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A NUMERICAL STUDY OF TURBULENT FORCED CONVECTION IN A SQUARE DUCT USING DIFFERENT TURBULENCE MODELS

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SUMMARY

The present work concerns development and application of turbulence models for forced convective heat transfer in ducts. The numerical approach is based on the finite volume technique and a non-staggered arrangement is employed. The SIMPLEC-algorithm is used for handling the pressure-velocity coupling. Cyclic boundary conditions are imposed in the main flow direction. The standard k-ε model with wall functions is used as a reference. The non-linear k-ε model of Speziale is applied to calculate the turbulent shear stresses. The turbulent heat fluxes are calculated by the simple eddy diffusivity concept, the GGDH method and the WET method. The overall comparison between the methods is presented in terms of the friction factor and average Nusselt number. In particular the secondary flow field is investigated.

1. INTRODUCTION

Ducts of various cross-section are frequently occuring in heat transfer equipment. Sometimes the ducts are curved, corrugated or wavy in the main flow direction. Many investigations have shown that the flow in noncircular straight ducts is accompanied by secondary motions in the plane perpendicular to the streamwise flow direction. The secondary motion may be caused by different mechanisms and depends on the Reynolds number, the precise geometry of the cross-section etc. The secondary flow distorts the axial flow and reduces the volumetric flow rate. The secondary flow may also affect the wall shear stress and the heat transfer at the walls. In [1], results for laminar and turbulent flow for a variety of ducts are provided.

The present investigation concerns turbulent flow and convective heat transfer in straight square ducts. The main purpose is to further develop, apply and evaluate a non-linear k-ε turbulence model for calculation of the turbulent shear stresses (ref. [2]) combined with the generalized gradient diffusion hypothesis (GGDH) and the wealth equals earning times time (WET) method for determination of the turbulent heat fluxes, ref. [3].

In the literature both experimental and numerical investigations have been presented on turbulent flow in straight square ducts. Since the secondary flow is weak and its velocity components are only a few percent of the main flow velocity, measurements of the secondary motion becomes very difficult. In this work, we are focusing on fully developed flow and the most relevant experimental investigations are those of Gessner [4] and Gessner and Emery [5].

Two-equations turbulence models have been applied by some investigators in the past but more recently the large eddy simulation method (LES) has been applied, see [6].

The combined modelling approach of this paper is new and the investigation thus provides a contribution in the field of numerical methods for turbulent convective heat transfer.

2. PROBLEM STATEMENT

In this study a straight duct with a square cross-section is considered. Symmetry conditions are being used as shown in Fig. 1.

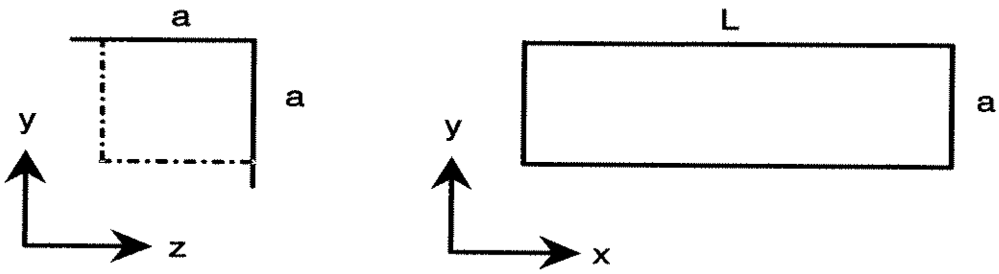

Figure 1. Duct under consideration.

The side length of the cross-section is 2a and the duct length is L. The investigation is for fully developed turbulent flow and periodic conditions are imposed at the inlet and outlet.

The overall performance of the duct in terms of the friction factor and Nusselt number is to be determined numerically. The secondary flow motion in the cross-sectional plane is also of major concern. Various turbulence models are applied.

3. GOVERNING EQUATIONS

The governing equations are the continuity, momentum and energy equations. Fully developed periodic turbulent flow is considered in this investigation. The following assumptions are employed: steady state, no-slip at the walls and no natural convection. One then has:

$$\frac{\partial}{\partial x_j}\left(\rho U_j\right)=0 \tag{1}$$

$$\frac{\partial}{\partial x_j}\left(\rho U_i U_j\right)=-\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\right]+\frac{\partial}{\partial x_j}\left(-\rho\overline{u_i u_j}\right) \tag{2}$$

$$\frac{\partial}{\partial x_j}\left(\rho U_j T\right)=\frac{\partial}{\partial x_j}\left[\frac{\mu}{Pr}\frac{\partial T}{\partial x_j}+\left(-\rho\overline{u_j t}\right)\right] \tag{3}$$

The turbulent shear stresses $-\rho\overline{u_i u_j}$ and turbulent heat fluxes $\left(-\rho\overline{u_j t}\right)$ are modeled as described in the following sections.

3.1 Turbulence models for shear stresses

Both the linear k-ε (standard) and non-linear k-ε models based on Speziale's method are used in this study, see [2].

The k-ε model for steady state is given by

$$\frac{\partial}{\partial x_j}\left(\rho U_j k\right)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_\tau}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k-\rho\varepsilon \tag{4}$$

$$\frac{\partial}{\partial x_j}\left(\rho U_j \varepsilon\right)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_\tau}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]+C_{\varepsilon 1}\frac{\varepsilon}{k}P_k-C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} \tag{5}$$

where P_k is the production term expressed as

$$P_k=\tau_{ij}\frac{\partial U_i}{\partial x_j}=-\rho\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} \tag{6}$$

In the linear k-ε, τ_{ij} is expressed as

$$\tau_{ij}=-\frac{2}{3}\rho k\delta_{ij}+2\mu_\tau S_{ij} \tag{7}$$

where

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \tag{8}$$

In the non-linear k-ε, the Speziale description is used. Thus

$$\tau_{ij} = -\frac{2}{3}\rho k \delta_{ij} + 2\mu_\tau S_{ij} + 4C_D C_\mu \mu_\tau \frac{k}{\varepsilon}\left(S_{ik}S_{kj} - \frac{1}{3}S_{mn}S_{nm}\delta_{ij} \right) + $$
$$+4C_E C_\mu \mu_\tau \frac{k}{\varepsilon}\left(\dot{S}_{ij} - \frac{1}{3}\dot{S}_{mm}\delta_{ij} \right) \tag{9}$$

where $\dot{S}_{ij}$ is the frame-indifferent Oldroyd derivative [7] of S_{ij} in the form of

$$\dot{S}_{ij} = \frac{\partial S_{ij}}{\partial t} + U_k \frac{\partial S_{ij}}{\partial x_k} - \frac{\partial U_i}{\partial x_k} S_{kj} - \frac{\partial U_j}{\partial x_k} S_{ki} \tag{10}$$

and

$$\mu_\tau = \rho C_\mu \frac{k^2}{\varepsilon} \tag{11}$$

k is the turbulent kinetic energy, ε the dissipation rate of the turbulent kinetic energy and C_μ is a constant. The values of the constants in equations (4), (5), (9) and (11) are given in Appendix 1.

3.2 Turbulence Models for Heat Flux

Three models are used to express the turbulent heat flux.

a) Simple Eddy Diffusivity (SED) based on the Boussinesq viscosity model as

$$\rho \overline{u_j t} = -\frac{\mu_\tau}{\sigma_T} \frac{\partial T}{\partial x_j} \tag{12}$$

b) Generalized Gradient Diffusion Hypothesis (GGDH) expressed by Daly and Harlow [8] as

$$\rho \overline{u_j t} = -\rho C_t \overline{u_j u_k} \frac{k}{\varepsilon} \frac{\partial T}{\partial x_k} \tag{13}$$

c) Wealth α Earnings×Time (WET) expressed by Launder [9] as

$$\rho \overline{u_j t} = -\rho C_t \frac{k}{\varepsilon}\left(\overline{u_j u_k} \frac{\partial T}{\partial x_k} + \overline{u_k t} \frac{\partial U_j}{\partial x_k} + \overline{f_j t} \right) \tag{14}$$

where $\overline{f_j t}$ is the buoyancy-driven heat flux which is zero in this case. The constant C_t is set to 0.3 in both cases.

3.3 Periodic Case

The pressure P is expressed by

$$P(x,y,z) = -\beta x + P^*(x,y,z) \tag{15}$$

where β is a constant which represents the non-periodic pressure gradient in the main flow direction. P^* is related to the detailed local manner and behaves periodically in the flow direction.

The dimensionless temperature θ is defined in the cyclic case as

$$\theta(x,y,z)=\frac{T(x,y,z)-T_w}{T_b(x)-T_w} \tag{16}$$

where T_W is the constant wall temperature and T_b is the fluid bulk temperature. Using this expression and inserting it into the energy equation (3) one obtains

$$\frac{\partial}{\partial x_j}\left(\rho U_j\theta\right)=\frac{\partial}{\partial x_j}\left[\frac{\mu}{Pr}\frac{\partial\theta}{\partial x_j}\right]+\gamma+\frac{\partial}{\partial x_j}\left(-\rho\overline{u_j\theta}\right) \tag{17}$$

where γ is

$$\gamma=\lambda\left[\Gamma\frac{\partial\theta}{\partial x}+\frac{\partial}{\partial x}(\Gamma\theta)-\rho U\theta\right]+\Gamma\theta\left(\lambda^2+\frac{\partial\lambda}{\partial x}\right) \tag{18}$$

In (18) λ is

$$\lambda=\frac{\partial(T_b-T_W)}{\partial x}/(T_b-T_W) \tag{19}$$

$\Gamma=\mu/Pr$ and both λ and γ are periodic parameters.

In the GGDH method , $\overline{u_j t}$ is calculated from

$$\overline{u_j t}=-c_t\frac{k}{\varepsilon}(T_b-T_w)\left\{\overline{u_j u}\left(\lambda\theta+\frac{\partial\theta}{\partial x}\right)+\overline{u_j v}\frac{\partial\theta}{\partial y}+\overline{u_j w}\frac{\partial\theta}{\partial z}\right\} \tag{20}$$

In the WET method , $\overline{u_j t}$ is determined from

$$\overline{u_j t}=-c_t\frac{k}{\varepsilon}(T_b-T_w)\left\{\overline{u_j u}\left(\lambda\theta+\frac{\partial\theta}{\partial x}\right)+\overline{u_j v}\frac{\partial\theta}{\partial y}+\overline{u_j w}\frac{\partial\theta}{\partial z}+\overline{u_k t}\frac{\partial U_j}{\partial x_k}\right\} \tag{21}$$

Since the energy equation contains two unknowns, $\theta(x,y,z)$ and $\lambda(x)$, an additional condition is needed to close the problem. This condition can be obtained from the definition of the bulk temperature. In dimensionless form one has

$$\int|U|\theta dA_c=\int|U|dA_c \tag{22}$$

where A_c is the cross-sectional area in the main flow direction. The shape of non-dimensional temperature profile $\theta(x,y,z)$ repeats itself in the fully developed periodic region.

4. BOUNDARY CONDITIONS

Periodicity conditions at the inlet and outlet are imposed.

$$\Phi(x,y,z)=\Phi(x+L,y,z)\qquad \Phi=U,V,W,\theta,P^*,k,\varepsilon,\overline{u_i u_j},\overline{u_j\theta},\lambda,S_{ij}$$

4.1 Wall Boundaries

For the near wall region, the law of the wall is assumed to be valid for both the flow and temperature fields. The procedure adopted here is similar to that presented in e.g. [10] but due to lack of space the details are omitted in this paper.

A non-uniform grid distribution is employed. Close to all walls the number of grid points or control volumes is increased to enchance the resolution and accuracy.

The pressure gradient perpendicular to any wall will be quite small close to the considered wall and is neglected at the wall proximity in the corresponding momentum equation. The extra terms $\frac{\partial}{\partial x_j}\left(\mu \frac{\partial U_j}{\partial x_i}\right)$, which appear due to the varying viscosity and often are neglected, will also be small close to a wall. At the grid point adjacent to the wall, these terms are neglected as the equation for the velocity component perpendicular to a wall is solved. The advantage of this procedure is that it enables more efficient convergence in achieving the secondary velocity vector parallel to the wall in the non staggered grid. However, the accuracy is not lost by this procedure.

5. NUMERICAL SOLUTION PROCEDURE

In order to deal with complex geometries, a general finite-volume technique is employed. A boundary fitted coordinate method is also applied.

The method allows us to map the complex flow domain in the physical space to a rectangular domain in the computational space by using a curvilinear coordinate transformation. The Cartesian coordinate system in the physical space is replaced by a general non-orthogonal coordinate system. The method is implemented in CALC-BFC [11].

The momentum equations are solved for U, V and W in the Cartesian coordinate system on a non-staggered grid. The Rhie-Chow interpolation method is used for the velocity components at the control volume faces. The pressure velocity coupling is handled by the SIMPLEC-method. TDMA and SIP based algorithms are employed for solving the equations. The convective terms are treated by the hybrid, van Leer, QUICK and Upwind schemes while the diffusive terms are treated by the central-difference scheme.

5.1 Sample Calculations

The Prandtl number was set to 0.73 and the Reynolds number was varied from 6500 to 65000 by choosing appropriate values of β, the per-cycle pressure gradient. The computations were terminated when the sum of absolute residuals normalized by the inflow was below 10^{-5} for all variables. To achieve this convergence criterion, the under-relaxation factor was set to 0.6 for all variables. The calculations were carried out on a DEC 3000/400 AXP computer. 21 grid points were chosen in the y- and z-direction and 3 in the x-direction. The calculations show that the number of grid points in the y- and z-direction should not be chosen less than 10. The calculations show also that choosing more than 21 grid points does not improve the results (Nu-number and friction factor) significantly.

6. ADDITIONAL EQUATIONS

The Re-number is defined by

$$Re = \frac{\rho U_m D_h}{\mu} = \frac{\dot{m} D_h}{A_{cross} \cdot \mu} \tag{23}$$

where A_{cross} is the cross sectional area, U_m is the mean velocity which is related to $\dot{m}$ as

$$\dot{m} = \int_A \rho U dA = \rho U_m A_{cross} \tag{24}$$

The pressure drop per cycle is defined by

$$\Delta p = \beta L = 4f \frac{L}{D_h} \frac{\rho U_m^2}{2} \tag{25}$$

where f is the Fanning friction coefficient. From (25) one has

$$f = \frac{\beta D_h / 4}{\rho U_m^2 / 2} \tag{26}$$

This friction factor is compared with the Prandtl friction law [11]

$$\frac{1}{\sqrt{4f}} = 2 \log\left(Re \sqrt{4f}\right) - 0.8 \tag{27}$$

The Nu-number can be found to follow

$$Nu_x = Pr \frac{\rho U^* D_h}{\mu T_b^+} \tag{28}$$

and

$$T_b^+ = \frac{\iint |U^+| T^+ dA}{\iint |U^+| dA} \tag{29}$$

By these equations and the definition of the T_b^+, one finds

$$Nu_x = \frac{Pr D_h}{\mu} \frac{1}{T_w - T_b} \frac{q_w}{C_p} \tag{30}$$

a) If $y^+ \leq 11.63$ at the wall adjacent grid point (index p) Nu_x is found from

$$Nu_x = D_h \frac{\theta_p}{y} \tag{31}$$

where θ_p is the dimensionless temperature .

b) If $y^+ \geq 11.63$ at the wall adjacent grid point (index p) , Nu_x follows

$$Nu_x = \frac{Pr\, D_h}{\mu} \frac{\rho U^* \theta_p}{\sigma_t \left[U^+ + P\left(\frac{Pr}{\sigma_t}\right) \right]} \tag{32}$$

where the P-function according to Jayatillika is defined as

$$P\left(\frac{Pr}{\sigma_t}\right) = 9.24 \left[\left(\frac{Pr}{\sigma_t}\right)^{0.75} - 1 \right] \left[1 + 0.28 e^{\left(-0.007/\left(\frac{Pr}{\sigma_t}\right)\right)} \right] \tag{33}$$

The overall heat transfer coefficient is calculated by

$$h_{ov} = \frac{\dot{Q}}{A_w \Delta T_{wb}} \tag{34}$$

where

$$\dot{Q} = \dot{m} C_p \left(T_{b2} - T_{b1} \right) \tag{35}$$

$$\Delta T_{wb} = \frac{1}{2} \left\{ \left(T_w - T_{b2} \right) + \left(T_w - T_{b1} \right) \right\} \tag{36}$$

Combining these equations one finds

$$Nu_{ov} = \frac{2 A_{cross}}{A_w} \frac{(1-\gamma)}{(1-\gamma)} Pr\, Re \tag{37}$$

where γ in the cyclic case can be derived as

$$\gamma = \exp\left(\int_0^L \lambda dx \right) \tag{38}$$

In non cyclic cases one has

$$\gamma = \frac{T_w - T_{b2}}{T_w - T_{b1}} \tag{39}$$

The Nu-number is compared to the Dittus-Boelter [12] equation for circular ducts:

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.4} \tag{40}$$

7. RESULTS and DISCUSSION

The secondary flow velocity vectors predicted by Speziale's non-linear k-ε model in a square duct in the fully developed region are shown in Fig. 2b.

The corresponding grid is shown in Fig. 2a. This secondary flow is predicted at all the considered Reynolds numbers. The agreement with the results in [5,6] is very good.

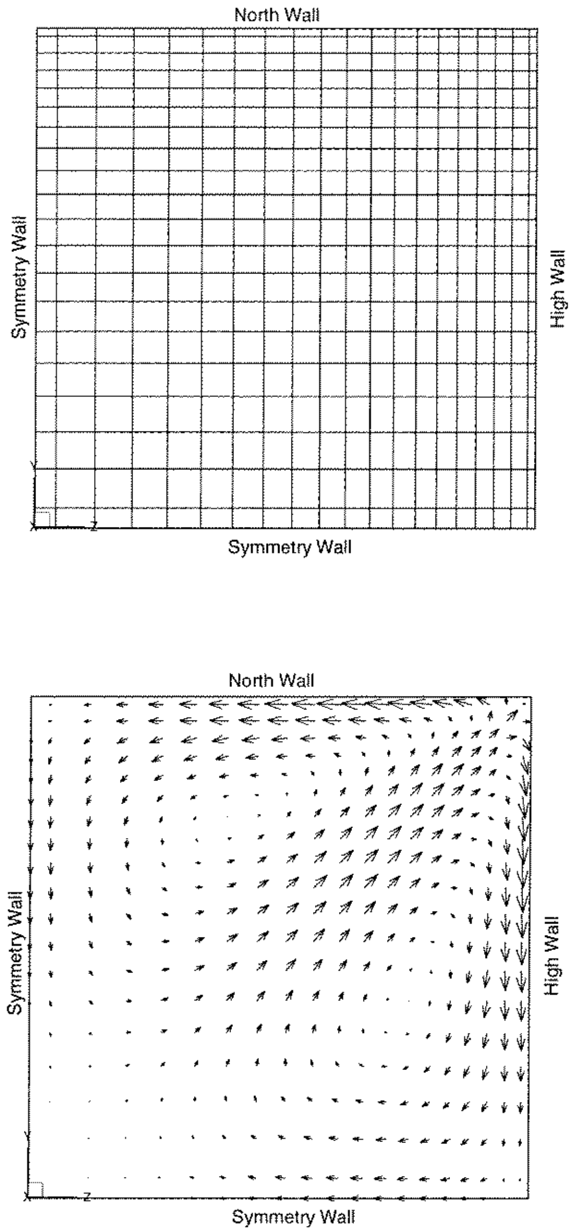

Figure 2. a) Grid specification,   b) Secondary velocity vectors.

Table 1 shows the calculated Nu_X (local)- and overall Nu-numbers. It should be noted that the turbulent shear stresses used in the GGDH and WET models are calculated by the Speziale's non-linear model. The results are compared with the Nu-number in circular ducts, because the experimental investigations show that the Nu-number in the square duct can be calculated by the formula for a circular one, see [13,14].

Table 1. Local (Nu_x) and overall Nu-number

Type	Re	Nu_x	Nu (cir)	diff[3] %	Nu_over-all	diff[3] %
k-ε[1]	64769.14	154.89	147.85	-4.76	157.38	-6.65
Speziale[1]	64344.91	155.40	147.07	-5.66	157.17	-6.86
GGDH[2]	"	147.67	"	-0.41	149.11	-1.38
WET[2]	"	148.97	"	-1.29	150.44	-2.29
k-ε[1]	49259.87	123.77	118.77	-4.21	126.08	-6.15
Speziale[1]	48977.45	124.24	118.08	-5.08	125.97	-6.55
GGDH[2]	"	118.68	"	-0.38	120.07	-1.56
WET[2]	"	119.72	"	-1.26	121.06	-2.40
k-ε[1]	29499.51	81.559	78.809	-3.54	83.706	-6.62
Speziale[1]	29332.02	82.005	78.451	-4.53	137.20	-6.55
GGDH[2]	"	79.307	"	-1.09	80.639	-2.79
WET[2]	"	79.964	"	-1.92	81.265	-3.59
k-ε[1]	20953.09	62.234	59.941	-3.83	64.160	-7.04
Speziale[1]	20859.87	62.539	59.727	-4.71	63.987	-7.13
GGDH[2]	"	61.030	"	-2.18	62.240	-4.21
WET[2]	"	61.502	"	-2.97	62.704	-4.98
k-e[1]	12473.28	41.628	39.583	-5.17	43.234	-9.22
Speziale[1]	12480.39	41.723	39.601	-5.36	42.990	-8.56
GGDH[2]	"	41.539	"	-4.89	42.632	-7.65
WET[2]	"	41.777	"	-5.49	42.867	-8.25
k-ε[1]	6514.167	25.766	23.540	-9.45	26.641	-13.17
Speziale[1]	6527.677	25.756	23.579	-9.23	26.613	-12.87
GGDH[2]	"	26.388	"	-11.9	27.126	-15.04
WET[2]	"	26.417	"	-12.0	27.168	-15.22

[1]Nu-number calculated by Simple Eddy Diffusivity (SED) model.
[2]The turbulent shear stresses in the GGDH and WET used from Speziale's non-linear k-ε
[3]Diff % = $\frac{Nu(cir) - Nu(calculated)}{Nu(cir)} \times 100$

Table 1 shows that the combination of Speziale's non-linear k-ε model and GGDH gives best result for Re-number greater than 10000. It shows also that GGDH and WET give better result than Simple Eddy Diffusivity (SED). y_p^+ for all calculations is between 42 and 44. It should also be noted that the results calculated by the hybrid, QUICK, van Leer and upwind schemes are almost the same. The results in the table 1 are calculated by the hybrid scheme.

Figure 3 shows the Fanning friction factor calculated by the linear k-ε and non-linear Speziale's model. The figure shows that both models give almost identical results for all Re-numbers. The predicted Fanning friction factor is much higher than expected. However, the friction factor depends strongly on the y_p^+ -value for the point adjacent to the wall. This y_p^+ -value is between 42 and 44 for the calculations in Fig.3.

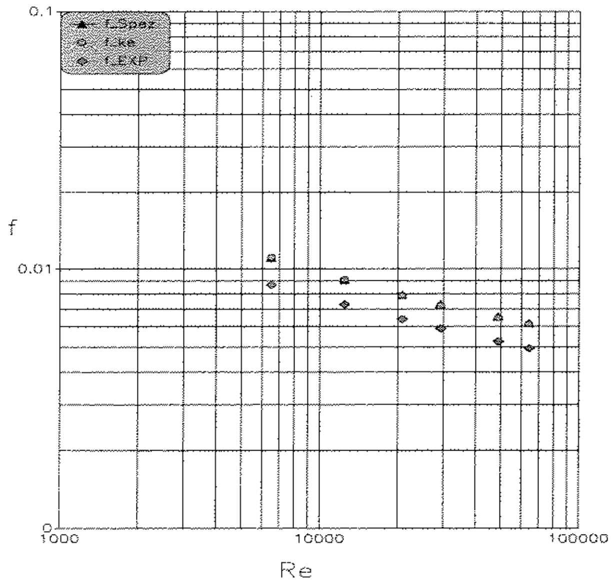

Figure 3. Fanning friction factor as function of Re-number.

Appendix 1

The values of the coefficients in equations (4), (5), (9) and (11) are: $\sigma_k=1.0$, $\sigma_\varepsilon=1.314$, $C_{\varepsilon 1}=1.44$, $C_{\varepsilon 2}=1.92$, $C_D=C_E=1.68$ and $C_\mu=0.09$. The turbulent Prandtl-number (σ_τ) is set to 0.8.

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