as $k \rightarrow 0$. The k-ω model is thus potentially capable of predicting low-Re-number ventilation flows, e.g. ventilation flows by displacement, where laminar, transitional and turbulent flow phenomena co-exist not only near the wall but also in regions far away from the wall.

The computational efficiency has also been investigated. The models perform very differently in convergence. The SKT model is very sensitive to the initial values of k and particularly of τ. Too-small initial values for τ (e.g. 10^{-10}) usually lead to an unstable, even, diverged solution procedure. To reach a faster and more stable convergence, the initial τ value often needs to be of the order of 1 or 10. The SKE, SKW and MKW models are much less sensitive to the initial values.

The source terms in the SKT model (Equation (4)) and in the MKW model (Equation (5)) can also affect the convergence procedure. These terms must be correctly linearized to avoid giving rise to negative values of k, ω or τ. The turbulent cross-diffusion term in the MKW model usually increases the diagonal dominance of the resulting matrix when solving for the ω-equation, since this term in the near-wall region is often negative. The solution procedure thus becomes more stable. The source term in the SKT model, however, usually does not provide this advantage. Although the always-negative term in Equation (4) (the second term) increases the diagonal dominance for solving τ-equation, the near-wall cross-diffusion term (the first term) is often positive (note that $\tau \sim 1/\omega$, and thus $(\partial\tau/\partial y) \sim -(\partial\omega/\partial y)$). Together, they tend to actually decrease the diagonal dominance. As a result, the solution procedure becomes relatively unstable and convergence is slowed.

When using the SKE, SKW and MKW models, Peng et al (1996a) compared the iteration numbers needed to reach a converged solution for the flow in the ventilation enclosure. With the wall-function method, the MKW model gave the fastest convergence. Here, the SKT model is also involved, and gives the slowest convergence. With consistent computational conditions, the SKT model needs about 75% more iteration numbers than the MKW model does, and about 55% more than the SKE model. With the extended-to-wall method, the solution with the k-ω model converges much faster than with the LRN k-ε model, because the boundary condition for ω is fixed at the near-wall first grid point.

4 CONCLUDING REMARKS

Three types of two-equation turbulence models are compared for predicting recirculating flows relevant to room ventilation, and their performance is discussed. The solutions produced by these models differ with the variations in the scale-determining equations. The main variation is usually not in the recirculating region, but in the wall-jet and near-wall regions.

The traditional SKE model gives reasonable results for the applications considered here. The SKT model has a relatively poor performance in both computational accuracy and computational efficiency. This model gives the slowest convergence. The SKW model also fails to give satisfactory predictions, particularly for the reattachment length, x_r, when solving the separated flow behind the backward-facing step; it overpredicts x_r by more than 10% with either the wall-function method or the extended-to-wall method. The modified k-ω model, i.e. the MKW model, improves the prediction, and has a performance similar to the SKE model when used in conjunction with the wall functions. The MKW model gives the fastest convergence.

Both the SKW and MKW models can be used with the extended-to-wall method. The inaccuracy with the SKW model can be reduced when using the MKW model. The latter gives results comparable to the LBKE model, while requiring much less computational effort to reach a converged solution.

The modified k-ω model, i.e. the MKW model, turns out to be an attractive option and alternative to the k-ε model for the numerical simulation of indoor air flows. Particularly, when the resolution of the near-wall mean flow profile becomes important, using this model with the extended-to-wall method has practical advantages.

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