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On the Assessment of Ventilation Performance with the Aid of Numerical Simulations

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The assessment of ventilation performance is discussed. New local indices are developed with the aid of numerical simulations to quantify air diffusion and contaminant dispersion. The local purging effectiveness, A_{sp}, is an index for evaluating the contribution of each inlet in a multi-inlet system. The local specific contaminant-accumulating index, α, can be used to indicate the tolerance of a ventilation flow to contaminants. A_{sp} and α can be derived from transport equations. A method based on age-variation analysis is used to define A_{sp} and the Expected Contaminant Dispersion Index (ECDI). The latter is an index for forecasting contaminant dispersion emitted at a specific location with unknown source strength. These new scales and methods can be used to assess ventilation performance. © 1997 *Published by Elsevier Science Ltd.*

NOMENCLATURE

A	local purging effectiveness
C	mean concentration
$\langle C \rangle$	mean room concentration
C_e	mean exhaust concentration
D	diffusivity for air or tracer gas
ECDI	expected contaminant dispersion index
M	contaminant content in a room or a volume
P	transfer probability
q	contaminant amount released per unit time
Q	supply air flow rate
r	contribution ratio
R_p	local residual turnover flow rate, $R_p = W_p - U_p$
$s1,...,sm$	notation of inlets
S_c	contaminant source strength per unit volume
t	time
$\mathbf{u}$	velocity vector
U_p	local purging flow rate
V	volume for the room space
W	local turnover flow rate
x,y,z	directions in Cartesian coordinates

Greek letters

α	local specific contaminant-accumulating index
β	residence time
γ	local age-integrated exposure (equation (21))
δV	volume of compartment
ε_{ap}	local air quality index, C_e/C_p
ε_c	contaminant removal effectiveness, $C_e/\langle C \rangle$
ε_p	local air change index, τ^n/τ_p
λ	residual time
τ	local mean age of air
τ^n	nominal time constant, V/Q
τ_t	turnover time of contaminants
ϕ	age frequency function
Φ	cumulative age distribution

Other symbols and subscripts

∇	nabla operator
c	contaminant
e	exhaust
jp	from j to p
p,j	locations within flow system
s	supply

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INTRODUCTION

The primary objective of ventilation is to improve indoor air quality and remove pollutants in rooms. A ventilation system achieves this by mechanically-driven air motion, where fresh air is delivered to the occupied zone and contaminants are removed or diluted. The performance of a ventilation system greatly depends on the air flow within a room created by the system itself. An understanding of the behavior of ventilation flow, therefore, is essential to assess ventilation performance.

In order to analyze the transport properties of the continuum flow in a system, one of the most effective and straightforward methods is to analyze the velocity distributions. In addition, great efforts have been made to explore the various parameters available for characterizing ventilation flows. The aim is to find scales that are applicable for indicating *ventilation efficiency*. Such scales should be relevant for determining the degree to which fresh air is dispersed, recirculated and mixed within a ventilated space, as well as for determining how ventilating air interacts with pollutants.

The scales for assessing ventilation performance can be either local or global. The local scales are usually able to reveal the details of ventilation flows at desired locations; the global scales, by contrast, yield general descriptions for flow systems. The quantitative determination of a ventilation scale is, in general, achieved with experimental measurements or numerical simulations. In addition, the compartmental (multi-chamber/zone) method has been used [1–4]. Most of the existing scales were originally devised in terms of concentration at a steady state and/or a time-dependent concentration sequence recorded by means of tracer experiments. Some scales have been defined by using the concepts of local mean age and residence time of the air. These concepts are virtually derived from concentration sequence through tracer experiments.

When using a scale to characterize a ventilation flow system, this scale should meet two criteria according to Sandberg [1]: it should be generally able to assess system performance under different operating conditions; and it should be measurable. These two criteria are not always met. In particular, some scales are difficult to measure in experiments, e.g. the local purging flow rate [5] and some of the scales proposed in Refs [6–8]. Nonetheless, some means must be available for quantifying these scales, such as numerical methods. On the one hand, many scales have been proven to be available for characterizing ventilation air flows. On the other hand, these scales are usually unsuitable for indicating contaminant dispersion and removal that depends not only on ventilation flows but also on problem-dependent contaminant source(s). Some scales sometimes even fail to show the level of contaminant due to a specific contaminant source. For example, a high local air change index at one position does not promise a low contaminant concentration, since this depends also on the contaminant source (e.g. its location and strength). A scale that is able to bridge these two aspects, i.e. characterize air flow (the general) and indicate contaminant dispersion from specific source (the specific), is thus needed for ventilation applications.

In this study, the ventilation scales are classified into three groups. In order to set up the background of the analyses, some basic concepts used for quantifying ventilation flows are briefly described, as well as the methods used for determining ventilation indices. These methods include the experimental method, the compartmental method and the numerical method. Imaginary tracer experiments are used to examine some relations and to analyze ventilation air flows.

Numerical simulation can usually provide fundamental and local flow structure in a ventilated space. This work therefore concentrates on discussing and developing local scales that can be explored with the aid of numerical simulations. The *local purging effectiveness of inlet*, A_{sp}, is proposed to evaluate the effect of each inlet when delivering fresh air to a location in a multi-inlet flow system. When using "reverse tracing" flow (see e.g. Ref. [7]), this concept can also be applied to systems with multiple outlets. Another new scale is the *local contaminant-accumulating index*, α, which can be used to indicate the capacity of a ventilation flow to account for specific contaminant sources. These local quantities can be derived from the transport equations given in this study. It is thus convenient to use numerical simulations to determine them. In addition, by using the *age-variation analysis*, a scale independent of source strength, termed the Expected Contaminant Dispersion Index (ECDI), is defined to forecast the dispersion of contaminants emitted at specific locations. The applications of these new proposals are demonstrated with the aid of numerical simulations.

ASSESSMENT OF VENTILATION PERFORMANCE

To assess ventilation performance, the evaluation of a flow system is usually carried out by means of ventilation scales or indices. These can be classified into the following three groups.

(a) *Ventilation air-diffusing efficiency*. This includes parameters that evaluate ventilation performance by indicating how efficiently fresh air has been supplied and delivered into a zone.
(b) *Ventilation effectiveness*. This includes parameters that evaluate ventilation performance by indicating how effectively passive contaminants in a zone can be removed or diluted by ventilating air flows.
(c) *Specific ventilation effectiveness*. This includes parameters that evaluate the ability of ventilating air to remove or dilute contaminants for a specific application.

Scales in (a) and (b) are the general indices that can be used to indicate ventilation performance under various operating conditions. These have been classified from two aspects of ventilation, i.e. the ability to supply fresh air into and remove contaminants from a space. Scales in (c) are the specific indices that often depend on the property of the contaminant source (strength and location). They are thus usually unsuitable as general indices.

In the following section, some indices and the methods used to determine them are briefly described. It should not, however, be viewed as a review. Comprehensive reviews of these indices can be found in several references, see e.g. Refs [9–13].

Basic ventilation indices

The concepts described here have usually been used as basic indices for quantifying ventilation systems, and used to define the other ventilation scales.

Concentration is the most fundamental concept, and has been used to derive almost all the other ventilation parameters. It expresses the amount of contaminant or tracer gas contained in a unit volume of fresh air. A detailed description of concentration has been given in, e.g., Ref. [5]. The local concentration is governed by the following transport equation:

$$\frac{\partial C}{\partial t} + \nabla \cdot (\mathbf{u}C) = \nabla \cdot (D\nabla C) + S_c, \quad (1)$$

where S_c is the contaminant source strength per unit volume. The concentration is a scale in both (b) and (c), with the aforementioned classification. It can be used to indicate both the ventilation effectiveness and the specific ventilation effectiveness.

Local mean age of air, τ. The local mean age of the air, τ, expresses statistically the mean time of the air to reach an arbitrary point after entering into a system. It is a transportable scalar quantity, and is governed by a transport equation [14]:

$$\nabla \cdot (\mathbf{u}\tau) = \nabla \cdot (D\nabla\tau) + 1. \quad (2)$$

Equation (2) can be derived from equation (1) by means of a passive tracer experiment with the source homogeneously distributed in the space. This parameter can be used to passively track the air flow. It represents the freshness of the air in a ventilated space. It can thus be used to indicate the ventilation air-diffusing efficiency.

Purging flow rate, U_p. The purging flow rate, U_p, expresses the net flow rate at which the passive contaminant at an arbitrary location (region) flushes towards the outlet. It also represents the net flow rate at which the fresh air is supplied to this region. The definition of U_p and its determination can be found in, e.g., Refs [4, 15, 16]. Note that U_p is not a quantity at a *point*. Instead, it is a quantity for a *region* within a flow system. This concept has thus been re-termed the *regional purging flow rate* [4]. The purging flow rate is a scale for both the ventilation air-diffusing efficiency and ventilation effectiveness.

Contribution ratio, *r*. The contribution ratio reflects the effect of each inlet and outlet for a multi-inlet/outlet system. Kato *et al.* [7] proposed and demonstrated the use of this concept. The contribution ratio of a supply opening, r_s, indicates the fraction of the fresh air delivered to a location. For an exhaust opening, the contribution ratio, r_e, indicates the influence of the exhaust opening. The contribution ratios r_s and r_e are thus scales available for the ventilation air-diffusing efficiency, and r_e can also be used to quantify the ventilation effectiveness if the contaminant is passive. The contribution ratio is less useful for recirculating regions. Peng and Davidson [4] recently proposed a Markov chain model to determine the transfer probabilities from a supply opening to an arbitrary location, and from an arbitrary location to an exhaust opening. When used together with the compartmental method (multi-chamber method), the contributions of the inlet and outlet to an arbitrary region in a ventilated space can be determined.

Residence time distribution (RTD). This concept was first explored in chemical engineering, see e.g. Refs [17–20], and introduced into ventilation practice by Chen *et al.* [21] and Sandberg and Sjöberg [5]. It characterizes the flow structure within a system, see e.g. Ref. [22]. This concept can thus be used for evaluating ventilation air-diffusing efficiency. When the contaminant within a flow system can be treated as passive, the RTD can be used to quantify the contaminant dispersion. This makes the RTD a scale suitable for indicating both the ventilation effectiveness and specific ventilation effectiveness. One example is the residence time of a contaminant released at an arbitrary location, which equals the residual time of the air passing through the same location. Another concept derived from the RTD and used in ventilation practice is the so-called "segregation", which expresses the variation between a flow pattern and a perfect mixing flow [17]. The relationships between different time distributions have been discussed in Ref. [5]. The expressions and equations for different time distributions for quantifying ventilation air flows have been summarized in Ref. [23].

Methods for determining ventilation scales

Three methods have usually been used to quantitatively determine various local ventilation scales: the experimental method, the compartmental method, and the numerical method. They each have advantages and disadvantages.

Experimental method. The experimental method can be further divided into two parts, the direct and indirect methods [5]. The direct method examines ventilation performance for specific situations by means of field or mock-up measurements. The specific ventilation effectiveness is thus evaluated. The indirect method is generally used to quantify the properties of ventilation air flows. It characterizes the capability of a system to evacuate contaminants, or to supply fresh air, or both. This method can thus be used to estimate both ventilation air-diffusing efficiency and ventilation effectiveness. The indirect measurement usually involves tracer experiments.

Compartmental method. This method is also termed the multi-chamber/zone method [2, 3]. Using the principle of mass conservation, this method can effectively analyze the performance of ventilating air in various zones or compartments. In particular, when used for buildings with multiple rooms, the ventilation air-diffusing efficiency and the ventilation effectiveness for each room can be evaluated. Peng and Davidson [4] recently used this method together with a Markov chain model to determine the regional purging flow rate (for each compartment) and other ventilation scales.

Numerical method. The advantages of using the numerical method to analyze ventilation performance have been pointed out in, e.g., Refs [8, 24]. Liddament [24] concluded that the predictions from numerical methods have enabled the concepts of ventilation efficiency and contaminant removal effectiveness to be applied at the design stage, while the value of the experimental method has been restricted to evaluation and diagnostic studies on existing structures. In numerical methods, a flow system is divided into a number of cells. A series of conservative equations is then solved for each cell with well-specified boundary conditions. The flow structure and the contaminant transport within the flow system are therefore simulated with the predicted spatial distribution of the air velocity and the contaminant concentration. The numerical method can be used together with the compartmental method, i.e. the CN method [4]. This can make the determination of the interconnecting flows between different compartments quite efficient.

The experimental method is usually costly and time-consuming, but often provides results closest to practice. The numerical method is the most efficient means of providing local detailed characteristics for both the air flow and the contaminant distribution. The compartmental method (multi-chamber/zone method) provides a tool for analyzing a ventilation system by means of quantifying the regional ventilation performance for various compartments/zones. The numerical method usually analyzes ventilation indices by solving the mass transport equation. The basic principle for both the compartmental method and the numerical method is thus that of mass conservation. When the size of the divided compartments in a flow space is close to that of the cells used in numerical simulations, the compartmental method approaches the numerical method. The key with the compartmental method is to accurately determine the interchanging flows. The numerical or compartmental methods can be used to determine some ventilation scales that are difficult to measure in experiments, e.g. the purging flow rate.

Tracer experiments

Tracer experiments have been widely used to characterize ventilation flow systems. Three well-known methods are often used in practice, the step-up method, the step-down method, and the pulse method, see e.g. Refs [1, 25–27]. In tracer experiments, the air flow in a ventilated space is tracked by a passive tracer gas released at the inlet or at a location within the space. When the contaminant generated in a room can be treated as passive, the measurement also indicates the characteristics of the contaminant dispersion. Tracer experiments are particularly useful for determining the RTDs of the air and passive contaminant within a flow system, as well as for determining the air exchange efficiency and air flow rate in buildings.

When the tracer gas is *passively* used to track an air flow, the flow should not be disturbed during the experiment. However, complete pre-mixing has often been recommended in many step-down experiments by using mixing fans. With complete pre-mixing, the measured tracer concentration decays exponentially. This is convenient when tracer experiments are carried out to measure the overall ventilating air flow rate. Caution must be taken, however, when tracer experiments are used to measure the local scales that characterize ventilation air flow patterns, e.g. the local mean age of air. Mixing fans give a uniform initial concentration and cause the air flow to deviate from a steady state. The concentration at a point within the space is that of an unsteady ventilation flow compared to the real one.

Several imaginary tracer experiments are discussed here, and introduced into analyses for steady ventilation flows. Although the experiments may be impossible to carry out in practice, they are theoretically useful for deriving some relations.

Case 1: *Step-down method.* In order not to disturb the air flow field, there is no pre-mixing. The compartmental method is used, so the room is divided into a number of compartments (regions). The mean concentration in an arbitrary compartment p is $C_p(t)$, and the volume of p is δV_p. By specifying a constant tracer concentration at the inlet, a steady mean initial concentration is created in each compartment, say $C_p(0)$ for compartment p. The step-down procedure is then switched on by re-specifying a zero concentration at the inlet.

With the compartmental method, which makes the flow field discretized, the spatial distribution of $C_p(t)$ at time t is approximated to be stepwise. The mean concentration in each compartment can approximately be assumed to decay exponentially [1], i.e.

$$C_p(t) \approx C_p(0)\exp(-E_p t). \tag{3}$$

In equation (3), E_p is a function of the mean position of the compartment (x_p, y_p, z_p). The recorded decay of $C_p(t)$ gives the complementary cumulative age distribution function, $(1-\Phi_p)$, for compartment p. This yields

$$\Phi_p = 1 - \frac{C_p(t)}{C_p(0)}. \tag{4}$$

Note that Φ_p is the *mean* cumulative age distribution for the air passing through the compartment. The mean age of the air in compartment p can then be derived from

$$\tau_p = \int_0^\infty t\left(\frac{\partial \Phi_p}{\partial t}\right)\mathrm{d}t. \tag{5}$$

Substituting equations (3) and (4) into equation (5) gives $E_p = 1/\tau_p$. The mean concentration for compartment p during a step-down procedure can then be expressed approximately as

$$C_p(t) \approx C_p(0)\exp\left(-\frac{t}{\tau_p}\right). \tag{6}$$

With a similar argument for a step-up procedure, the mean transient concentration for an arbitrary compartment can be written approximately as

$$C_p(t) \approx C_p(\infty)\left[1-\exp\left(-\frac{t}{\tau_p}\right)\right]. \tag{7}$$

Equations (6) and (7) provide an approximate way to estimate the transient mean concentration for a compartment. The mean age τ_p is thus the average value of the air passing through the compartment. It is not advisable, however, to use these equations to determine other ventilation scales. The inaccuracy in equations (6) and (7) lies in the assumption of equation (3). The expression for the mean concentration in a compartment or zone is actually much more complicated than equation (3), as shown with the two-zone model [1, 2]. The experimental data given by Sandberg [1] with a two-zone model showed that the mean zonal concentration in the initial period does not change exponentially.

Case 2: *Homogeneously distributed source in space.* The source is homogeneously distributed within a ventilated space, and tracer gas is continuously released at a constant rate $S_c = q/V$ (per unit volume). The compartmental method is used to make the analysis. The inlet and outlet are assigned imaginary compartments. This means that a part or whole of an inlet connected to an interior compartment is treated as an individual boundary compartment (with zero volume) that supplies air into its neighboring interior compartments. A similar treatment is given to the outlet.

For an interior arbitrary compartment p, using the principle of mass conservation gives the following relation:

$$S_c\delta V_p + \sum_{j(j\neq p)} W_{jp}C_j(\infty) = W_pC_p(\infty). \tag{8}$$

In equation (8), W_p is the turnover flow rate for compartment p (total air flow which is leaving p), W_{jp} $(j\neq p)$ is the flow rate from j to p (air flow entering p), and $W_p = \sum W_{jp}$ $(j\neq p)$. Note that $\tau_p \equiv C_p(\infty)/S_c$ for a homogeneous source, which can easily be shown by dividing both sides of equation (1) by S_c, and comparing with equation (2). Equation (8) can thus be rewritten as

$$\tau_p = \frac{1}{W_p}\left(\delta V_p + \sum_{j(j\neq p)} W_{jp}\tau_j\right). \tag{9}$$

In equation (9), W_p and W_{jp} $(j\neq p)$ are the flow rates induced due to air convection, this equation then turns out to be the discrete form of the differential equation for τ (equation (2)), with the diffusion term dropped. The local mean age derived from the compartmental method is thus an approximation that does not consider diffusion. This approximation has proved to be acceptable when solving equation (2) with the diffusion term excluded, see e.g. Ref. [28]. If the diffusion conductance is incorporated into the flow rates in equation (9), W_p and W_{jp} $(j\neq p)$ are then equivalent to the coefficients of equation (2) in its discrete form. Equation (9) is then the discretized τ-equation.

In equation (9), let $\tau_{np} = \left(\sum_{j(j\neq p)} W_{jp}\tau_j\right)/W_p$, which is a flow-rate-weighted average age of the air passing into other compartments. Then equation (9) becomes

$$\tau_p = \frac{\delta V_p}{W_p} + \tau_{np} = \tau_p^n + \tau_{np}, \tag{10}$$

where $\tau_p^n = \delta V_p/W_p$ is regarded as the local nominal time constant for the air passing through compartment p. Equation (10) (or equation (9)) indicates that the local mean age of the air is always larger than the local nominal time constant for a compartment, i.e. $\tau_p > \tau_p^n$.

With the purging flow rate, U_p, τ_p can also be expressed as [4, 5]

$$\tau_p = \frac{1}{U_p}\left(\delta V_p + \sum_{j(j\neq p)} P_{jp}\delta V_j\right), \tag{11}$$

where P_{jp} is the transfer probability from compartment j to p, and $P_{jp}\leq 1$.

The turnover flow rate for a compartment p is composed of two parts: the *net* flow rate at which the air is flushing towards the outlet, i.e. the purging flow rate U_p, and *the residual turnover flow rate*, R_p, that may recirculate and rejoin compartment p after leaving it. This suggests

$$U_p = W_p - R_p. \tag{12}$$

Further, using equations (9) and (11) gives

$$R_p = \left(\frac{1}{\tau_p}\right)\sum_{j(j\neq p)} (W_{jp}\tau_j - P_{jp}\delta V_j). \tag{13}$$

Equation (13), together with equation (12), provides an alternative way to describe the purging flow rate U_p.

Case 3: *Step-up method.* Tracer gas is released at the inlet at a constant rate q. The compartmental method is used again in the following analysis. By using the mean value of the concentration, $C_p(t)$, for an arbitrary compartment p at time t, the mass conservation of tracer gas requires that

$$\sum_{j(j\neq p)} W_{jp}C_j(t) - W_pC_p(t) = \delta V_p\frac{\partial C_p(t)}{\partial t}. \tag{14}$$

Dividing by $C_p(\infty)$ on both sides of equation (14) yields

$$\sum_{j(j\neq p)} W_{jp}\Phi_j(t) - W_p\Phi_p(t) = \delta V_p\frac{\partial \Phi_p(t)}{\partial t}, \tag{15}$$

where $\Phi_j(t) = C_j(t)/C_j(\infty)$ and $\Phi_p(t) = C_p(t)/C_p(\infty)$. Note that $C_j(\infty) = C_p(\infty)\equiv q/Q$ for any compartments with step-up tracer release at the inlet. Differentiating equation (15) with respect to t yields

$$\sum_{j(j\neq p)} W_{jp}\phi_j(t) - W_p\phi_p(t) = \delta V_p\frac{\partial \phi_p(t)}{\partial t}, \tag{16}$$

where ϕ is the age frequency distribution.

Equation (9) can be reproduced by integrating the product of t and equation (16) from 0 to ∞. Furthermore, equations (15) and (16) suggest that both the mean cumulative age distribution and the mean frequency distribution for an arbitrary compartment hold relations similar to that held by the mean concentration in the compartmental analysis. The mean cumulative age distribution and mean frequency distribution are properties of the air flow itself, and independent of tracer experiments, though the resultant concentration field varies. When the boundary conditions and initial conditions for Φ and ϕ are known, their discrete solutions can then be determined by numerically solving equations (15) and (16).

New ventilation scales

Local purging effectiveness of inlet, A. For systems with multiple inlets, understanding the respective contribution of each supply opening is important for ventilation design and optimization. Kato *et al.* [7] and Murakami [8] proposed the concept of contribution ratio, as mentioned above. The contribution ratios of inlet and outlet, r_s and r_e, can indicate the territories affected by the supply jet and extract sink. They are less useful in recirculating regions of the flow field. The contribution ratio of the outlet, r_e, is determined by a "reverse tracing" of the air flow. This method is numerically stiff. By inserting a negative velocity into equation (1) (with $S_c = 0$), the diffusion term becomes negative. A positive diffusion term in transport equations often enhances the numerical stability. With the negative diffusion induced by the reverse tracing method, the solution could become numerically unstable.

A Markov chain model, which is used together with the compartmental method, has recently been proposed [4]. This model is able to yield the transfer probabilities from a supply opening to an interior region, and from an interior region to an extract. The contribution of each supply and exhaust opening can thus be analyzed.

Here, a new local index is proposed to clarify quantitatively the effect of each supply opening for a multi-inlet system. The openings are denoted by $s1, s2, ..., sp, ..., sm$. For an arbitrary opening, say sp, its contribution to an arbitrary location within this system is analyzed as follows.

(a) The nominal time constant, τ^n, for the flow system is first calculated, $\tau^n = V/Q$, which is the mean age of the air leaving the system through the extract.
(b) *Old air* is supplied into the system through each supply opening, e.g. the exhausted air is fed back to all the supply openings. This can be done in numerical calculations by simply setting the boundary condition of the local mean age at each supply opening to τ^n. Equation (2) is then solved to get the

local mean age of the *old air*, τ_{old}. Note that τ_{old} can also be calculated as the sum of τ^n and the local mean age of air predicted with zero condition at all the supply openings.

(c) *Fresh air* is supplied into the system through one specific inlet, *sp*, while the old air continues to be supplied through the rest of the inlets. This means that the boundary condition of the local mean age at supply opening *sp* is re-specified as 0, and the boundary conditions for the other supply openings remain the same as at step (b), i.e. τ^n. The calculated local mean age of air under this condition is denoted τ_{new}.

The property of the air supplied through the inlets, except *sp*, and the flow field remain unchanged for (b) and (c). The variation between τ_{old} and τ_{new} at any location is, therefore, *purely* due to the contribution of inlet *sp*. At an arbitrary point, the decrease in mean air age, i.e. $\delta\tau = (\tau_{\text{old}} - \tau_{\text{new}})$, indicates the system's capability of purging the *old air* by fresh air supplied through inlet *sp*. In other words, it reflects the *freshening* ability of the supply opening *sp* to the point in question. The relative decrease in mean age at an arbitrary point is defined here as the *local purging effectiveness* of inlet *sp*, A_{sp}. It thus yields

$$A_{sp} = \frac{\delta\tau}{\tau_{\text{old}}} = \frac{(\tau_{\text{old}} - \tau_{\text{new}})}{\tau_{\text{old}}}. \qquad (17a)$$

At the supply opening considered, the local purging effectiveness is equal to unity, and zero for other inlets. A large A_{sp} means a large capability of a supply opening *sp* to diffuse fresh air into a location for purging or diluting the contaminant there. This index can thus indicate the effect of a specific supply opening in a multi-inlet ventilation system. The local purging effectiveness of a specific inlet can be measured with tracer experiments by altering the induced tracer concentration at the inlet considered. The use of this scale is demonstrated with numerical simulations in the next section. When using the "reverse tracing" flow, equation (17a) can also be used to indicate the effect of an exhaust opening for systems with multiple outlets.

The local ages in equation (17a) (calculated under two different conditions) can be replaced by the local concentration, by carrying out two calculations for concentration. This gives

$$A'_{sp} = \frac{(C_{\text{old}} - C_{\text{new}})}{C_{\text{old}}}, \qquad (17b)$$

where A'_{sp} has the same physical background as A_{sp}.

Equations (17a) and (17b) provide a new method for analyzing ventilation air flows, which is termed here the *age/concentration-variation analysis*. The use of this method in conjunction with numerical simulations is convenient. The principle involved in this method is straightforward: the age/concentration difference, $\delta\tau$ or δC, at one location reveals the effect of the factor changed to induce this variation. The method can also be used to analyze the influence by contaminant sources in a ventilated space. Further, the age/concentration variation actually obeys a transport equation that can easily be derived from equation (1) or equation (2):

$$\nabla \cdot [\mathbf{u}(\delta f)] = \nabla \cdot [D\nabla(\delta f)], \qquad (18)$$

where δf represents $\delta\tau$ or δC. The boundary condition for the inlet considered, say *sp*, should be $\delta f_{sp} = \tau^n$, and 0 for other inlets.

Expected contaminant dispersion index (ECDI). For air flows in ventilated rooms, three time quantities are of interest at an arbitrary point: the local mean age of the air, τ, the residual time (life expectancy), λ, and the residence time, β. The age and the residual time together comprise the residence time, i.e. $\tau + \lambda = \beta$. The population must be carefully specified for these three time concepts. The residence time associated with a particular fluid element is unique, wherever this *tagged* fluid element passes. This indicates that β should *not* be viewed as a function of position. The age and residual time of a fluid element change as it passes through the system. Their spatial distributions are thus of practical interest. Discussing the residence time of the air at a position without distinguishing the population is theoretically inappropriate, because there are different residence times at the same point for different populations.

For a contaminant generated at an arbitrary position in a ventilated space, all three time concepts described above can be used. When the contaminant can be treated as passive, its residence time is the residual time of the air passing through the same position. In tracer experiments, this time distribution can be surveyed by recording the concentration sequence at the exhaust opening after the release of a tracer gas at the position considered. For a source located at a specific position, the residence time of the contaminant, β_c, turns out to be its turnover time, τ_t. It can be calculated from

$$\beta_c = \tau_t = \frac{\langle C \rangle V}{q}. \qquad (19)$$

Kato *et al.* [7] suggested a method using reverse tracing flow to numerically work out the residence time of a passive contaminant at any position within the flow system.

In practice, the contaminant source is often distributed at a specific location, which is often a region/zone. It is thus necessary to explore the effect of a ventilating air flow on such a specific regional source as a whole. When the source strength is known, the spatial distribution of the concentration has usually been used to indicate the contaminant dispersion. With unknown source strength, the expected contaminant dispersion for a location-specified source may need to be evaluated as well. In other words, a source-independent index is needed for quantifying *ventilation flows* to forecast the dispersion of contaminants produced at a specific location. The local mean age of contaminant, τ_c, appears to be an alternative to such an index. Theoretically, τ_c can be calculated with a method similar to the determination of local mean age of air. The location where contaminants are released is treated as an inlet of the contaminant. The contaminant is *fresh* at this location, and its age is zero. However, numerical difficulties arise at the inlet of the flow system. The local mean age of the contaminant is infinite at air supply openings, and the boundary condition for τ_c is thus difficult to specify there.

In this work, a new scale, termed the local Expected

Contaminant Dispersion Index (ECDI), is proposed. This scale indicates how the air flow in a ventilated space transports contaminants emitted at a specific location, when the source strength of the contaminant is unknown. The ECDI can be derived from the age-variation analysis, and is defined as

$$\mathrm{ECDI} = \frac{(\tau_{c2}-\tau_{c1})}{\tau_{c2}}, \tag{20}$$

where τ_{c2} is the local age calculated by specifying an age τ^n at the specific location, and zero age at the supply opening; τ_{c1} is the local age calculated by specifying zero age at both the specific location and the supply opening.

The spatial distribution of the ECDI can then be used to estimate how contaminants produced at a specific location are dispersed by the air flow. The use of this index is discussed below.

Local specific contaminant-accumulating index, α. Many of the existing ventilation scales have been used to quantify either ventilating air flows (by indicating the ventilation air-diffusing efficiency), or contaminant removal/dilution (by indicating the ventilation effectiveness) for a ventilation system. In a specific situation, a scale used as a general index (indicating ventilation air-diffusing efficiency or ventilation effectiveness) can be inconsistent with a scale used for indicating specific ventilation effectiveness. For example, a location with a low mean age of air can be a location with a high contaminant concentration, since the contaminant distribution is related not only to the air flow but also to the property of the source. Attempts are thus made in this work to develop an index for assessing the capability of a ventilation flow to tolerate the contaminant source in a specific situation. This index should function as a bridge between the general scale and the specific scale. In other words, it should be able to reflect how the ventilation air flow and the specific contaminant source interact with each other.

With a steady air flow, the distribution of contaminant concentration can be obtained by numerically solving equation (1), if the properties of contaminant source (its location and strength) are specified. The resultant concentration is a straightforward index that indicates the contaminant dispersion within the system for this specific situation. However, this indication is not sufficient to characterize the potential and general capability of an air flow to dilute/remove contaminants. The local contaminant level due to a specific source is not capable of indicating the local *freshness* of the ventilating air itself, which is usually represented with the local mean age of the air. The contaminant concentration cannot, therefore, be adopted as a general index to describe the interaction between air flow and pollution for a specific situation. A new local parameter termed the *local age-integrated exposure*, γ, is thus proposed as the following:

$$\gamma = \int_0^\tau C(t)\,\mathrm{d}t. \tag{21}$$

Equation (21) shows that γ expresses the local accumulation of the contaminant at an arbitrary position over a time equal to the local mean age of the air passing this position. This parameter thus reflects the diluted/removed contaminant amount at a position when the air has *grown up* to the local mean age (representing freshness) τ, or simply the total exposure to the contaminant over a time period of τ. A too large γ suggests, therefore, that the contaminant is *overloaded* by the air with an age of τ at the location considered. By comparing this index with a value specified for limiting the time-integrated exposure, a scale can be obtained for evaluating the capacity of an air flow to dilute/remove the contaminant in a ventilated zone. A low value of γ implies either that a small amount of contaminant has been transported to the position in question, or that the air flow has been quickly supplied to this position, or both.

The local age-integrated exposure is measurable in practice. First, the local mean age of the air at an arbitrary point is detected by tracer experiments. The concentration sequence of the contaminant is then recorded, with the specific contaminant source switched on. The area under the concentration curve during the time period 0–τ is then the value of γ.

When using numerical methods, a transport equation for this quantity can be derived from equations (1) and (2). Integrating both sides of equation (1) with respect to time from 0 to τ at each point yields the following transport equation for γ:

$$\nabla\cdot(\mathbf{u}\gamma) = \nabla\cdot(D\nabla\gamma) - 2D[\nabla\tau\cdot\nabla C(\tau)] + S_c\tau, \tag{22}$$

where $C(\tau)$ is the concentration at the time equal to the local mean age τ. For an arbitrary position, the local concentration after a period τ tends to be steady, since the local mean age expresses statistically a period during which the air has passed through all its upstream points and brought about upstream effects. It is thus arguable that $C(\tau)$ is (approximately) a quasi-steady concentration, i.e. $C(\tau) \approx C(\infty)$ at each position. This makes the implementation of equation (22) easier numerically.

Equation (21) represents the *start-up* accumulation of the contaminant at an arbitrary location. In practice, the accumulation at steady state is of more importance and interest. Equation (21), therefore, becomes $\gamma = C(\infty)\tau$, and the transport equation takes the following form:

$$\nabla\cdot(\mathbf{u}\gamma) = \nabla\cdot(D\nabla\gamma) - 2D[\nabla\tau\cdot\nabla C(\infty)] + C(\infty) + S_c\tau. \tag{23}$$

This equation can be solved by coupling it with equation (1) and equation (2) at a steady state. The boundary condition of γ at the inlet is zero, and its first derivative is assumed to be zero normal to the wall surface and the outlet.

The mean exposure of the whole space to contaminant during one air change is $\langle C\rangle\tau^n$. This mean nominal time-integrated exposure can then be used to normalize the age-integrated exposure, γ. The logarithm (to the base 10) of this normalized quantity is used to define the *local specific contaminant-accumulating index*, α, i.e.

$$\alpha = \log\left(\frac{\gamma}{\tau^n\langle C\rangle}\right). \tag{24}$$

When $\alpha = 0$, then $\gamma = (\tau^n\langle C\rangle)$; $\gamma < (\tau^n\langle C\rangle)$, as $\alpha < 0$; and $\gamma > (\tau^n\langle C\rangle)$ as $\alpha > 0$. A negative α indicates a small amount of contaminant accumulation, and thus implies a large contaminant-diluting capability at the location considered. For complete mixing, α is zero, which forms the basic scale of this quantity. When the steady-state concentration is used for γ in equation (21), equation (24) can be expressed as

$$\alpha = \log\left(\frac{\varepsilon_c}{\varepsilon_p \varepsilon_{ap}}\right), \tag{25}$$

where ε_c is the contaminant-removal effectiveness, $\varepsilon_c = C_e/\langle C\rangle$, ε_p is the local air change index, $\varepsilon_p = \tau^n/\tau_p$, and ε_{ap} is the local air quality index, $\varepsilon_{ap} = C_e/C_p$. Equation (25) shows that α refers to both the delivering of fresh air to a location and the removal of contaminants from this location.

The local specific contaminant-accumulating index, α, is a scale combining the effects of both the specific source (through concentration of contaminant) and the ventilation air flow (through local mean age of air). This index is thus able to reflect the interaction of a specific contaminant source and the ventilation flow. It can be used as a general scale to indicate the ventilation performance under different situations, and as a scale to bridge the ventilation air diffusing efficiency (or ventilation effectiveness) and specific ventilation effectiveness. The use of this index is discussed in the following section.

APPLICATION AND DISCUSSION

In this section, the new scales and methods are analyzed and demonstrated with an example by using numerical simulations. For convenience, a two-dimensional simulation is carried out. Nonetheless, these new proposals can be applied to three-dimensional ventilation flows.

The configuration of the ventilated space is shown in Fig. 1a. This ventilation flow system has two supply openings in the ceiling and two exhaust outlets at floor level. The size of the ventilated space is 8 m × 3.6 m. A block (2 m × 1 m) is used to simulate the working platform, and a passive contaminant source is uniformly distributed above this platform with $S_c = 1.0\,\text{mg/s m}^2$. The nominal time constant, τ^n, is 168 s, and the turnover time of contaminant, τ_t, is 158 s.

The turbulent air flow is simulated with the conventional two-equation turbulence k–ε model, in conjunction with wall functions. The resultant velocity field,

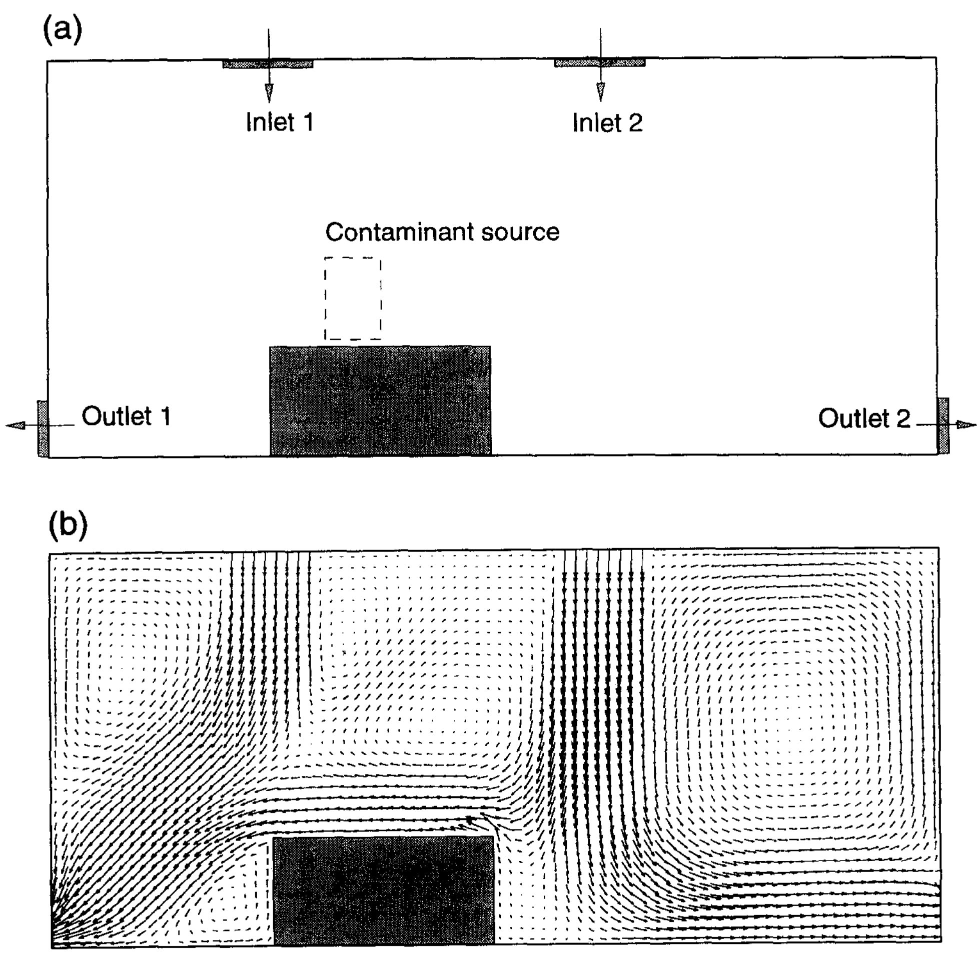

Fig. 1. The ventilation flow system used for demonstration: (a) configuration of a multi-inlet ventilation flow system; (b) air flow pattern simulated with numerical method.

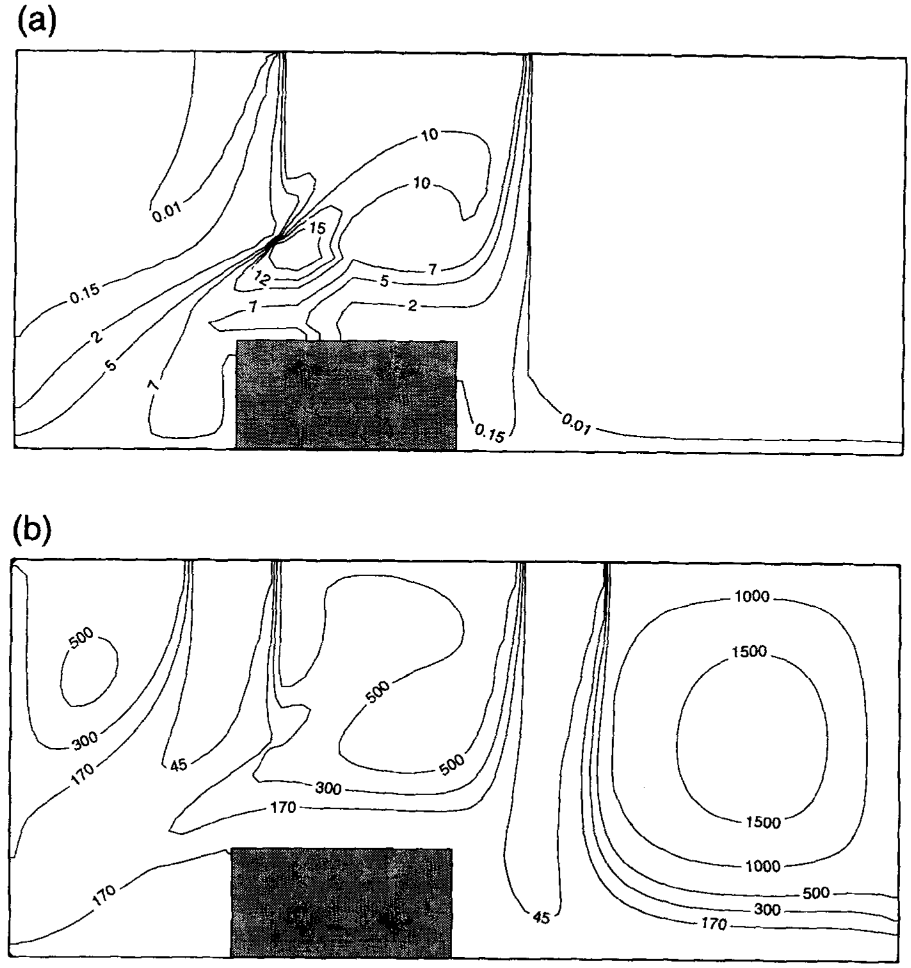

Fig. 2. Distributions for contaminant concentration and local mean age of air: (a) local contaminant concentration $C(\infty)$ (mg/m^2); (b) local mean age of the air τ (s).

shown in Fig. 1b, is then used to solve the transport equations for the concentration C (equation (1)), the local mean age τ (equation (2)), the local age-integrated exposure γ at steady state (equation (23)), and the age/concentration variation (equation (18)). Figure 2a and b show the calculated distributions of the local contaminant concentration and the local mean age of the air. The flow pattern within the space is rather complicated, with several recirculating regions. Owing to air convection, the contaminant above the platform has been entrained into three recirculating regions, i.e. two regions between the inlets, and one behind the platform. The concentration there is thus particularly high. As expected, the local mean age of the air within all the recirculating regions is much higher than τ^n, since the fresh air is not directly delivered into these zones.

Figure 3a and b show the spatial distributions of the local purging effectiveness for the two supply openings, A_{s1} and A_{s2}. The effects of the two supply openings, inlet 1 ($s1$) and inlet 2 ($s2$), are clear. There is always a high purging effectiveness within the supply jet. The contaminant source above the platform is purged or diluted mainly by the air from inlet 2, which has a much higher purging effectiveness than inlet 1. This index is also capable of indicating the influence of the air supply on the recirculating regions. The more fresh air reaching a region, the larger is the purging effectiveness there. A high local age usually corresponds to a low purging effectiveness. The local value of the purging effectiveness indicates the capability of the supply opening considered to provide fresh air to a location, and to remove or dilute contaminants there. A very small value, e.g. 0.05, can be used to indicate the approximate boundary the fresh air reaches. The local purging effectiveness is able to reflect the contribution of each supply opening to a position. This index indicates the ability of a specific supply opening to remove or dilute the contaminated air at a location and replace it with fresh air.

Figure 4 gives the spatial distribution of the ECDI, which is an index independent of source strength. Its calculation can thus be carried out by knowing only the source location. For the purpose of comparison, the location used here is the source location used to compute the contaminant concentration. Note that the ECDI is different from the contaminant concentration that depends on the release rate of contaminants. The ECDI

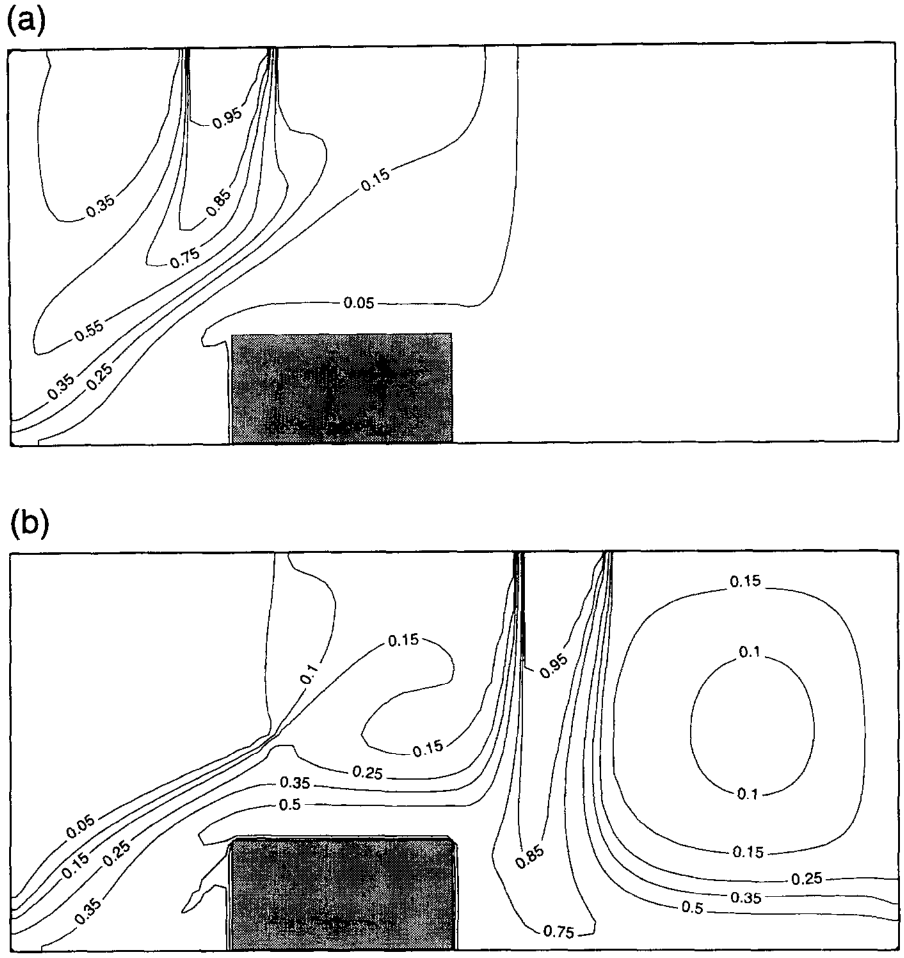

Fig. 3. Distributions of local purging effectiveness for inlets 1 and 2: (a) local purging effectiveness of inlet 1, A_{s1}; (b) local purging effectiveness of inlet 2, A_{s2}.

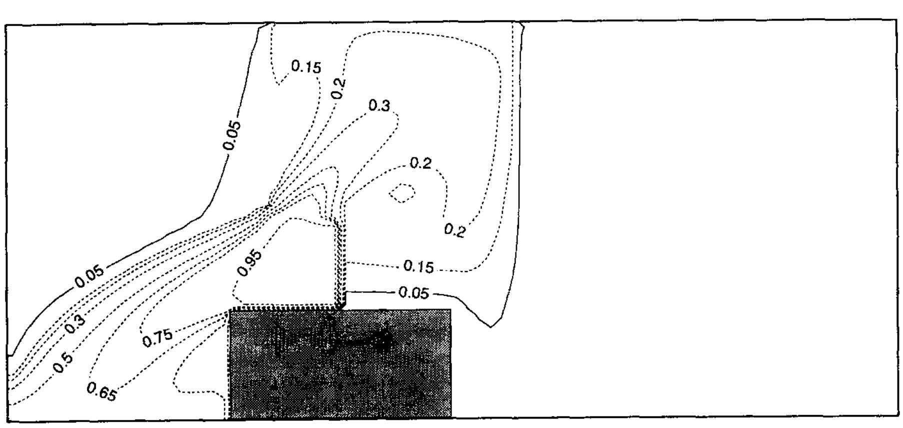

Fig. 4. Distribution of expected contaminant dispersion index, ECDI.

distribution indicates how a contaminant is dispersed by the air flow *if* a source exists above the platform.

The spatial distributions of the age-integrated exposure γ and the specific contaminant-accumulating index α are given in Fig. 5a and b. The dotted lines in Fig. 5b correspond to negative values of α. The two recirculating regions, on the upper left-hand side and right-hand side, have negative values of α due to the low concentration (see Fig. 2a), though the local mean age there is rather high (see Fig. 2b). The region enclosed by the zero contour line in Fig. 5b is the domain with high exposure to the contaminant and old air.

CONCLUSIONS

Studies of scales for assessing ventilation performance are important when designing and optimizing room ven-

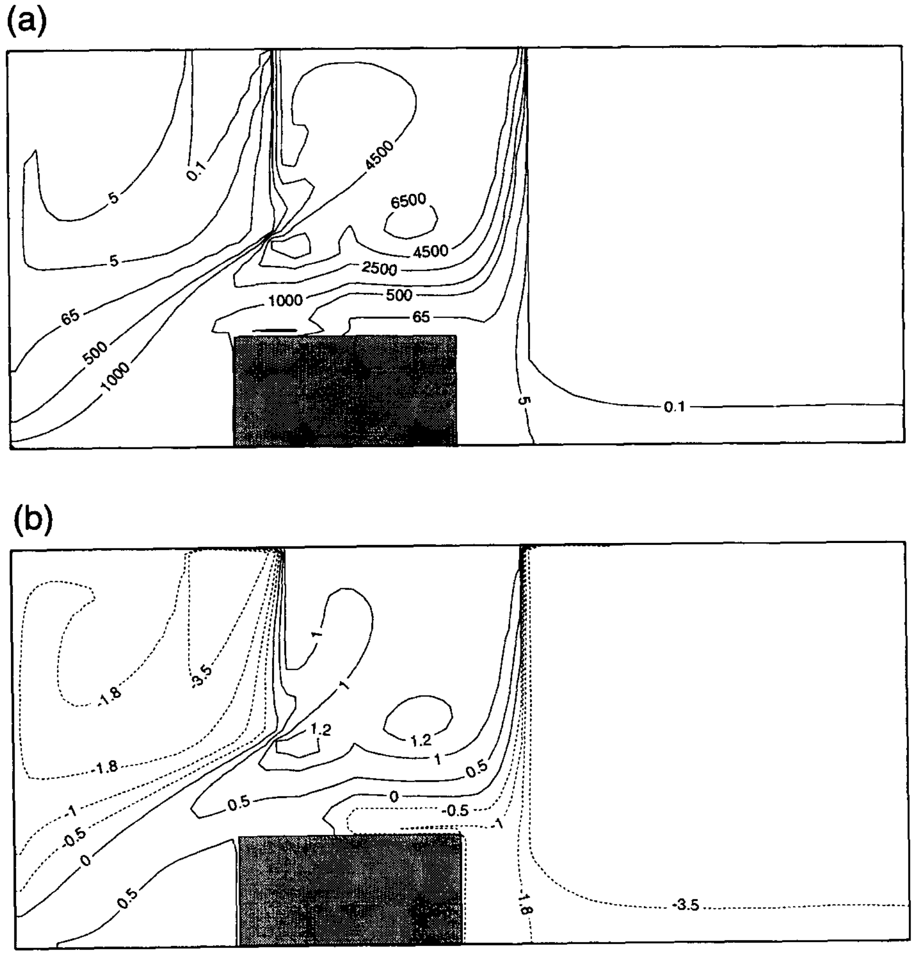

Fig. 5. Calculated distributions of γ and α: (a) age-integrated exposure γ (mg s/m^2); (b) contaminant-accumulating index α.

tilation systems. This work classifies the scales used in practice for characterizing ventilation performance into three groups: *ventilation air-diffusing efficiency* for indicating the ability to provide fresh air to occupants; *ventilation effectiveness* for indicating the ability to remove contaminants from ventilated space; and *specific ventilation effectiveness* for indicating the ability to deal with specific situations. Attempts have been made to develop new ventilation scales to account for these aspects. Imaginary tracer experiments have been used to derive relations useful for analyzing indoor air flows.

Numerical simulations can often provide details of local air flow structure and contaminant dispersion. Several local indices, which can be calculated with numerical methods, have been proposed. The local purging effectiveness of an inlet, A_{sp}, can be used to indicate the effect of each supply opening with multi-inlet systems. This quantity reflects the capability of one supply opening to purge the old air at a location by delivering fresh air there. The same concept can be used to assess the effect of an exhaust opening by using a reverse-tracing air flow for a multi-outlet system.

The specific contaminant-accumulating index, α, or the age-integrated exposure, γ, have been proposed as a scale to combine the general index, τ, with the concentration of contaminant produced by a specific source. This index can be used to assess the ability of a ventilation flow system to tolerate contaminants produced by specific sources. It can also be used to indicate the regions where the air is fresh (low age) but contaminated (high concentration) due to a specific source, or *vice versa*. This scale, as a general index, is thus capable of reflecting the interaction between the ventilation flow and a specific contaminant source.

The new indices proposed here, A_{sp} and α, are both experimentally measurable and numerically solvable. The age/concentration variation (to get A_{sp}) and γ (to get α) are governed by transport equations, which have been derived in this work. They can thus be conveniently analyzed by numerical simulations.

The age/concentration-variation analysis appears to be useful to indicate the influence of a single input in a multi-input system. This method has been used to define A_{sp} and another new scale termed the Expected Contaminant Dispersion Index (ECDI). The latter evaluates the possible dispersion of contaminants produced at a specific location in an air flow. The ECDI is independent of source strength, and can be used to quantify ventilation air flows.

These new parameters and methods have been demonstrated with the aid of numerical simulations. They are useful to quantify and assess indoor air flows and contaminant dispersion. They can thus be used when designing, diagnosing and optimizing ventilation systems. Future work will incorporate these new proposals into measurements and practical applications.

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