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Towards the Determination of Regional Purging Flow Rate

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This paper deals with the description and determination of the purging flow rate, U_p, for ventilation systems or equivalent flow systems. The regional purging flow rate and its use are discussed and proposed. By using the mass conservation principle, U_p is embodied in various accessible mathematical expressions in terms of the transfer probability. Some U_p-related parameters are described. A Markov chain model is proposed for determining the transfer probability and exploring several useful ventilation indices. An effective CN method is proposed for calculating the interchanging flow rates between various regions. The application of these proposals is demonstrated, and they appear to be promising for analyzing and assessing ventilation performance. © 1997 *Published by Elsevier Science Ltd.*

NOMENCLATURE

- a the mean number of visiting times of the particle to a specified region
- $\boldsymbol{A}$ matrix whose entries are a
- b entries in matrices $\boldsymbol{B}$ and $\boldsymbol{B}_O$
- $\boldsymbol{B}$ matrix whose non-diagonal entries are the transfer probabilities, and diagonal entries are the back-mixing probabilities for the interior regions
- $\boldsymbol{B}_O$ matrix whose entries are the transfer probabilities to the outlet
- C concentration
- $C(\infty)$ steady concentration
- $C(t)$ transient concentration at time t
- $C_e(t)$ transient concentration in the exhaust at time t
- $\boldsymbol{D}$ submatrix in $\boldsymbol{F}$
- e number of outlets
- $\boldsymbol{E}$ unit matrix
- f transition probability
- $\boldsymbol{F}$ matrix whose entries are f
- $\boldsymbol{H}$ submatrix in $\boldsymbol{F}$
- $\boldsymbol{I}$ state space/set for the initial distribution of a particle
- $I(s)$ member in $\boldsymbol{I}$, with the initial state at the inlet
- m amount of a pulse contaminant release
- $\dot{m}$ release rate for a continuous contaminant release
- M total contaminant amount borne by the purging flow
- n number of compartments (regions) divided within a flow system
- $\boldsymbol{O}$ zero matrix whose entries are zero
- P transfer probability
- q constant release rate for a continuous contaminant release
- Q total volumetric flow rate supplied to the flow system
- R_p residual turnover flow rate for region p
- s inlet state, $s \in \boldsymbol{S}$
- $s1,...,sm$ notations for the inlets in a multi-inlet flow system
- $\boldsymbol{S}$ a space/set formed by all states the particle may have
- $\boldsymbol{S}_I$, $\boldsymbol{S}_O$ subsets of state space $\boldsymbol{S}$, $\boldsymbol{S}_I+\boldsymbol{S}_O=\boldsymbol{S}$
- t time
- T transfer index
- U purging flow rate
- V volume of flow system
- V_{sp} volume swept by the purging flow passing through region p
- W turnover flow rate passing through a region
- X notation for the station visited by the particle

Greek letters

- α_p equivalent Peclet number for region p
- β_p back-mixing index/probability for region p
- δq contaminant release rate per unit volume
- δV volume of compartment (region)
- Δt time interval
- ε_p air exchange index for region p
- τ mean age of the air
- τ^n nominal time constant of flow system, V/Q
- τ_{rp} residual time of the air passing through region p
- $\psi(t)$ probability density function for τ_{rp}, $\psi(t)=\Psi'(t)$
- $\Psi(t)$ cumulative distribution function

Subscripts and other symbols

- e,O outlet
- i,j,p,r referred to the location of a point or a region in the flow system
- ip from region (point) i to region (point) p
- I interior
- s referred to supply
- $s1,...,sm$ referred to the related inlets
- (j,p) from state (region) j to state (region) p

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1. INTRODUCTION

Indoor air quality and thermal environment are closely related to room ventilation. Good indoor air quality requires efficient dilution and removal of pollutants. For a comfortable thermal environment, excess heat usually needs to be removed. The removal of both requires effective ventilation. An understanding of the air flow behavior is therefore essential to minimize exposure to contaminants and/or excess heat.

Analyses of an internal continuous flow system can usually be directed towards evaluating either the local or the mean global flow behaviors, or both. In chemical engineering, for example, the internal/external residence time distributions have been widely used to characterize flow patterns in chemical and biochemical reactors [1–4]. These concepts are also applicable for analyzing ventilation flows [5]. Sandberg *et al.* [6–9] have introduced and developed a series of concepts, such as the local mean age of air and the moments of concentration histories. These have become one part of the scales now used for assessing the performance of ventilation.

A mean parameter used for a general evaluation of room ventilation, e.g. the external residence time distribution, often fails to account for the flow details. These details are essential for revealing the ventilation effectiveness in specific regions, e.g. the occupied zone and the breathing zone. One of the most promising parameters being used as an indication of the local ventilation performance is the local mean age of air, τ, which represents the mean time interval since the air passing through the location considered has been supplied into a room through the inlet. It is a measure of the air *freshness*, and thus the local dilution capability. This parameter is a passive transportable quantity, and governed by a transport equation [10].

The local mean age of air, however, does not reflect the local potential contaminant-removing capability of the ventilation air. To represent this capability, the so-called "purging flow rate" can be used [7]. It was originally defined as the net flow rate at which contaminants present at a location are expelled from the flow system [11]. This definition makes the purging flow rate a somewhat *artificial* and *imaginary* quantity. A more physical description is required.

The purging flow rate is, in general, defined as a local flow property, which describes the nature of the purging process in a flow system. Applied to ventilation flows, it helps to reveal the characteristics of the air flow pattern and the local pollutant-purging capability. A small purging flow rate for a region means that this region is weakly connected with the rest of the system. Such a region is stagnant. The limits to the lower and upper bounds of the purging flow rate were discussed by Sandberg and Sjöberg [7] and Sandberg [8]. Sandberg [12] later described its use for quantifying the performance of a general ventilation system. The purging flow rate appears to be effective in characterizing the distributions of both ventilation air and contaminant in a room. Unfortunately, a quantitative determination of the local purging flow rate is either currently difficult with experimental methods, or very tedious with numerical methods as shown by Davidson and Olsson's calculations [13], which were limited to a two-dimensional ventilation flow because of huge computational requirements.

In addition to the numerical method, the compartmental method has been used. This method is also termed the multi-chamber/zone method (see e.g. Ref. [8]), and can be used for both a ventilation room divided into a number of compartments and for buildings with multiple rooms. The analysis here is done for a one-room system with multiple divided compartments/regions, unless otherwise stated. However, this does not mean that the analysis is not available for a multi-room system. With the compartmental method, an equation system is set up by using the mass conservation principle for each compartment. The key in the compartmental method is to determine the flow fluxes between interconnecting regions. This is often a very tedious job, particularly when the number of compartments is large, and the compartments are randomly divided and arranged for a ventilated space with a complex geometry. In practice, the application of this method has usually been limited to a few subregions [6, 8, 14]. Too few compartments could amplify the inaccuracy when evaluating flow properties in large enclosures.

Using the elegant matrix theory with the compartmental method, Sandberg [8] derived a set of useful relations between different ventilation parameters, including the purging flow rate. Such a matrix analysis may be called the *deterministic method*, in which all the quantities are algebraically expressed. In chemical engineering, the *stochastic method* has been widely used to analyze mixing within chemical reactors, e.g. Refs [15, 16]. A stochastic method visualizes the fluid in a flow system as being composed of discrete entities. This visualization provides a greater insight into the underlying mechanism than deterministic methods do, thereby facilitating our understanding of the flow characteristics of the system. One of the objectives of this paper is to use stochastic theory to re-examine the compartmental model, and explore some useful concepts that are not included when using the deterministic method.

As the starting point of this work, the concept of the purging flow rate, U_p, and its mathematical derivation are discussed by means of imaginary tracer experiments and the mass conservation principle. It is shown that these expressions are useful for calculating U_p and analyzing ventilation flow systems. The concept of the regional purging flow rate is discussed and proposed for use in ventilation practice. Some useful U_p-related quantities are also described. As with the compartmental method, a Markov chain model is proposed to determine the transfer probabilities between different interior regions, from the air supply inlet to interior regions, and from interior regions to the exhaust outlet. These transfer probabilities can be used to analyze the contributions to various parts of the ventilated space due to supplying and exhausting, and to compute the purging flow rate. An effective method that combines the compartmental method with the numerical method is proposed (the CN method) in order to determine the interchanging flow rates between various compartments (regions). These flow rates are needed to obtain the transfer probabilities in the Markov chain model. The application of the model and the methods developed in this work is then demonstrated.

2. THE PURGING FLOW RATE

The concept of purging flow rate was originally proposed by Zvirin and Shinnar [11] for analyzing two-phase flow systems, and used to distinguish between well-purged and stagnant regions. This concept is defined for the local continuum motion in a flow system, and is an inherent local property to continuous internal flows, including ventilation flows. However, the local purging flow rate, unlike the local mean age of air, is not a passive transportable quantity. There is no transport equation for this quantity.

2.1. *Regional purging flow rate*

The definition of the purging flow rate refers to a local property. It is, therefore, usually called the *local* purging flow rate. The local purging flow rate expresses the fraction of the total flow through the system that passes one location in the system on its way to the outlets. In other words, it represents the net flow rate at which the passive contaminant at this location is flushed towards the exhaust opening. The local purging flow rate is originally defined through a pulse or step tracer experiment [7, 9, 11]. When a pulse tracer is released at an arbitrary location p, the local purging flow rate, U_p, is expressed as

$$U_p = \frac{m_p}{\int_0^\infty C_p(t)\,\mathrm{d}t}. \tag{1}$$

With a step tracer experiment, U_p is defined as

$$U_p = \frac{q_p}{C_p(\infty)}. \tag{2}$$

It is stressed here that the local purging flow rate is an *artificial* concept. Addressing U_p for a point will make this concept ambiguous and even useless, since a flow rate at a *point* is zero. The definition in equation (1) or equation (2) shows that U_p is simply a local property. It varies with local flow behavior. In practice, a *point* in a flow field is often represented by a small *volume* around this point. In both the compartmental and numerical methods, using discrete subregions (compartments or cells) to represent a continuous flow system is the usual means to analyze air flow patterns. With this numerical treatment, the purging flow rate is thus a net flow rate passing through a *region*, instead of a *point*, and all the contaminant contained in this region is purged out from the system at this net flow rate. Therefore, the purging flow rate, computed for a number of regions comprised in a system, is called here *the regional purging flow rate*. With the numerical method, the grid used is often very fine, the computed purging flow rate for each cell used to be called the local purging flow rate. With the compartmental method, by contrast, the compartment used is often rather large. It is then appropriate to term the resulting U_p for a compartment the regional purging flow rate. The regional purging flow rate is related to the volume of the region considered. The dependence on the cell size used in the numerical calculations has been numerically demonstrated in Ref. [13]. The same is true when using the compartmental method, as will be shown below.

So far, the numerical method has mostly been used to determine the purging flow rate, but it is not efficient. In practice, it is often not necessary to know the purging flow rate for such small cells as in numerical calculations. Instead, some specific regions usually need to be emphasized, such as the occupied zone and the breathing zone. The regional purging flow rate, therefore, is of practical importance for characterizing the performance of ventilation flow in a region. A specific region is usually much larger than the cell volume used in numerical computations, so large that the compartmental method can be effectively applied.

The regional purging flow rate depends on both the location and the form (shape and size of the territory) of the region considered in a flow system. This quantity is not an integrated value over a region. It is more of a mean value, since the concentration in equation (1) or equation (2) can be approximately replaced with a mean value for a region. The regional purging flow rate is expected to be larger with a larger region, because the larger the region is, the more can be the amount of contaminant present in this region. A larger net flow rate is therefore needed to purge the contaminant within the region towards the outlet, according to the principle of mass conservation. To experimentally determine U_p within an arbitrary region of a room, Etheridge and Sandberg [9] suggested using the spatial-averaged concentration in equation (1) or equation (2) to estimate U_p. The accuracy of U_p then depends on the uniformity of the concentration within the volume considered. When complete mixing occurs in this volume, the resultant U_p is the regional purging flow rate.

The purging flow rate has been defined through tracer experiments, but it is an inherent property of air flow and not influenced by passive contaminant (tracer). A straightforward description of the regional purging flow rate can resort to the turnover flow rate, which is the total local flow rate passing through the region considered. The turnover flow rate for region p, W_p, includes two parts: the net flow rate at which the air leaving p flows towards the outlet, i.e. U_p; and the remaining flow rate (termed here the *residual turnover flow rate*), R_p, at which the air may recirculate and rejoin p after leaving it. In this way, the regional purging flow rate is then defined as $U_p = W_p - R_p$, and thus $U_p \le W_p$. These will be shown to be useful relations for analyzing the purging flow rate. By means of this definition, the purging flow rate for a flow system, as a whole, is therefore the total supplied or exhausted air flow rate through the inlet or outlet. Distinguishing between U_p and R_p thus opens a way to make the regional purging flow rate measurable with tracer techniques. It should be pointed out that the other quantities used along with the regional purging flow rate in a calculation must also be regional in order to match the regional flow properties.

2.2. *Mathematical derivations of the purging flow rate*

The purging flow rate corresponds to the flow state (steady or unsteady), since it is a property of the air flow itself. The purging flow rate is taken as transient in transient flows. With an unsteady flow, however, each transient flow state can be treated separately as steady at one time. With no loss of generality, the flow field is therefore assumed to be steady in this work. The purging flow rate is thus time independent.

The flow space is divided into n parts. With a step tracer experiment, at time $t = 0$, a passive contaminant

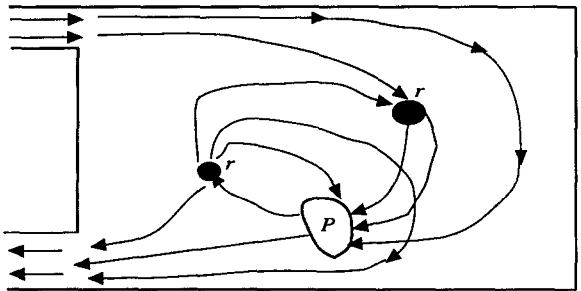

Fig. 1. Illustration of the transport of passive contaminant.

is continuously released at a rate $\dot{m}_r$ at location r $(0 < r \leq n)$, see Fig. 1. The release rate, $\dot{m}_r$, can be constant. To be more general, it can also be a transient rate. In order to reach a steady state, however, $\dot{m}_r$ should satisfy $\mathrm{d}\dot{m}_r/\mathrm{d}t = 0$ as $t \to \infty$. This means that a time-dependent start-up tracer release can be used in experiments, and then the release rate should approach a constant value after a period of time.

The released passive contaminant follows the air flow, undergoing diffusion and convection. A fraction P_{rp} is brought through an arbitrary location p $(p = 1,2,...,n)$, where p is a receiver. The total amount of contaminant ever reaching location p after time t, from all contaminant sources in the system, is then

$$\sum m_p = \sum_r \left(P_{rp} \int_{t_{0r}}^{t} \dot{m}_r \,\mathrm{d}t \right), \tag{3}$$

where P_{rp} is the transfer probability. It reflects the transport ability possessed by the flow, and represents the fraction of passive contaminant transported from the source-bearing location r to an arbitrary location p; t_{0r} is the time when the first fraction of contaminant emerges at location p after its release at r; and r denotes the locations having the contaminant source. Note that the contaminant is passive, and used only for *tracking* the air flow. The transfer probability, P_{rp}, is thus simply a property of the flow.

As $t \to \infty$, according to the mass conservation principle, the total amount of contaminant ever reaching p after its release, Σm_p as $t \to \infty$, must be balanced by an amount of contaminant M_p carried in the net air flow passing through p and flushing out of the system, to make the concentration at p steady. Thus in general

$$M_p = \sum m_p \quad \text{as } t \to \infty. \tag{4}$$

The net flow rate needed to hold the contaminant at location p, and to carry it towards the outlet, is the purging flow rate U_p. Both U_p and P_{rp} are independent of the contaminant-releasing procedure, which has no influence on the air flow. At time t after release, M_p can be written as

$$M_p = U_p \int_{t_0}^{t} C_p(t)\,\mathrm{d}t, \tag{5}$$

where t_0 is the time when the first fraction of contaminant appears at location p, $C_p(t)$ is the mean transient concentration at location p, and $C_p(t) \equiv 0$ for $t \leq t_0$. Substituting equations (3) and (5) into equation (4) yields

$$U_p = \lim_{t \to \infty} \frac{\sum_r \left[P_{rp} \int_{t_{0r}}^{t} \dot{m}_r \,\mathrm{d}t \right]}{\int_{t_0}^{t} C_p(t)\,\mathrm{d}t}. \tag{6}$$

With a pulse release, equation (4) can be used to give an expression for U_p, similar to equation (6):

$$U_p = \frac{\sum_r (P_{rp} m_r)}{\int_0^{\infty} C_p(t)\,\mathrm{d}t}, \tag{7}$$

where m_r is the contaminant amount released at location r by a short burst.

Theoretically, with a continuous release of passive contaminant, equation (6) or equation (7) provides one way to determine U_p experimentally, if the measurement is possible in practice. For a constant release rate for each source, i.e. $\dot{m}_r \equiv q_r$ at any time for all r $(r \leq n)$, equation (6) can be rewritten in terms of a steady mean concentration, $C_p(\infty)$, as

$$U_p = \frac{\sum_r [P_{rp} q_r]}{C_p(\infty)}. \tag{8}$$

It is shown that the mass conservation principle, formulated in equation (4), forms the physical basis for deducing U_p. It is thus possible to embody this artificial quantity in straightforward mathematical expressions. With the aid of tracer experiments, equation (7) and equation (8)/equation (6) express the purging flow rate in general forms. When only one passive contaminant source is active in the system, say at location i, the expression for U_p with equation (7) or equation (8) then takes the same form as described in Refs [7, 8]. In particular, when the single passive contaminant source is located at p, then $P_{pp} \equiv 1$, and equations (7) and (8) turn out to be identical with the definition in equations (1) and (2), respectively. Since U_p is a property of the flow, equation (1) or equation (7) and equation (2) or equation (8) suggest that, for a steady flow field, the relation between the amount of passive contaminant and the resulting mean concentration response is always linear at any location within the system.

2.3. *Expressions for the purging flow rate from two special situations*

Equations (6), (7) and (8) provide basic mathematical expressions of U_p for a procedure from an unsteady state approaching a steady state. For a situation with continuous release, after a steady state is reached, the amount of purged contaminant for an arbitrary location p, M_p, over a time period Δt can be written as

$$M_p = U_p C_p(\infty) \Delta t. \tag{9}$$

With a pulse release, M_p is obtained by accumulating from time $t = 0$ to $t \to \infty$. A pulse release needs to be described with a transient procedure (for concentration). A continuous constant release can often be described with its steady state; this is convenient and efficient for numerical calculations, with no need for solving a time-consuming transient problem. For a pulse release, the counterpart of the following results can be analyzed in a similar way.

Equation (2) gives an expression for the purging flow rate at an arbitrary location, where a passive contaminant is continuously released. Now two other *imaginary* tracer experiments are used to derive expressions that are more accessible both computationally and physically.

(a) *Step-up release at the inlet.* At time $t = 0$, the passive contaminant is released at the inlet at a constant rate q_s. The transfer probability from the inlet to an arbitrary interior location p is P_{sp}. The total contaminant ever reaching p over a time period Δt, at a steady state, is then $\Sigma m_p = P_{sp} q_s \Delta t$. This amount is balanced by M_p in equation (9). This gives

$$U_p = \frac{P_{sp} q_s}{C_p(\infty)}. \qquad (10a)$$

A continuous release at the inlet will eventually give a uniform concentration in the system as $t \to \infty$. This concentration equals the concentration at the inlet, i.e. $C_p(\infty) \equiv q_s/Q$ for all locations. Equation (10a) thus becomes

$$U_p = P_{sp} Q. \qquad (10b)$$

Equation (10b) provides a convenient method for obtaining U_p in terms of the transfer probability from the inlet to an arbitrary interior region, without requiring the transfer probabilities between different interior locations. Note that P_{sp} depends only on the flow pattern, and represents the fraction of fresh air from the inlet contributing to a region p. The purging flow rate, therefore, also represents the net flow rate at which the fresh air is supplied to a location. The purging flow rate for the region around the inlet is thus the supplied air flow rate, Q.

(b) *Overall step-up release within flow system.* The flow space is divided into n regions, and each has a volume, δV_p ($p = 1,2,...,n$). The passive contaminant is released at each region. The release rate, δq_p ($p = 1,2,...,n$), is the contaminant amount released per unit time and unit volume, i.e. with a dimension of [kg/s m³]. When $i \neq j$ ($i, j = 1,2,...,n$), δq_i need not be equal to δq_j. The situation is again analyzed at a steady state. Within an arbitrary region p, the total contaminant amount during a time period Δt includes two parts: the amount released from the source in this region and the total amount during Δt from other sources in the rest of the system. This gives

$$\sum m_p = \sum_{i=1}^{n} (P_{ip} \delta q_i \Delta t \delta V_i)$$
$$= \Delta t \left[\delta q_p \delta V_p + \sum_{i=1(i \neq p)}^{n} (P_{ip} \delta q_i \delta V_i) \right]. \qquad (11)$$

According to the principle of mass conservation, this amount is equal to M_p in equation (9). Consequently

$$U_p = \frac{\delta q_p \delta V_p + \sum_{i=1(i \neq p)}^{n} (P_{ip} \delta q_i \delta V_i)}{C_p(\infty)}. \qquad (12)$$

Note that, as n is large enough, the results obtained from equation (12) are then equivalent to those calculated with numerical methods [18].

When continuous release occurs in only a few regions, equation (12) turns out to be equation (8). Unlike a step-up release at the inlet, the situation considered here is impossible to achieve in practice. Equation (12), however, is theoretically helpful for the analysis of the purging flow rate. If the source distribution is spatially homogeneous, i.e. $\delta q_i \equiv$ constant everywhere, the local mean age of air, τ_i, can then be expressed in terms of δq_i and $C_i(\infty)$, i.e. $\tau_i = C_i(\infty)/\delta q_i$. Introducing this relation into equation (12) gives

$$U_p = \frac{\delta V_p + \sum_{i=1(i \neq p)}^{n} (P_{ip} \delta V_i)}{\tau_p} = \frac{V_{sp}}{\tau_p}, \qquad (13)$$

where V_{sp} is the volume swept by purging flow on its way from the supply opening to region p, see also Ref. [7]. The same equation was derived in Ref. [8] by means of matrix analyses. Equation (13) gives an important relationship between the regional purging flow rate and the mean age of the air passing through an arbitrary region p. This equation also shows that the regional purging flow rate is related to the volume of the region considered, as mentioned above. The regional purging flow rate can be determined by means of equation (13), as well as equations (6), (7), (8) and (10b). The spatial-averaged concentration should be used when using equation (8), equation (10a) or equation (12). Here, the use of transfer probability is recommended to calculate the purging flow rate with equation (10b) or equation (13). This avoids the use of the release rate-dependent concentration, $C_p(\infty)$, which can largely affect the accuracy of U_p, owing to its non-uniformity in the region considered.

Additionally, a relation between the probabilities can be derived by combining equations (13) and (10b), i.e.

$$P_{sp} = \frac{\tau^{n}}{\tau_p} \frac{\delta V_p + \sum_{i=1(i \neq p)}^{n} (P_{ip} \delta V_i)}{V}, \qquad (14)$$

where τ^{n} is the nominal time constant for the flow system, and $\tau^{n} = V/Q$.

3. SOME PARAMETERS RELATED TO PURGING FLOW RATE

When applied to ventilation flows, the purging flow rate indicates the potential capability of the ventilating flow to expel the contaminants at a location out of the system, or the capability to supply fresh air to a region. The purging flow rate can be used to derive some useful parameters, which are applicable for analyzing and assessing ventilation flow systems.

3.1. *Regional air exchange index*, ε_p

The mean age of the air, τ_p, either local or regional, is used to define the local or regional air exchange index:

$$\varepsilon_p = \frac{\tau^{n}}{\tau_p}. \qquad (15)$$

Here, τ_p is the local mean age of the air at point p, or the mean age of the air for region p. From equation (13) or equation (14), a relation can be derived between the regional air exchange index and the regional purging flow rate:

$$\varepsilon_p = \frac{U_p}{Q}\frac{V}{\left(\delta V_p + \sum_{i=1(i\neq p)}^{n}(P_{ip}\delta V_i)\right)} = \frac{P_{sp}V}{\left(\delta V_p + \sum_{i=1(i\neq p)}^{n} P_{ip}\delta V_i\right)}. \quad (16)$$

This equation suggests that any region having a zero (or very small) purging flow rate is stagnant, where $\varepsilon_p = 0$ and the local mean age of the air tends to be infinite (see equation (13)). From equation (16) it is easy to show that $\varepsilon_p \equiv U_p/Q = 1.0$ for complete mixing.

When the system is divided into n equal compartments, i.e. $\delta V_i = V/n$ $(i = 1,2,...,n)$, equation (16) provides

$$\tau_p = \frac{V}{U_p}\frac{\left(1+\sum_{i=1(i\neq p)}^{n} P_{ip}\right)}{n}. \quad (17a)$$

The local mean age of air at point p is then

$$\tau_p = \lim_{n\to\infty}\frac{V\left(1+\sum_{i=1(i\neq p)}^{n} P_{ip}\right)}{nU_p}. \quad (17b)$$

Further, equation (16) can be used to provide a relation for the region into which the air passing through the remaining regions of the system is converged (sunk). The transfer probabilities from all the remaining regions to such a particular region are so large that the right-hand side of equation (16) can be approximated by U_p/Q. This gives

$$U_p \approx \frac{\tau^n Q}{\tau_p}. \quad (18)$$

This equation holds true conditionally. One region for which equation (18) is valid is the region around the outlet, where the purging flow rate is equal to the exhausted air flow rate Q, since the mean air age is equal to τ^n there.

3.2. *Transfer index*, T_{ip}

This concept was introduced by Lidwell [17], see also Refs [7, 8]. It can be written as

$$T_{ip} = \frac{P_{ip}}{U_p}. \quad (19)$$

T_{ip} can be used as an "index of exposure to contaminant" at a location p when a contaminant is released at location i. A smaller T_{ip} means a better capability both to isolate and purge a contaminant. If the probability in equation (19) is related to the inlet, the transfer index from the inlet s to any location is $T_{sp} \equiv 1/Q$.

3.3. *Equivalent regional Peclet number*, α_p

Zvirin and Shinnar [11] originally proposed this concept, which is expressed as

$$\alpha_p = \frac{2V}{\delta V_p}\frac{U_p}{W_p}, \quad (20)$$

where W_p is the total local air flow rate, also called the turnover flow rate, passing through region p. W_p expresses the exchanging ability of a region with its surroundings. The quantity U_p/W_p thus indicates the fraction of purging air flow in the turnover flow rate passing through region p, see also Ref. [18]. α_p can be used as an indication of the uniformity of mixing. It can also be used to represent the segregation between the flow in the region considered and ideal plug flow. Note that α_p is a parameter that depends on the volume of the region considered.

3.4. *Back-mixing index/probability*, β_p

As discussed above, for an arbitrary region p within a flow system, the turnover flow rate W_p is composed of two parts, i.e. the purging flow rate U_p and the residual turnover flow rate R_p. This gives

$$W_p = U_p + R_p. \quad (21)$$

The residual turnover flow rate, R_p, is the remaining net flow rate at which the air leaving region p may return back. The back-mixing index or probability, β_p, is thus defined in terms of R_p and W_p, i.e.

$$\beta_p = \frac{R_p}{W_p} = 1 - \frac{U_p}{W_p}. \quad (22)$$

The back-mixing index indicates the probability of the air rejoining p after leaving it. This index, therefore, reflects the degree of air recirculation for region p. It will be further explored in the next section.

3.5. *Residual time of the air*, τ_{rp}

The probability density function $\psi_p(t)$ for the local remaining (residual) time distribution at an arbitrary point p can be obtained by injecting a tracer (a pulse) at p and measuring over the entire system. Zvirin and Shinnar [11] showed that

$$\psi_p(t) = -\frac{1}{m_p}\frac{\mathrm{d}M(t)}{\mathrm{d}t}, \quad (23)$$

where $M(t)$ is the hold-up of tracer at time t within the system. It can be expressed by

$$M(t) = m_p - Q\int_0^t C_e(t)\,\mathrm{d}t. \quad (24)$$

The probability density function can then be rewritten as

$$\psi_p(t) = \frac{QC_e(t)}{m_p}. \quad (25)$$

Equation (25) is the same as the probability density function for the residence time of the tracer released at point p, see Ref. [7]. This means that the residual time of the air at a point is equal to the residence time of the passive contaminant released at the same point. Equations (23), (24) and (25) can also be used equivalently for a region. Taking the first moment of $\psi_p(t)$, and using equation (1), gives the residual time for the air passing through region p as

$$\tau_{rp} = \frac{Q}{U_p} \frac{\int_0^{\infty} tC_e(t)\,dt}{\int_0^{\infty} C_p(t)\,dt}. \quad (26)$$

For the situation with a continuous step-up release, q_p, it can be shown that the cumulative distribution function for $\psi_p(t)$ is

$$\Psi_p(t) = \frac{QC_e(t)}{q_p}. \quad (27)$$

Note that $\psi_p(t) = \Psi_p'(t)$. The mean residual time for the air passing through region p can then be written as

$$\tau_{rp} = \frac{Q}{U_p} \frac{\int_0^{\infty} [C_e(\infty) - C_e(t)]\,dt}{C_p(\infty)} = \int_0^{\infty} (1 - F(t))\,dt, \quad (28)$$

where $F(t)$ is the cumulative distribution function for the contaminant leaving the system, and $F(t) = C_e(t)/(q_p/Q)$. Equation (28) thus indicates that the mean residual time for the air passing through region p is equal to the turn-over time of the contaminant released at p.

4. A MARKOV CHAIN MODEL FOR TRANSFER PROBABILITIES

The equations derived in Section 2 can be used to compute the purging flow rate. The key is to determine the transfer probabilities between different regions within the system. Equation (10b) is the most convenient method, as only the transfer probability from the inlet to the interior region in question is needed. Sandberg [8] used a deterministic method, where the compartmental model was analyzed with matrix theory. The deterministic method can be used to find the transfer probabilities between different interior regions, by solving a set of algebraic equations built up with the mass conservation principle. The transfer probabilities from the inlet to the interior regions, plus the transfer probabilities from the interior regions to the outlet, are not included. These parameters are important indicators of the contributions of the inlet and outlet to an arbitrary interior region, particularly for multi-inlet/outlet flow systems. A stochastic method is proposed here, where a Markov chain model is developed. This model can be used in combination with the compartmental method to calculate the desired transfer probabilities mentioned above. In addition, the back-mixing probability can be further explored with this model.

4.1. *Transition probability between various regions of the flow field*

Stochastic theory can be found, e.g. in the textbooks by Feller [19], Cinlar [20] and Chiang [21]. It has been used in chemical engineering for many years, applied mainly to analyses of mixing and residence time distributions in chemical reactors, e.g. Refs [15, 16].

The flow is investigated by *tracking* an imaginary fluid element (or a passive particle) within a flow field that is divided into a number of regions. Each region denotes a state of the local flow, and thus also of the particle when it passes there. As the particle passes through the system to the exhaust, it experiences various states. The Markovian process is suitable for analyzing a general continuous ventilation flow system. The flow field is assumed to be steady, and divided into n regions with interconnecting flows. The flow system has one inlet and multiple outlets (the multi-inlet situation will be discussed later). The interior regions are numbered continuously from 1 to n. Let the inlet be denoted by the letter s, and the outlets numbered $n+1, n+2, \ldots, n+e$, where e is the total number of outlets. The inlet and outlets are treated as special regions with zero volume (i.e. $\delta V_s = \delta V_{n+1} = \ldots = \delta V_{n+e} = 0$, and $V = \delta V_1 + \delta V_2 + \ldots + \delta V_n$). Each region (including the inlet and outlets) represents a state of the particle. A state space $\boldsymbol{S}$ is then formed, and

$$\boldsymbol{S} = \{s, 1, 2, \ldots, n, n+1, n+2, \ldots, n+e\}. \quad (29)$$

$\boldsymbol{S}$ consists of two subspaces: the interior states (including the inlet and interior regions) form $\boldsymbol{S}_I$, and the outlet states (recurrent states) form $\boldsymbol{S}_O$, i.e. $\boldsymbol{S} = \boldsymbol{S}_I \cup \boldsymbol{S}_O$ ($\boldsymbol{S}_I \subset \boldsymbol{S}$ and $\boldsymbol{S}_O \subset \boldsymbol{S}$).

When the particle is released at a state p ($p \in \boldsymbol{S}$) in the beginning of the tracking, its initial state is denoted by $X_0 = p$. Let $I(p) = P\{X_0 = p, \exists p \in \boldsymbol{S}\}$. A set for the initial states is then formed:

$$\boldsymbol{I} = \{I(s), I(1), \ldots, I(n+e)\}. \quad (30)$$

For one particle, $\Sigma I(i) \equiv 1$. It is important to understand the contribution of the inflow to different interior regions. The particle is therefore released at the inlet, i.e. $I(s) = P\{X_0 = s\}$. After entering the room, the particle follows the air flow and visits a set of regions before leaving through an outlet. Each visit to a region is counted as a station X_i, where i denotes the number of stations the particle has ever visited since its release at initial state $I(s)$. The station $X_i = k$, when $k \geq n+1$, means that the particle leaves the system. The sequence $\{X_i; i = 0,1,2,\ldots\}$ is thus a discrete space, and a stochastic process. Its trajectories give a complete picture of the particle movement in terms of the flow regions it visits. Since all the possible states in the system are covered by the state space $\boldsymbol{S}$, and the flow pattern is assumed to be steady, the state for the particle's current station X_i is only affected by its state at the last station $X_i - 1$. This is thus a typical Markov process. Its present state alone is therefore all that is needed to forecast its future, see Refs [15, 19]. The statistical sequence of a Markov process is governed entirely by the probabilities of transition from one state to another. In the parlance of statistics, for $i \geq 0$

$$f\{X_i = p | X_0, X_1, \ldots, X_{i-1}\} = f\{X_i = p | X_{i-1}\} \ (\forall p \in \boldsymbol{S}). \quad (31)$$

Furthermore, it is assumed that the particle movement is a Markov chain with stationary transition probabilities. Then for all $i \geq 0$

$$f(j,p) = f\{X_i = p | X_{i-1} = j\} = f\{X_1 = p | X_0 = j\} \ (\forall j, p \in \boldsymbol{S}), \quad (32)$$

where $f(j,p)$ is the transition probability from state j to state p. With all members in the state space, $\boldsymbol{S}$, the transition probabilities between any two states form the entries of a matrix. This is called here the $\boldsymbol{F}$-matrix, which is the matrix for the transition probability

$$\boldsymbol{F}=\begin{bmatrix} f(s,s) & f(s,1) & \dots & f(s,n+e) \\ f(1,s) & f(1,1) & \dots & f(1,n+e) \\ \vdots & \vdots & \dots & \vdots \\ f(n+e,s) & f(n+e,1) & \dots & f(n+e,n+e) \end{bmatrix}. \tag{33}$$

The transition probability, $f(j,p)$, expresses the probability for a particle leaving a state j, and immediately entering another state p. In other words, it indicates the fraction of air flow at state j tending to leave and to be transferred, by one step, into state p ($j,p\in\boldsymbol{S}$). The $\boldsymbol{F}$-matrix can be partitioned into four submatrices, i.e.

$$\boldsymbol{F}=\begin{pmatrix} \boldsymbol{D} & \boldsymbol{H} \\ \boldsymbol{O} & \boldsymbol{E} \end{pmatrix}. \tag{34}$$

Submatrix $\boldsymbol{D}$ is a block with $(n+1)\times(n+1)$ elements that represent the transition probabilities between interior states. Submatrix $\boldsymbol{H}$ is a block with $(n+1)\times e$ elements that represent the transition probabilities from interior states to outlet states. Submatrix $\boldsymbol{O}$ is a zero block with $e\times(n+1)$ elements that represent the transition probabilities from outlet states to interior states. $\boldsymbol{E}$ is a unit matrix with $e\times e$ elements that represent the transition probabilities from outlet states to outlet states.

Special attention must be paid to the diagonal entries of $\boldsymbol{F}$. Without exception, $f(p,p)\equiv 1$ ($\forall p\in\boldsymbol{S}_O$), for matrix $\boldsymbol{E}$; and the probability $f(p,p)$ ($p\in\boldsymbol{S}_I$) is usually 0 for matrix $\boldsymbol{D}$. If there is any bypassing flow which flows back to the same state without experiencing any other states in $\boldsymbol{S}_I$, then $f(p,p)\neq 0$, and is the fraction of the bypassing flow. For ventilation flows, this situation seldom occurs. An additional way to deal with the bypassing flow is to extend the state space by assigning one or more states to the bypassing region(s).

4.2. *Transfer probability between various regions of flow field*

A new matrix, $\boldsymbol{A}$, can be derived from matrix $\boldsymbol{D}$, whose entries, $a(j,p)$, are the mean number of visiting times of the particle to a region p with a last state j ($\forall j,p\in\boldsymbol{S}_I$):

$$\boldsymbol{A}=(\boldsymbol{E}_D-\boldsymbol{D})^{-1}, \tag{35}$$

where $\boldsymbol{E}_D$ is the unit matrix with the same dimension as the $\boldsymbol{D}$-matrix. Let $\boldsymbol{B}$ be another probability matrix with the same dimension as $\boldsymbol{A}$. Its non-diagonal entries $b(j,p)$ ($\forall j,p\in\boldsymbol{S}_I$ and $j\neq p$) express the probability of the particle ever reaching state p when its last state is j. This probability is thus the transfer probability from state j to state p, i.e. $b(j,p)=P_{jp}$ ($\forall j,p\in\boldsymbol{S}_I$ and $j\neq p$). The diagonal elements $b(p,p)$ for all $p\in\boldsymbol{S}_I$, represent the probability that the particle ever returns to p after it leaves. Assume that no upstream diffusion occurs at the inlet (i.e. $b(s,s)\equiv 0$). Then for any states j and p($j,p\in\boldsymbol{S}_I$), we have

$$b(p,p)=1-\frac{1}{a(p,p)}, \tag{36}$$

$$b(j,p)=\frac{a(j,p)}{a(p,p)}\quad (j\neq p). \tag{37}$$

For a ventilation flow, $b(p,p)$ ($\forall p\in\boldsymbol{S}_I$) is the fraction of air ever returning to region p after first leaving this region and before being exhausted through the outlet. It is thus the back-mixing probability, β_p, defined in Section 3, i.e. $\beta_p\equiv b(p,p)$. A larger $b(p,p)$ means a higher degree of recirculation and back-mixing for region p. Equations (35), (36) and (37) provide a method for determining the transfer probabilities through stochastic theory. It is interesting to point out that the present method can be shown to be identical with the result derived from the deterministic method in Ref. [8], when the particle's initial state is at an interior region instead of at the inlet (see Appendix). This means that the result given by the deterministic method is covered in the present model.

The contribution of the outlet to the interior regions is reflected by the transfer probability from an interior region j ($j\in\boldsymbol{S}_I$) to an outlet k ($k\in\boldsymbol{S}_O$), i.e. the probability $b(j,k)$. This can be calculated by means of matrices $\boldsymbol{H}$ and $\boldsymbol{A}$, i.e.

$$\boldsymbol{B}_O=\begin{bmatrix} b(s,n+1) & b(s,n+2) & \dots & b(s,n+e) \\ b(1,n+1) & b(1,n+2) & \dots & b(1,n+e) \\ \vdots & \vdots & \dots & \vdots \\ b(n,n+1) & b(n,n+2) & \dots & b(n,n+e) \end{bmatrix} = \boldsymbol{A}\cdot\boldsymbol{H}. \tag{38}$$

The elements of each row in the $\boldsymbol{B}_O$-matrix indicate the fractions of the air in a region eventually exhausted by the various outlets. The sum of these elements should therefore be unity.

Equations (35), (36), (37) and (38) provide a method to compute the desired transfer probabilities: from the inlet to the interior regions; from one interior region to another; and from an interior region to an outlet. These transfer probabilities can be used either to calculate the regional purging flow rate, or to analyze the effects of the inlet and outlets on the interior region considered. The back-mixing probability furthermore provides a new index to explore the flow behavior in the interior regions of a flow system.

4.3. *Flow systems with multiple inlets*

When air is supplied into a space through multiple inlets, the Markov chain model can be used to calculate the transfer probabilities from each inlet to any interior region. Once a steady flow pattern is set up, the effects of multiple inlets are then completely included. With all the inlet states included in the state space $\boldsymbol{S}$, the initial state for the tracked particle needs to be changed from one inlet to another in order to account for the effect of each inlet. However, the transition probability between any two interior regions remains unchanged. By changing only the transition probabilities in the $\boldsymbol{F}$-matrix from the inlet to the interior regions, the calculation can be carried out in the same way as for a system with only one inlet. With equations (33), (34), (35), (36), (37) and (38), the transfer probabilities between any two states (the inlet, the outlet and the interior region) can be obtained. Note that the transition probability between any two inlets is always zero.

With multiple inlets, all the inlets contribute to the regional purging flow rate. With the aid of equation (10b), the regional purging flow rate for a region p can be determined by

$$U_p = b(s1,p)Q_{s1} + b(s2,p)Q_{s2} + \cdots + b(sm,p)Q_{sm}, \quad (39)$$

where $s1,s2,\ldots,sm$ are used to denote the inlets, and $Q_{s1},Q_{s2},\ldots,Q_{sm}$ are the flow rates supplied through each inlet. Equation (39) shows also the separate contribution of each inlet. A similar evaluation can be made for the outlets. In addition, it should be pointed out that equation (13) can also be used to calculate the regional purging flow rate for systems with multiple inlets, since no transfer probabilities related to inlets are involved in this equation.

5. CALCULATION OF REGIONAL PURGING FLOW RATE

To determine the regional purging flow rate, the transfer probabilities need to be calculated first, whether using equation (10b) or equation (13). It is thus essential to determine the transition probability matrix $\boldsymbol{F}$. This requires information on the interconnecting flows between any two regions. An effective method is to combine the compartmental method with numerical simulations. A numerical simulation contains sufficient details, and can effectively provide the interchanging flow rates with a predicted velocity field. The transition probabilities can consequently be obtained. The proposed Markov chain model can then be used for determining the transfer probabilities, and thus the regional purging flow rate.

The use of the present stochastic method is first demonstrated by applying it to a multi-room building (Case 1). Sandberg [8] has used this building as an example in his analysis with the deterministic method. The interconnecting flow rates have been given in Ref. [8]. In Case 2, the stochastic method is applied to a ventilation flow that is simulated with the numerical method, and the interconnecting flows are analyzed from the predicted velocity field. This is a combination of the compartmental method with the numerical method. For convenience, it is termed here the hybrid CN method, to distinguish it from the combination of the compartmental method and measurement (e.g. in Case 1).

Case 1. The interchamber flow rates and infiltration/exfiltration rates for a three-storey office building are given in Fig. 2 (from Ref. [8]). With the present method, the infiltrations are treated as supply inlets ($s1$, $s2$, $s3$), and the exfiltrations as exhaust outlets (4, 5, 6). The interior regions are numbered 1 (ground floor), 2 (first floor) and 3 (second floor). From the flow rates given in Fig. 2, the transition probabilities in the $\boldsymbol{F}$-matrix are obtained in Table 1, where the transition probability is expressed from a region in the first column to a region in the first row. Note that in Ref. [8] the transfer probability was termed "transition probability". Here, they are used as two different concepts, the latter referring to the terminology used in the statistics.

From Table 1, the submatrices $\boldsymbol{D}$ and $\boldsymbol{H}$ in equation (34) are obtained for each inlet by varying the first row and column in the $\boldsymbol{F}$-matrix (equation (33)). Equations (34), (35), (36), (37) and (38) are then used to calculate

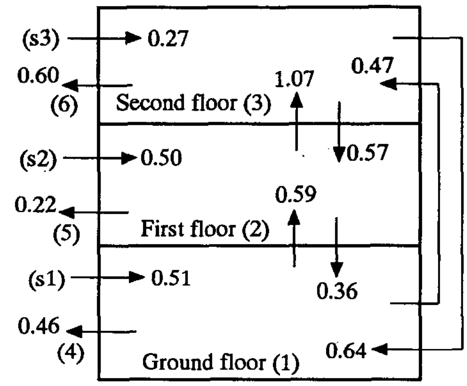

Fig. 2. Interconnecting flow rates (m^3/s) between various regions for an office building.

the transfer probabilities in the $\boldsymbol{B}$-matrix and the $\boldsymbol{B}_O$-matrix, as shown in Table 2, where the transfer probabilities are from a region in the first column to a region in the first row.

In Table 2, the transfer probabilities from the inlets to the interior regions are those in italic in the top left block. Using equation (39) to summarize the contribution of each inlet gives the regional purging flow rates: $U_1 = 3366\,m^3/h$ (ground floor), $U_2 = 3301\,m^3/h$ (first floor), $U_3 = 3506\,m^3/h$ (second floor). They agree well with the results calculated with the deterministic method in Ref. [8]. The back-mixing probabilities for the chambers (1, 2, 3) are expressed in bold numbers. The contributions of the outlets (4, 5, 6) are reflected by the transfer probabilities in italic in the bottom right block.

The transfer probabilities from inlet $s1$ to its unconnected regions ($P_{s12},P_{s13},\ldots,P_{s16}$) equal the transfer probabilities from chamber 1 to these regions ($P_{12},P_{13},\ldots,P_{16}$), respectively. This is because all the air flow supplied through inlet $s1$ first enters chamber 1, and is then delivered into other regions. The same can be found for inlets $s2$ and $s3$.

Case 2. The compartmental method requires only the interchanging flow rates between various regions, and is independent of the orientation of the geometry considered. The analysis here is thus applied to a two-dimensional flow, but it can also be applied to three-dimensional flows.

To demonstrate the use of the hybrid CN method, a two-dimensional mixing room ventilation flow is analyzed. The ventilated room is shown in Fig. 3. The air is supplied through an inlet at ceiling level, and exhausted through two outlets near the floor. The room is divided into six interior regions (numbered from 1 to 6). The inlet is denoted by s, and the two outlets are numbered 7 and 8. The block in region 2 is used to mimic a working platform, with region 3 as the (usual) occupied zone for a worker.

The flow through the room is solved with the standard k–ε model, in conjunction with wall functions to deal with the near-wall viscous effects. Figure 4 shows the numerically predicted ventilation flow pattern.

With the simulated velocity field, the interchanging flow rates between any two neighboring regions within the room can readily be calculated, see Fig. 5. For each region, according to the principle of mass conservation,

Table 1. The F-matrix, transition probabilities between various regions

Region	$s1$	$s2$	$s3$	1	2	3	4	5	6
$s1$	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
$s2$	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
$s3$	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.388	0.309	0.303	0.0	0.0
2	0.0	0.0	0.0	0.218	0.0	0.649	0.0	0.133	0.0
3	0.0	0.0	0.0	0.354	0.315	0.0	0.0	0.0	0.331
4	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

Table 2. The B- and B_O-matrix, transfer probabilities calculated with the Markov chain model

Region	$s1$	$s2$	$s3$	1	2	3	4	5	6
$s1$	—	—	—	*1.0*	*0.545*	*0.613*	0.491	0.131	0.378
$s2$	—	—	—	*0.563*	*1.0*	*0.783*	0.276	0.241	0.483
$s3$	—	—	—	*0.531*	*0.508*	*1.0*	0.261	0.122	0.617
1	0.0	0.0	0.0	**0.383**	0.545	0.613	*0.491*	*0.131*	*0.378*
2	0.0	0.0	0.0	0.563	**0.448**	0.783	*0.276*	*0.241*	*0.483*
3	0.0	0.0	0.0	0.531	0.508	**0.463**	*0.261*	*0.122*	*0.617*

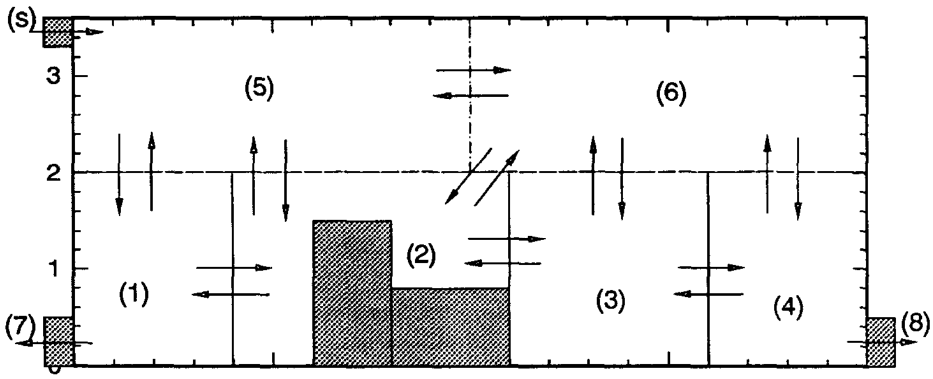

Fig. 3. Configuration of the ventilated room and the divided regions.

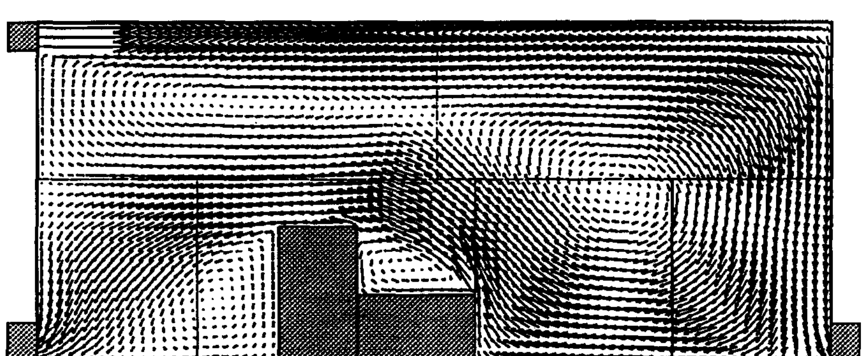
Fig. 4. The ventilation flow pattern simulated by numerical method.

the inflow and outflow should be equal. The slight error in the results is due to numerical inaccuracy in the calculation of velocity field. The error is so small, however, that it will hardly affect the calculations of transition probability. Numerical simulation, indeed, appears very effective, compared to costly and time-consuming measurements, for obtaining the interchanging flow rates between various regions.

With the interchanging flow rates given in Fig. 5, the transition probability between any two regions is then computed. The transition probability, say from region i to region j, is the fraction of turnover flow rate for region i, which leaves i and immediately enters j. All the transition probabilities form the transition probability matrix, i.e. the $\boldsymbol{F}$-matrix, see equations (33) and (34). For the case considered here, the $\boldsymbol{F}$-matrix takes the following form:

$$\boldsymbol{F}=\begin{pmatrix}\boldsymbol{D} & \boldsymbol{H}\\ \boldsymbol{O} & \boldsymbol{E}\end{pmatrix}=\left[\begin{array}{ccccccc|cc} 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0\\ 0 & 0 & 0.128 & 0 & 0 & 0.031 & 0 & 0.841 & 0\\ 0 & 0.624 & 0 & 0 & 0 & 0.201 & 0.175 & 0 & 0\\ 0 & 0 & 0.670 & 0 & 0.009 & 0 & 0.321 & 0 & 0\\ 0 & 0 & 0 & 0.756 & 0 & 0 & 0 & 0 & 0.244\\ 0 & 0.064 & 0.089 & 0 & 0 & 0 & 0.847 & 0 & 0\\ 0 & 0 & 0 & 0.045 & 0.828 & 0.127 & 0 & 0 & 0\\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]. \quad (40)$$

From equation (40), the submatrices $\boldsymbol{D}$ and $\boldsymbol{H}$ are then used in equations (35), (36), (37) and (38) to calculate the transfer probabilities. Using equations (35), (36) and (37) gives the transfer probabilities between the interior regions, and between the inlet and the interior region, which form the $\boldsymbol{B}$-matrix. Using equation (38) produces the transfer probabilities from the interior regions to the outlets, which form the $\boldsymbol{B}_O$-matrix. Merging them into one gives a matrix, $\boldsymbol{B}_T$, as follows:

$$\boldsymbol{B}_T=[\boldsymbol{B}\,\boldsymbol{B}_O]=\left[\begin{array}{ccccccc|cc} \mathbf{0} & 0.65 & 0.68 & 0.67 & 0.86 & 1.00 & 0.89 & 0.63 & 0.37\\ 0 & \mathbf{0.13} & 0.15 & 0.06 & 0.08 & 0.07 & 0.08 & 0.97 & 0.03\\ 0 & 0.86 & \mathbf{0.35} & 0.31 & 0.39 & 0.30 & 0.40 & 0.83 & 0.17\\ 0 & 0.78 & 0.90 & \mathbf{0.45} & 0.58 & 0.31 & 0.60 & 0.75 & 0.25\\ 0 & 0.59 & 0.68 & 0.76 & \mathbf{0.44} & 0.24 & 0.45 & 0.57 & 0.43\\ 0 & 0.65 & 0.68 & 0.67 & 0.86 & \mathbf{0.32} & 0.89 & 0.63 & 0.37\\ 0 & 0.60 & 0.69 & 0.76 & 0.96 & 0.34 & \mathbf{0.51} & 0.58 & 0.42 \end{array}\right] \quad (41)$$

The diagonal elements in submatrix $\boldsymbol{B}$ are the back-mixing probabilities (the bold elements), which indicate the fraction of air ever returning to the same region after leaving. $b(s,s)\equiv 0$ for the inlet, assuming no upstream diffusion. The other elements in the $\boldsymbol{B}$-matrix and the elements in the $\boldsymbol{B}_O$-matrix are the transfer probabilities used to compute the regional purging flow rate, and to analyze the contributions of the inlet and the outlets.

With this example, the working platform (usually also a contaminant source) is located in region 2. Region 3 is assumed to be the occupied zone for the worker. To minimize occupant exposure to the pollutant released from region 2, the transfer probability from 2 to 3, i.e. P_{23}, should be as low as possible. For this case, $P_{23} = 0.31$, which is much less than P_{32} ($P_{32} = 0.90$). The transfer probability from region 2 to region 1, P_{21}, however, is rather large ($P_{21} = 0.86$). This implies that the pollutant in region 2 is removed mostly through region 1, and exhausted from outlet 7. The contribution of each outlet is reflected by the transfer probabilities in the $\boldsymbol{B}_O$-matrix. Because most of the room air is exhausted through outlet 7 (see Fig. 5), the contribution of this outlet to the interior regions is generally larger than that of outlet 8.

By using the transfer probabilities from the inlet to the interior regions, P_{sp}, the regional purging flow rate, U_p, is then calculated by using equation (10b). The result is shown in Table 3, where the regional air exchange index, ε_p, and the regional equivalent Peclet number, α_p, have been calculated by means of equations (16) and (20), respectively.

The inlet is directly connected to region 5. The purging flow rate for this region is therefore equal to the supply flow rate. Most of the fresh air flows into region 3 through regions 5, 6 and 4. The purging flow rates for regions 4 and 6 are thus larger than that for region 3. The U_p values for regions 1, 2 and 3 are similar. The purging flow rate for region 1 is only slightly higher than the exhausted flow rate through outlet 7, which implies that contaminant removal in region 1 depends mostly on outlet 7. This has also been reflected by the transfer probability from region 1 to outlet 8, which is very small ($P_{18} = 0.03$). The parameters ε_p and α_p are also shown to be reasonable for

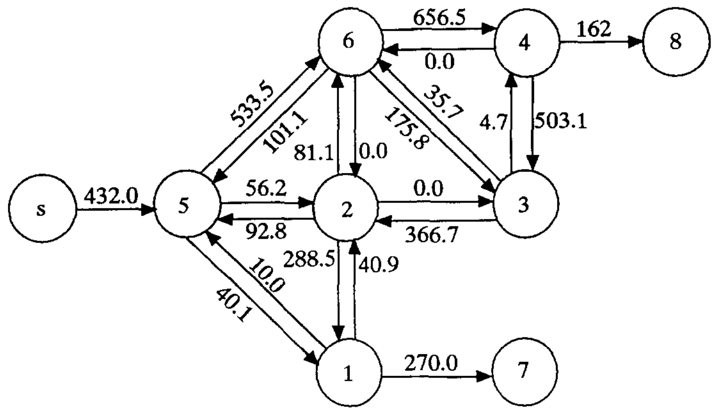

Fig. 5. The interconnecting flow rates between regions (unit: m^3/h).

Table 3. Calculated regional ventilation parameters for Case 2

Parameter	Region 1	Region 2	Region 3	Region 4	Region 5	Region 6
P_{sp}	0.65	0.68	0.67	0.86	1.00	0.89
U_p (m^3/h)	281	294	289	372	432	385
ε_p	0.90	0.98	1.06	1.22	2.25	1.35
α_p	14.4	9.8	7.1	9.3	5.7	4.1

indicating the flow characteristics in the various regions. A high ε_p, in general, corresponds to a high transfer probability from the inlet to the region considered, as well as a high regional purging flow rate, but corresponds to a low α_p, which indicates air motion approaching plug flow. With the wall jet near the ceiling, both regions 5 and 6 have a relatively low α_p.

The analysis of ventilation flows by the present method is able to provide quantitative indication of the flow properties and system performance. The method can be used to improve ventilation system designs. The transfer probability, the back-mixing probability and the purging flow rate can be used to characterize flow behaviors and the potential contaminant-removing capability in various ventilated zones/regions. The hybrid CN method provides convenience for effectively applying the compartmental analysis.

6. CONCLUSIONS

The purging flow rate, U_p, and some U_p-related ventilation indices have been described and discussed. The regional purging flow rate and its use for characterizing ventilation flows are proposed and demonstrated. Based on the principle of mass conservation and with the aid of imaginary tracer experiments, the purging flow rate is reformulated with various mathematical expressions in terms of transfer probability. These expressions are useful for analyzing ventilation flow systems. The U_p-related parameters are relevant ventilation indices for quantifying regional flow properties. These parameters include the regional air exchange index, the transfer index, the equivalent regional Peclet number, the back-mixing index and the residual time of the air.

The emphasis is imposed on the compartmental method (multi-chamber/zone method) for analyzing tracer experiments. When using this method to compute the regional purging flow rate, U_p, the use of the transfer probability is recommended. A convenient method of computing U_p is to use the transfer probability from the inlet to the region considered (e.g. Equation (10b)). This method is particularly effective for multi-inlet flow systems.

Stochastic theory has been applied in combination with the compartmental method. A Markov chain model is proposed for calculating the transfer probability. The model can, as with single-inlet/outlet flow systems, be applied to multi-inlet/outlet flow systems with no difficulties. Moreover, several indices useful for analyzing and assessing ventilation performance can be explored by this model, including the back-mixing probability of the air passing through an arbitrary region; the transfer probabilities from an inlet to an arbitrary region, and from an arbitrary region to an outlet. These indices are absent from the deterministic method.

An effective hybrid CN method is proposed for computing the interconnecting flow rates between two compartments (regions). These flow rates are needed when using the Markov chain model to calculate the transition probability, and thus the transfer probability. This method is a combination of the compartmental method and numerical simulations.

The model and the method, as well as the ventilation indices proposed in this work, can be used to diagnose, improve and optimize ventilation designs. The regional purging flow rate can also be used to assess the ability of a ventilation system to supply fresh air to and remove contaminants from any region within the flow field. Future work will incorporate the present studies into diagnoses and designs of ventilation systems.

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APPENDIX

In this appendix, the stochastic model presented in this paper is shown to be identical with the result derived from the deterministic analysis in Ref. [8], when this model is applied to a special situation. The symbols used in this analysis should also be referred to the $\boldsymbol{Q}$-matrix and its inverse matrix, the $\boldsymbol{Q}^{-1}$-matrix, in Ref. [8]. However, the subscript indices used here are opposite to those in Ref. [8]. Therefore, the first index is the origin, and the second is the destination.

The flow system is divided into n compartments, which are denoted in the same way as in Ref. [8] (and the inlet is thus not included). The passive particle has now an initial state at a region within the system (not at the inlet). Let $X_0 = 1$, the $\boldsymbol{D}$-matrix in equation (34) can then be expressed as

$$\boldsymbol{D} = \boldsymbol{E}_D - \boldsymbol{H}^{-}\boldsymbol{Q}^{\mathrm{T}}, \tag{A1}$$

where $\boldsymbol{Q}^{\mathrm{T}} = [Q_{ij}]_{n\times n}$, and $\boldsymbol{H} = \mathrm{diag}[Q_{11}, Q_{22}, \ldots, Q_{nn}]$.

From equation (A1) and equation (35), the following relation can readily be derived:

$$\boldsymbol{Q}^{-1} = (\boldsymbol{AH}^{-1})^{\mathrm{T}}. \tag{A2}$$

Note that $\beta_p = b(p,p)$ and $Q_{pp} = W_p$. Substituting equation (22) into equation (36) gives

$$a_{jj} = [A]_{jj} = \frac{Q_{jj}}{U_j}. \tag{A3}$$

Introducing equation (A3) into equation (37) yields the non-diagonal elements for the $\boldsymbol{A}$-matrix:

$$a_{ij} = [A]_{ij} = \frac{P_{ij}Q_{jj}}{U_j}. \tag{A4}$$

Note that $P_{ij} = b(i,j)$.

By inserting equation (A3) and equation (A4) into equation (A2), the inverse matrix of $\boldsymbol{Q}$ is obtained:

$$\boldsymbol{Q}^{-1} = \left[\frac{P_{ji}}{U_i}\right]_{n\times n}. \tag{A5}$$

Note that $P_{ji} \equiv 1$ for $i = j$. Equation (A5) is therefore identical to equation (52b) of Ref. [8].