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A Modified Low-Reynolds-Number k-ω Model for Recirculating Flows

A modified form of Wilcox's low-Reynolds-number k-ω model (Wilcox, 1994) is proposed for predicting recirculating flows. The turbulent diffusion for the specific dissipation rate, ω, is modeled with two parts: a second-order diffusion term and a first-order cross-diffusion term. The model constants are re-established. The damping functions are redevised, which reproduce correct near-wall asymptotic behaviors, and retain the mechanism describing transition as in the original model. The new model is applied to channel flow, backward-facing step flow with a large expansion ratio ($H/h = 6$), and recirculating flow in a ventilation enclosure. The predictions are considerably improved.

1 Introduction

Turbulence modeling is one of the most important aspects in numerical simulations of fluid flow and heat transfer. In conjunction with empirical wall functions, the conventional k-ϵ model (Launder and Spalding, 1974) has been widely used in engineering practice, and has turned out a success in many applications. Nevertheless, some problems exist when using the wall-function method (Patel et al., 1984). The lack of universality of the wall functions has been frequently criticized. Extensive research has thus been made to develop near-wall low-Reynolds-number (LRN) corrections.

Most of LRN models have been developed based on the k-ϵ model. A major deficiency with the k-ϵ-based LRN models is the uncertainty of specifying ϵ at the wall. There are other deficiencies with some LRN k-ϵ models (Peng et al., 1996a). The damping functions used in LRN models usually rely on wall-proximity-dependent variables (e.g., $y^+ = u_\tau y/\nu$ and $R_y = k^{1/2}y/\nu$ etc.). This gives rise to complications in numerics when solving for internal flows with non-planar wall geometries. Further, Savill (1995) has indicated that LRN models that use damping functions dependent on the turbulent Reynolds number, R_t, are more appropriate for predicting low-Reynolds-number transitions than those that only introduce a dependence on wall proximity. In addition, for some turbulent recirculating flows of engineering interest, e.g., low-velocity displacement ventilation flows, where laminar and transitional phenomena exist locally not only in near-wall regions but also in regions remote from the walls, using an LRN k-ϵ model often turns out unrealistic laminar solutions (Davidson, 1989).

In recent years, some new LRN two-equation models have been proposed as alternatives to the LRN k-ϵ models, e.g., the LRN k-τ model by Speziale et al. (1992). Wilcox has developed a standard two-equation k-ω model (Wilcox, 1988) and its LRN variant (Wilcox, 1994). The standard k-ω model has been validated for predicting boundary layer and free shear flows (Wilcox, 1988). Combining the standard k-ω model with the k-ϵ model, Menter (1994) developed two new models, and obtained predictions improved for adverse pressure gradient flows. Patel and Yoon (1995) used the standard k-ω model to solve separated flows over rough surfaces, and reported good accuracy. Abid et al. (1995) used the k-ω model in combination with an explicit algebraic stress model for recirculating flows, and obtained results in good agreement with experiments. Larsson (1996) applied the k-ω model to predictions of turbine blade heat transfer, and concluded that the k-ω model performs as well as or better than the k-ϵ model. Some other recent applications with the standard k-ω model can also be found, see e.g., Liu and Zheng (1994) and Huang and Bradshaw (1995). With both the wall-function method and the extended-to-wall method (integrating the model directly towards the wall surface), Peng et al. (1996b) recently applied the standard k-ω model to recirculating flows. It was found that this model overpredicted the reattachment length for the flow behind a backward-facing step with a large expansion ratio ($H/h = 6$).

With the LRN k-ω model, satisfactory results have been reported for simulating transitional boundary layer flows by means of the concept of numerical roughness strip (Wilcox, 1994). One of the attractive features of Wilcox's LRN model is that it uses damping functions that depend only on the turbulent Reynolds number, R_t. It is therefore convenient to apply this model to internal flows with complex geometries. Moreover, the k-ω model possesses a nontrivial laminar solution for ω as $k \to 0$. It can thus be expected to be able to capture flow characteristics for e.g. low-velocity displacement ventilation flows of which an LRN k-ϵ model fails to handle. However, the LRN k-ω model, as its standard form, yields significant inaccuracy in predictions for the flow over a large backward-facing step. Furthermore, this model does not reproduce correct asymptotic behavior for $-\overline{u'v'}$, with $-\overline{u'v'} \propto y^4$ as $y \to 0$. It has been argued that the correct wall-limiting condition for $-\overline{u'v'}$, as well as for ϵ, contributes to the improvement of predictions of by-pass transitions (Savill, 1995).

This paper presents an improved form of the LRN k-ω model. A turbulent cross-diffusion term is added to the modeled ω-equation, in analogy to its viscous counterpart in the exact transport equation. The model constants are re-established. New R_t-dependent damping functions are devised to make the model asymptotically consistent as the wall is approached. In addition, the mechanism for simulating boundary layer transitions is pre-

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served. The modified model is applied to channel flow, a backward-facing step flow with a large expansion ratio, and recirculating flow in a ventilation enclosure. The results are compared with DNS data, experimental data and predictions given by other LRN models. The effects due to the modification are discussed.

2 Development of the Modified *k-ω* Model

In the k-ω model, it is assumed that the turbulence is characterized by a velocity scale, $k^{1/2}$, and a length scale, $k^{1/2}/\omega$. The eddy viscosity is thus formulated as $\nu_t \sim k/\omega$. Wilcox termed ω as the specific dissipation rate of k, which is actually the reciprocal turbulent time scale, $1/\tau$. The transport equations for k and ω, together with the equations for continuity and momentum, form the mathematical description. In Wilcox's LRN k-ω model (1994), the k- and ω-transport equations are written as

$$\frac{\partial(\rho u_j k)}{\partial x_j} = P_k - c_k f_k \rho \omega k + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \quad (1)$$

$$\frac{\partial(\rho u_j \omega)}{\partial x_j} = c_{\omega 1} f_\omega \frac{\omega}{k} P_k - c_{\omega 2}\rho\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] \quad (2)$$

P_k is the production of turbulence energy, and for incompressible flows takes the form

$$P_k = -\rho\overline{u_i' u_j'}\frac{\partial u_i}{\partial x_j} = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} \quad (3)$$

With the Kolmogorov-Prandtl relation, the eddy viscosity, μ_t, is defined by

$$\mu_t = c_\mu f_\mu \frac{\rho k}{\omega} \quad (4)$$

In Wilcox's model, the closure constants are determined as $c_\mu = 1.0$, $c_k = 0.09$, $c_{\omega 1} = 0.56$, $c_{\omega 2} = 0.075$, $\sigma_k = \sigma_\omega = 2.0$, and the damping functions are

$$f_\mu = (0.025 + R_t/6)(1 + R_t/6)^{-1} \quad (5)$$

$$f_k = [0.278 + (R_t/8)^4][1 + (R_t/8)^4]^{-1} \quad (6)$$

$$f_\omega = (0.1 + R_t/2.7)[(1 + R_t/2.7)f_\mu]^{-1} \quad (7)$$

where R_t is the turbulent Reynolds number, and $R_t = k/(\omega\nu)$.

2.1 Near-Wall Asymptotic Behavior. In the vicinity of the wall, the fluctuating velocity components can be written with the Taylor series expansion as

$$u' = a_1 y + a_2 y^2 + \ldots \quad (8)$$

$$v' = b_2 y^2 + \ldots \quad (9)$$

$$w' = c_1 y + c_2 y^2 + \ldots \quad (10)$$

where a_i, b_i, and c_i are functions of x, z, t, and the coordinate y is normal to the wall.

With the aid of equations (8–10) and the definitions of k and ϵ, the asymptotic behavior of wall turbulence can be represented by the relations: $k \propto y^2$, $\epsilon \propto y^0$, $\mu_t \propto y^3$, $-\overline{u'v'} \propto y^3$, $P_k \propto y^3$ and $R_t \propto y^4$ as $y \to 0$. Note that $u^+ = y^+$ has been used near the wall and thus $(\partial u/\partial y) \propto y^0$. The specific dissipation rate ω can be expressed in terms of k and ϵ, i.e. $\omega \sim \epsilon/k$. This gives $\omega \propto y^{-2}$. From Eqs. (3) and (4), $f_\mu \propto y^{-1}$ is thus required to make the near-wall shear stress asymptotically consistent.

Close to the wall, the turbulent transport term is negligible in comparison to the dissipation and the viscous diffusion (Speziale et al., 1992). With the aid of DNS data, Mansour et al. (1989) further pointed out that the pressure diffusion term in the turbulent kinetic-energy budget remains negligibly small compared to the other terms. In the immediate wall proximity, Wilcox (1988) showed that the balance between the dissipation term and the molecular diffusion term holds in the equations for both k and ω, from which the asymptotic solutions for k and ω can be derived

$k \to cy^n$ as $y \to 0$, with

$$n = (1 + \sqrt{1 + 24 c_k f_k / c_{\omega 2}})/2 \quad (11)$$

and

$$\omega \to \frac{6\nu}{c_{\omega 2} y^2} \quad \text{as} \quad y \to 0 \quad (12)$$

From Eq. (12), it is apparent that $c_{\omega 2}$ must satisfy $c_{\omega 2} \propto y^0$, so as to keep the correct asymptotic behavior for ω. In equation (11), the damping function for coefficient c_k, i.e. f_k, should then asymptotically behave as $f_k \propto y^0$. Furthermore, the following relation is needed in order to ensure $k \propto y^2$

Nomenclature

$c_k, c_\omega, c_{\omega 1}, c_{\omega 2}$ = turbulence model constants
f_μ, f_k, f_ω = damping functions of turbulence model
h = height of inlet
H = height of computational domain
k = turbulent kinetic energy
L_t = turbulent length scale
p = pressure
Re = inlet-based Reynolds number, $U_0 h/\nu$
Re_τ = Reynolds number, based on friction velocity, $u_\tau(H/2)/\nu$
R_t = turbulent Reynolds number
T = height of outlet
U_0 = inlet velocity
u_k = Kolmogorov velocity scale
u_m = mean velocity of channel flow
u_i = mean velocity component in direction x_i
u_τ = friction velocity, $(\tau_w/\rho)^{1/2}$
u', v', w' = fluctuating velocities in x, y, z directions, respectively
u^+ = dimensionless velocity, u/u_τ
W = height of backward-facing step
x_i = Cartesian space coordinate
x_r = reattachment length of backward-facing step flow
y^+ = dimensionless distance from wall surface, $u_\tau y/\nu$
ϵ = dissipation rate of k
η_k = Kolmogorov length scale
κ = von Kármán constant
μ = molecular dynamic viscosity
μ_t = turbulent dynamic viscosity
ν = kinematic viscosity, μ/ρ
ν_t = turbulent kinematic viscosity, μ_t/ρ
ρ = density of air
σ_k, σ_ω = model constants
τ = turbulent time scale
τ_w = wall shear stress
ω = specific dissipation rate of k

$$(c_k f_k)/c_{\omega 2} = 1/3 \quad \text{as} \quad y \to 0 \tag{13}$$

Since $D\omega/Dt = (1/k)(D\epsilon/Dt) - (\omega/k)(Dk/Dt)$, the exact ω-equation can readily be obtained from the exact k- and ϵ-equations (Speziale et al., 1992; Peng et al., 1996b)

$$\frac{\partial(\rho u_j \omega)}{\partial x_j} = \left(\frac{P_\epsilon}{k} - \frac{\omega P}{k}\right) + \left(\rho\omega^2 - \frac{\Pi_\epsilon}{k}\right) + \left(\frac{\omega D}{k} - \frac{D_\epsilon}{k}\right) + \left(\mu \frac{\partial^2 \omega}{\partial x_j \partial x_j} + \frac{2\mu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j}\right) \tag{14}$$

where P_ϵ, Π_ϵ and D_ϵ are the production, destruction and diffusive transport terms, respectively, in the exact ϵ-equation, while P and D are the production and diffusive transport terms in the exact k-equation. The exact production term for ω is $P_\omega = (P_\epsilon - \omega P)/k$, and $P_\epsilon \propto y$, $P \propto y^3$ as $y \to 0$. Therefore, $P_\omega \propto y^{-1}$ near the wall. Compared with the modeled production term in the ω-equation, this term will be asymptotically consistent if the damping function $f_\omega \propto y^0$ as $y \to 0$.

2.2 The Modified ω-Equation and Model Constants. In Wilcox's LRN model, the same model equations and constants are adopted as those in the standard k-ω model, but the inconsistent wall-limiting behavior of k is corrected by means of the damping functions. Wilcox's LRN model is capable of satisfactorily yielding the near-wall turbulence level for attached turbulent flows, e.g., channel flows. However, it is found in this study that the reattachment length is significantly overpredicted when this LRN model is applied to a separated flow over a backward-facing step with a large expansion ratio. To improve it, the ω-equation and the model constants need to be modified.

In analogy to its viscous counterpart, the turbulent diffusion term in the exact ω-equation (Eq. (14)) is modeled with two parts: a second-order diffusion term and a first-order cross-diffusion term. In the modeled ω-equation, however, the viscous cross-diffusion term has to be dropped to make the asymptotical solution of ω realizable as the wall is approached (Peng et al., 1996b). Otherwise, a negative ω value arises close to the wall unless $c_{\omega 2}$ is damped. Keeping the viscous cross-diffusion term, Speziale et al. (1992) showed that the near-wall asymptotical solution of ω will behave correctly if $c_{\omega 2}$ takes the form of $c_{\omega 2} = (\text{constant} \cdot f_2 - 1)$, where f_2 is a damping function and $f_2 \propto y^2$ as $y \to 0$. This decomposition, however, will destroy the mechanism preserved in the model for transition simulation according to Wilcox (1994). With the LRN k-ω model, the closure constants have been established to ensure that the production of k is amplified earlier than that of ω. This is an essential condition for the k-ω model to account for transition from laminar to turbulence. This condition consequently requires that a relation, as given by Eq. (17*a*) in Section 2.3, must be held for the coefficients of the production and dissipation terms in the k- and ω-equation. It is obvious that the above decomposition of $c_{\omega 2}$ does not satisfy this relation. Instead of damping $c_{\omega 2}$, we feel that it is more practical to preserve the mechanism for transition simulation in the modified model as well as in Wilcox's model by dropping the viscous cross-diffusion term. The modified ω-equation then takes the following form

$$\frac{\partial(\rho u_j \omega)}{\partial x_j} = c_{\omega 1} f_\omega \frac{\omega}{k} P_k - c_{\omega 2}\rho\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + c_\omega \frac{\mu_t}{k}\left(\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}\right) \tag{15}$$

Compared to the original ω-equation in Wilcox's model (Eq. (2)), an additional turbulent cross-diffusion term is included in Eq. (15). Note that the inclusion of this term makes the present model somewhat similar to the k-ϵ model, since a similar turbulent cross-diffusion term can be reproduced by transforming the modeled ϵ-equation into an ω-equation.

In order to eliminate the sensitivity of the k-ω model to the freestream value of ω when solving free shear flows, Menter (1992) has proposed an ω-equation similar to equation (15). A so-called "blending function" is used to determine c_ω. As a result, the turbulent cross-diffusion term disappears close to the wall surface, while being switched on in regions away from the solid boundaries. This approach becomes particularly functional when approaching the shear layer edge of a free shear flow. Wilcox (1993) has also given a simple and straightforward proposal that is nearly identical to Menter's blending function. When applying either Menter's or Wilcox's approach to internal recirculating flows, however, the role played by the cross-diffusion term in the ω-equation is negligibly small, since the gradients for both k and ω often tend to vanish in regions away from walls. These approaches then possess a behavior similar to Wilcox's standard model for internal recirculating flows. In contrast to both Menter's and Wilcox's approaches, the present modification, initiated from the exact ω-equation, makes the cross-diffusion term generalized in both near-wall and far-wall regions. Close to the wall, the gradients of k and ω are often of opposite sign. The turbulent cross-diffusion term as a whole reduces ω and thus raises k near the wall, as desired.

With the ϵ-equation derived from the original k-ω model (i.e., from Eqs. (1) and (2)), Wilcox (1993) has shown that an extra cross-diffusion term appears in the resultant ϵ-equation. This term, similar to the so-called "Yap correction", helps to suppress the rate of increase of the near-wall turbulent length scale, which is often overpredicted by the k-ϵ model for wall-bounded flows in the presence of adverse pressure gradient. When using the modified ω-equation (Eq. (15)), it is easy to show that a similar cross-diffusion term retains in the resultant ϵ-equation, only if $c_\omega < (1/\sigma_k + 1/\sigma_\omega)$. The effect of the turbulent cross-diffusion term on the turbulent length scale will be further discussed in Section 3, by investigating its distribution near the wall and its effects on the turbulent length scale near the reattachment zone for a separated flow.

It is desirable, for an LRN model, to adopt the same model constants as used in its high-Re-number form (the parent model). The closure constants for the high-Re-number form of the present LRN model were evaluated and discussed in a previous work (Peng et al., 1996b). For fully developed turbulence, the model constants were re-established as

$$c_\mu = 1.0, \quad c_k = 0.09, \quad c_{\omega 1} = 0.42, \quad c_{\omega 2} = 0.075,$$
$$c_\omega = 0.75, \quad \sigma_k = 0.8, \quad \sigma_\omega = 1.35 \tag{16}$$

2.3 Damping Functions. In order not to violate the mechanism for describing transitions contained in the k and ω equations, the following relations are required (Wilcox, 1994)

$$c_{\omega 1} f_\omega c_\mu f_\mu < c_{\omega 2} \quad \text{as} \quad R_t \to 0 \tag{17a}$$

$$(c_k f_k)/(c_\mu f_\mu) \to 1 \quad \text{as} \quad R_t \to 0 \tag{17b}$$

Equations (17*a*) and (17*b*), together with Eq. (13), form the lower bound when determining the model constants for viscous modification as the wall is approached ($y \to 0$ or $R_t \to 0$). Equation (16) gives the upper bound of the model constants for fully developed turbulence as $R_t \to \infty$. The coefficient $c_{\omega 1}$, as $R_t \to \infty$, is re-evaluated in the predictions of the reattachment length for backward-facing step flow. For local-equilibrium wall-turbulent flows, this coefficient must also satisfy

$$c_{\omega 1} = \frac{c_{\omega 2}}{c_k} - \frac{\kappa^2}{\sigma_\omega \sqrt{c_k c_\mu}} \tag{18}$$

To make numerical implementation convenient for internal flows with complex geometries, the turbulent Reynolds number,

R_t, is used as the only dependent variable in the damping functions.

In view of the above discussion, the following model functions are proposed

$$f_\mu = 0.025 + \left\{1 - \exp\left[-\left(\frac{R_t}{10}\right)^{3/4}\right]\right\} \times \left\{0.975 + \frac{0.001}{R_t}\exp\left[-\left(\frac{R_t}{200}\right)^2\right]\right\} \quad (19)$$

$$f_k = 1 - 0.722\exp\left[-\left(\frac{R_t}{10}\right)^4\right] \quad (20)$$

$$f_\omega = 1 + 4.3\exp\left[-\left(\frac{R_t}{1.5}\right)^{1/2}\right] \quad (21)$$

As the wall is approached, applying the Taylor series expansion to Eq. (19) gives $f_\mu \sim (1/R_t^{0.25} + \ldots)$, which complies with the correct asymptotic condition $f_\mu \propto y^{-1}$. Furthermore, using Eq. (4) gives

$$\mu_t \sim (k/\omega)R_t^{-1/4} \sim L_t u_k \quad (22)$$

where $L_t \sim k^{1/2}/\omega$ is the turbulent length scale, and u_k is the Kolmogorov velocity scale, $u_k \sim (\nu k\omega)^{1/4}$. Equation (22) thus suggests that the near-wall eddy viscosity is determined by the small-scale eddies. The turbulent length scale, L_t, is proportional to y^3 as the wall is approached, and thus decreases towards the wall surface. As L_t approaches the Kolmogorov length scale, $\eta_k \sim (\nu^3/k\omega)^{1/4}$, the eddy viscosity is reduced to the same order as the molecular viscosity. This is consistent with the analysis of Kolmogorov behavior in near-wall turbulence by Shih and Lumley (1993). With the present proposal, the turbulent effect is therefore suppressed by damping the turbulent velocity scale as the wall is approached.

With the damping functions in Eqs. (19–21), the asymptotic behavior in the modified model is consistently satisfied for near-wall turbulence, and the mechanism of simulating transitions, contained in the k- and ω-equations, is preserved.

3 Application of the Model

In this section, the present model is first applied to a fully developed channel flow, and the results are compared with DNS data. Two internal recirculating flows are then solved: the separated flow over a backward-facing step with a large expansion ratio, and the flow in a confined ventilation enclosure, see Figs. 1(*a*) and 1(*b*). The predictions are compared with experimental data. The effects of the modification are discussed.

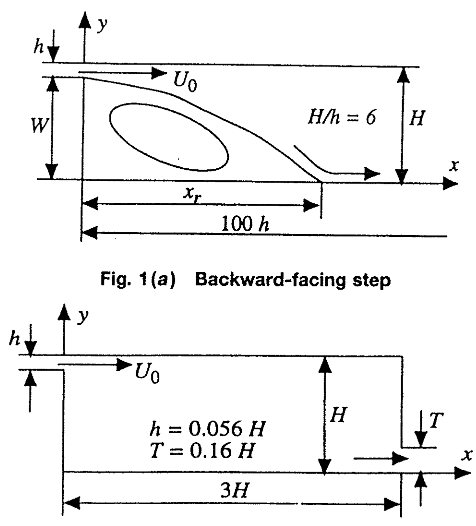

Fig. 1(*a*) Backward-facing step

Fig. 1(*b*) Confined ventilation enclosure

Fig. 1 Configurations for the computed recirculating flows

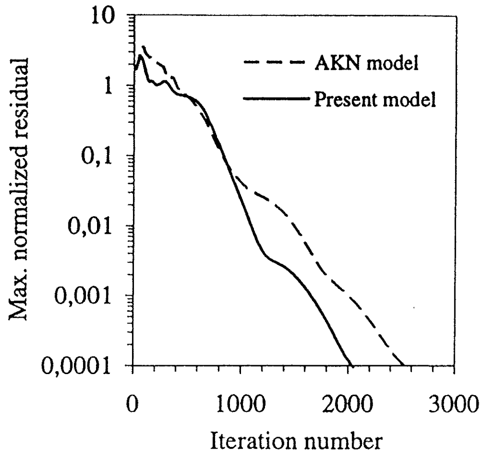

Fig. 2 Typical convergence procedure when solving for the flow in the ventilation enclosure

3.1 Numerical Procedure. The numerical procedure can affect the results of turbulence models. Attention was thus paid to the numerics so as to make the model appraisal meaningful. The calculation was performed with a finite-volume-based computer program (Davidson and Farhanieh, 1992). The third-order QUICK scheme (Leonard, 1979) was used for the convection terms in the momentum equations, and the hybrid upwind/central differencing scheme (Patankar, 1980) was employed in the turbulence-transport equations to ensure a stable solution procedure. The other terms were discretized with the second-order central difference. A collocated grid was used with the aid of Rhie-Chow interpolation (Rhie and Chow, 1983) to avoid non-physical oscillation. The SIMPLEC algorithm was used to handle the pressure-velocity coupling. When solving the discrete algebraic equation for ω, the turbulent cross-diffusion term was added to the right-hand side if it was positive, otherwise to the left-hand side to increase the diagonal dominance for the resulting coefficient matrix.

The iterative solution process was considered to be converged when the sum of absolute cell residuals for each equation, normalized by the respective inlet fluxes, was less than 10^{-4}. When solving the ventilation flow in Fig. 1(*b*) with the present model and the LRN k-ϵ model by Abe et al. (1994; AKN model), respectively, a typical convergence procedure is shown in Fig. 2, where the normalized residual is plotted versus the iteration number. A faster convergence is shown with the k-ω model. This has also been pointed out earlier by Peng et al. (1996b).

The boundary condition at the inlet was prescribed for all the variables. At the outlet, the streamwise derivatives of the flow variables were assumed to be zero. For the flow in the two-dimensional ventilation enclosure, the velocity component normal to the outlet was specified from global mass conservation. At the wall, $u = v = 0$ and $k = 0$. Equation (12) was used at the near-wall first grid point as the boundary condition of ω.

Adopting the asymptotical solution of ω as the boundary condition requires a very fine grid close to the wall. Extensive tests were made to establish the grid mesh. The grid was nonuniformly distributed, and controlled by the ratio of two successive cells, i.e. $\lambda = \Delta_{i+1}/\Delta_i$. The grid dependence was investigated by varying both λ and the number of mesh points. Various grids were tested for each case considered in this study. The relative difference, R_ϕ, between the solutions yielded with different grids was checked. It is required to be sufficiently small to

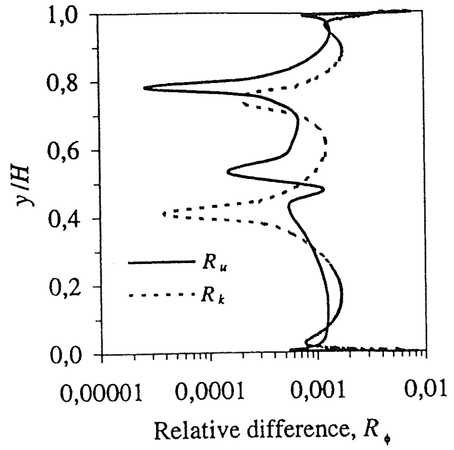

Fig. 3 Evaluation of the relative difference of solutions with two grids (102 × 132 and 204 × 264) for the flow in the ventilation enclosure (at $x = 2H$)

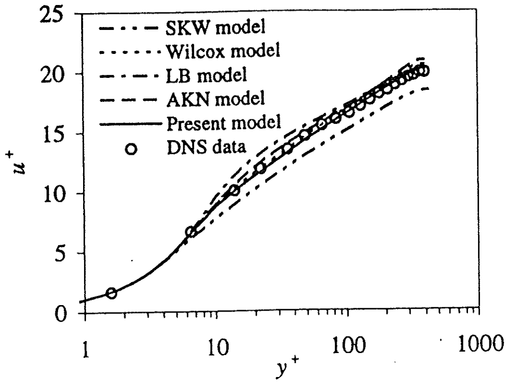

Fig. 4(a) Mean velocity

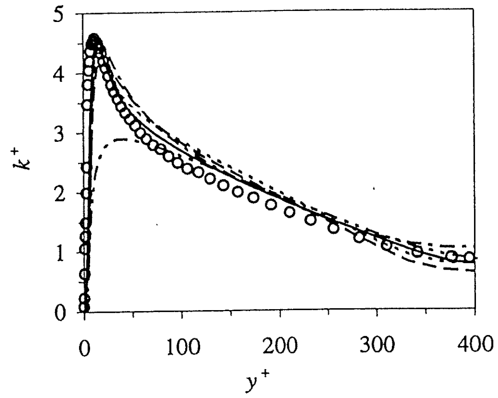

Fig. 4(b) Turbulent kinetic energy

Fig. 4 Predictions for turbulent channel flow at $Re_\tau = 395$ (Re = 13750)

minimize the dependence of the solution on the grid used. For the flow in the confined ventilation enclosure, the relative differences for u and k at $x = 2H$, i.e., R_u and R_k, respectively, are shown in Fig. 3 as an example to demonstrate the grid dependence. The solutions used for estimating R_u and R_k in Fig. 3 were obtained with two grids of 102 × 132 and 204 × 264. The solution hardly varies with refining grid when the grid used has already been sufficiently fine. Using $\lambda = 1.05$ for a channel flow, Yang and Shih (1993) showed that the solutions were almost identical as the cross-stream mesh points varied from 30 to 150. For the channel flow considered here, 100 cross-stream grid points were used. It was found that the results became rather insensitive to the number of the mesh points as long as there were two or more points arranged in $y^+ < 1$. This has also been observed by Yang and Shih (1993). A 202 × 86 mesh was used when solving the backward-facing step flow, and 102 × 132 grid points were used to calculate the recirculating flow in the ventilation enclosure. In general, two or more points were located within the range of $y^+ < 1$. For all the flows considered here, numerical tests disclosed that nearly grid-independent solutions were reached.

3.2 Fully Developed Channel Flow. To assess the modified model, the channel flow at $Re_\tau = 395$ was first solved. The results were compared with both the DNS data (Kim, 1990) and the predictions obtained with other LRN models, including the Lam-Bremhorst k-ϵ model (LB model; Lam and Bremhorst, 1981), the Abe-Kondoh-Nagano k-ϵ model (AKN model; Abe et al., 1994) and Wilcox's LRN k-ω model (Wilcox model; Wilcox, 1994). Wilcox's standard k-ω model (SKW model; Wilcox, 1988) was also included in the comparison, since this model can be used by integrating it directly to the wall surface without using the wall function as a bridge (termed here the *extended-to-wall* method).

Figure 4 shows the distributions of mean velocity and turbulent kinetic energy. The near-wall maximum (peak) values for k^+, as well as the friction velocity u_τ, are compared in Table 1. The present model shows reasonable agreement with the DNS data. Both Wilcox's and the present LRN models predict satisfactory profiles for the mean velocity and the turbulent kinetic energy, particularly in the near-wall region. The present model shows acceptable abilities for accommodating the near-wall low-Reynolds-number effect for attached turbulent flows.

When using Wilcox's standard k-ω model (the SKW model) with the extended-to-wall method, the result gives rise to the largest error, compared to the DNS data. This model considerably underpredicts the near-wall peak value of k. The SKW model also overpredicts the wall shear stress (and thus u_τ, see Table 1). The u^+-profile deteriorates in the far-wall region because u_τ is overpredicted, though the non-normalized mean velocity predicted there is actually in reasonable agreement with the DNS data. The dissatisfactory predictions for near-wall turbulence imply that the SKW model cannot satisfactorily account for near-wall viscous effects. It is therefore inappropriate to integrate this model directly towards the wall surface. However, when the extended-to-wall method was applied to internal recirculating flows with the *standard* form of the present model, i.e. with the damping functions set to unity, reasonable predictions were obtained, particularly for the mean flow profiles (Peng et al., 1996b). By using the SKW model with the extended-to-wall method, satisfactory simulations have been reported in other engineering applications, e.g., by Wilcox (1988), Patel and Yoon (1995), Menter (1994), and Liu and Zheng (1994).

The addition of the turbulent cross-diffusion term enhances the diffusion of ω. To make the diffusion of k compatible, it is thus necessary to set $\sigma_k < \sigma_\omega$, see Eq. (16). This also improves the near-centerline predictions, where the dissipations of both k and ω are mainly balanced by their diffusions, with the production terms negligible.

3.3 Backward-Facing Step Flow. The separated flow over a high backward-facing step ($H/h = 6$, see Fig. 1(a) is particularly relevant to ventilation flows (Restivo, 1979), which

Table 1 u_τ **and near-wall maximum (peak)** k^+**-value for channel flow at** $Re_\tau = 395$

Model	AKN	LB	Wilcox	SKW	Present	DNS
u_τ/u_m	0.0557	0.0560	0.0562	0.0621	0.0574	0.0571
k^+	4.21	4.51	4.52	2.90	4.48	4.58

Table 2 **Comparison of the predicted reattachment length, x_r**

Measured data	AKN model	Wilcox model	Present model
$6.12W$	$6.60W$	$8.24W$	$6.40W$

are usually induced by a wall-jet below the ceiling to create recirculating and mixing air motions. It has therefore been widely used in validations of turbulent models for predicting recirculating ventilation flows (Restivo, 1979; Skovgaard, 1991). With the experimental data available (Restivo, 1979), the flow at Re = 5050 is calculated here.

The reattachment lengths computed with different LRN models are compared in Table 2. Wilcox's LRN model considerably overpredicts the reattachment length by nearly thirty-five percent. By contrast, the modified model is capable of yielding an x_r whose error is less than five percent. By assuming the damping functions to be unity in the modified model, i.e. by using its standard form, a satisfactory result was also predicted with either the wall-function method or the extended-to-wall method (Peng et al., 1996b).

The reattachment length, x_r, is very sensitive to the coefficient $c_{\omega 1}$. It was found that x_r changes by about 1.4 times the step height, with a variation of 0.1 for $c_{\omega 1}$. The turbulent cross-diffusion term alters x_r by about ten percent. This suggests that the modified constant $c_{\omega 1}$, together with the damping function f_ω, plays a main role in raising k by suppressing the production of ω to improve the prediction of x_r.

Figure 5 shows the distributions of the mean streamwise velocity and the turbulent kinetic energy at five positions (x/h = 5, 10, 15, 20, 30). The largest error in the prediction of reattachment length corresponds to the largest inaccuracy in the prediction of velocity with the original model, which unrealistically keeps predicting a too-high velocity peak in the wall-jet along the upper wall. All the models predict reasonable and similar velocity distributions in the far-wall recirculating region. Near the step and close to the lower wall (at x/h = 5 and 10), the AKN model under-estimates the mean velocity. The AKN model also predicts a slightly higher velocity peak in the wall-

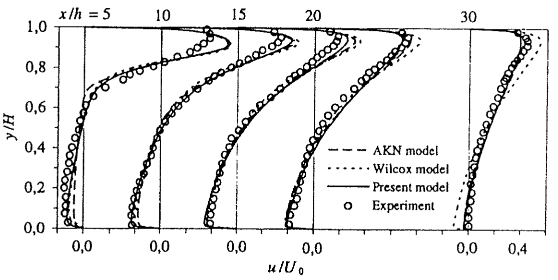

Fig. 5(*a*) Mean streamwise velocity

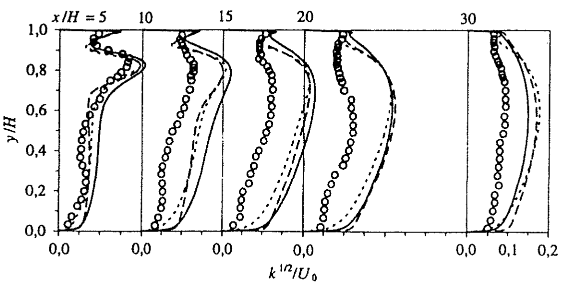

Fig. 5(*b*) Turbulent kinetic energy

Fig. 5 Distributions calculated for the flow over a backward-facing step (the experimental data used in the comparison of k are for $\sqrt{u'^2}/U_0$)

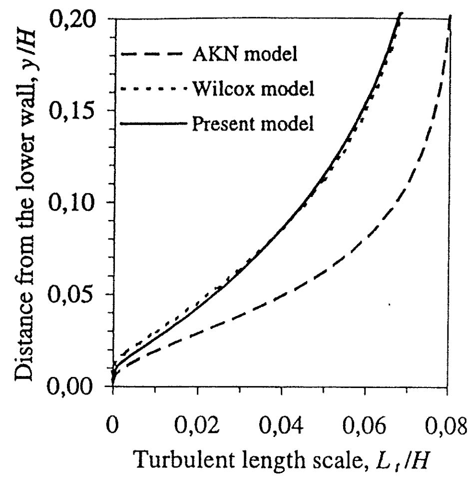

Fig. 6 Predicted turbulent length scales near the lower wall of the backward-facing step at $x = 30h$

jet than the present model does. As the flow approaches the reattachment position (near $x/h = 30$), the Wilcox model fails to capture the mean flow property.

Note that the measured data for $\sqrt{u'^2}$ have been used for comparison of k in Fig. 5. The turbulent kinetic energy computed by all three models is higher than the measured turbulence level. With the Launder-Sharma LRN k-ϵ model, similar predictions have been reported by Skovgaard (1991). For other backward-facing step flows, the predicted maximum value of k has usually been in a range of $(0.02 \sim 0.04)U_0^2$ near the reattachment position, see e.g., Abe et al. (1994). The same can be found in Fig. 5 from the predicted distributions of k near the reattachment zone ($x/h = 30$). In contrast to other backward-facing step flows, however, the measured $\sqrt{u'^2}$ data for the flow situation considered here are much smaller. In a recent calculation (not included here) the present model was applied to the backward-facing step flow with a lower expansion ratio of $H/h = 1.2$. In comparison with DNS data (Le et al., 1993), it was found that the present model improved significantly the prediction over Wilcox's original model, particularly for the turbulence kinetic energy.

To further explore the influence of the turbulent cross-diffusion term, the turbulent length scales, L_t, predicted with various models near the reattachment position ($x = 30h$), are compared in Fig. 6. L_t has been evaluated by means of relations: $L_t = C_\mu f_\mu k^{1/2}/\omega$ for the k-ω model and $L_t = C_\mu f_\mu k^{3/2}/\epsilon$ for the k-ϵ model. The present model yields a turbulent length scale very close to that given by Wilcox's model, and the AKN model predicts the largest L_t in the region near the lower wall. It is known that Wilcox's model performs well for boundary layers with adverse pressure gradient (Wilcox, 1994), because this model possesses the same effect as of the Yap correction used often in the k-ϵ model. The Yap correction was originally invented to suppress the turbulent length scale in the reattachment zone after a sudden pipe expansion (Yap, 1987). The result in Fig. 6 suggests that the addition of the turbulent cross-diffusion term does not change the behavior retaining in Wilcox's model for predicting the turbulent length scale near the reattachment zone for separated flows.

3.4 Recirculating Flow in a Ventilation Enclosure. The flow through a two-dimensional ventilation enclosure (see Fig. 1(*b*)) is similar to the recirculating flow behind a backward-facing step, but with a nearly closed end opposite the inlet. The recirculation in the enclosure depends on the depth reached by the wall-jet, and thus relies on the inlet-based Reynolds number. The present calculations were carried out for the flow at Re = 5000, where experimental data are available.

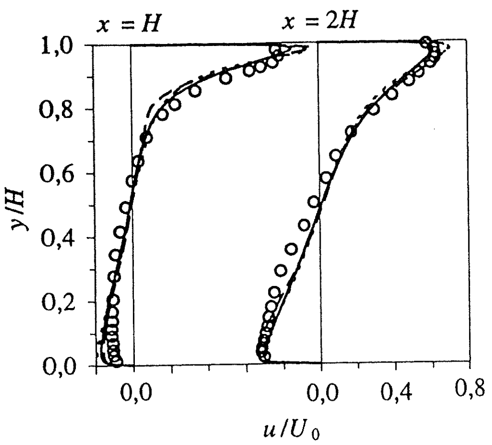

Fig. 7(*a*) Mean streamwise velocity (at $x = H$ and $x = 2H$)

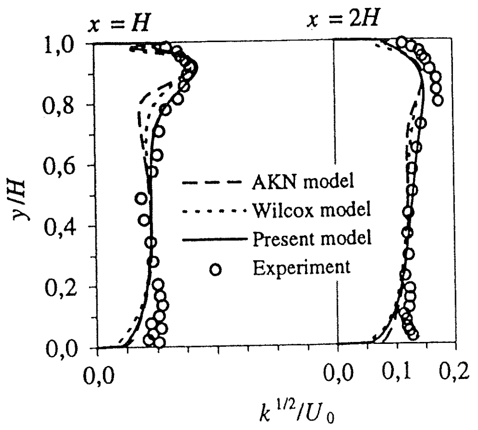

Fig. 7(*b*) Turbulent kinetic energy (at $x = H$ and $x = 2H$)

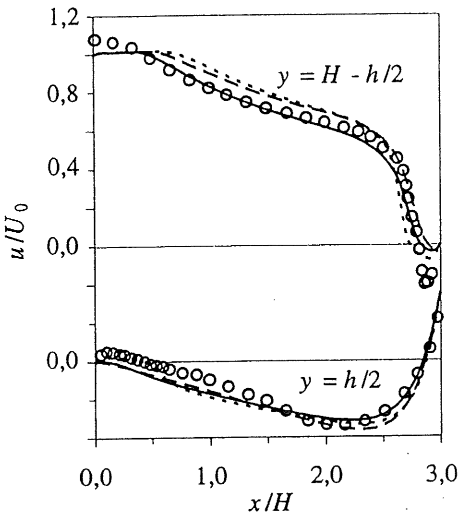

Fig. 7(*c*) Mean streamwise velocity (at $y = h/2$ and $y = H - h/2$)

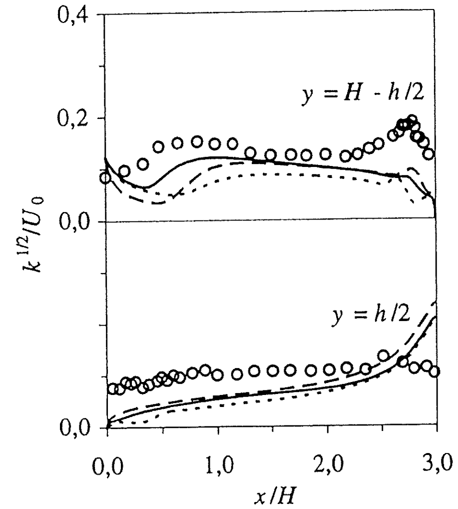

Fig. 7(*d*) Turbulent kinetic energy (at $y = h/2$ and $y = H - h/2$)

Fig. 7 Distributions calculated for the flow in the ventilation enclosure (the experimental data used in the comparison of k are for $\sqrt{u'^2}/U^0$)

In Fig. 7, the calculated distributions are compared with the experimental data of Restivo (1979). In general, the present model gives improved predictions for both the mean streamwise velocity and the turbulent kinetic energy. Wilcox's model and the AKN model overpredict the velocity peak, and under-estimate the near-wall turbulence level in the wall-jet. Near the floor, there is no large variation in the results (see the distributions at $x = H$ and $x = 2H$). At the edge of the wall-jet, however, the turbulent kinetic energy, which is underpredicted by both the AKN model and Wilcox's model, is enhanced by the present model. The enhancement of the turbulent kinetic energy is partly due to the addition of the turbulent cross-diffusion term. This term increases the velocity scale in the near-wall region by reducing the near-wall specific dissipation rate when the gradients of k and ω are of opposite sign, which often holds in regions close to the wall. The constant $c_{\omega 1}$ also makes contributions to the increment of k by reducing the generation of ω, since this constant is decreased comparing to that in Wilcox's model.

On the one hand, it is desired to enhance the near-wall turbulent kinetic energy that is under-estimated by the original LRN model. On the other hand, as the principal aspect for any LRN models, the turbulence level close to the wall needs to be suppressed so as to accommodate viscous effects. This means that an *equilibrium* between the two contrary requirements, damping the turbulent effect and enhancing the turbulent kinetic energy in the near-wall region, must be properly achieved. The present model seems to achieve this equilibrium, and is capable of yielding improved predictions when applied to the recirculating flow considered.

To further investigate the influence of the turbulent cross-diffusion term, we devised an additional group of damping functions and model constants when the turbulent cross-diffusion term was dropped in the ω-equation. These closure functions and constants were capable of yielding reasonable predictions for both channel and backward-facing step flows. However, it was found that the velocities predicted in the wall-jet and near the floor were unsatisfactory (not shown here). Indeed, the model preserves the wall-jet better with the turbulent cross-diffusion term than without it. In the wall-jet, with increasing wall distance, the turbulent kinetic energy k increases in the immediate proximity of the ceiling, then decreases close to the maximum velocity where $\partial u/\partial y \rightarrow 0$, before it increases again in the outer shear layer (see Figs. 7(*a*) and 7(*b*)). The gradient of k thus changes sign in the wall-jet. Accordingly, the contribution of turbulent-cross diffusion can be either positive or negative across the wall-jet, depending on the change in ω. The largest contribution, usually, occurs in the immediate proximity of the wall, where the gradients for both k and ω are rather large and of opposite sign. Figure 8 shows that the addition of the cross-diffusion term in the modified model, together with the decreased coefficient $c_{\omega 1}$, in general, reduces ω in the wall-jet. Compared to Wilcox's model, the modified model thus gives a higher eddy viscosity there. This, in turn, enhances the turbulent diffusion for the momentum, and predicts a lower and wider velocity peak in the wall-jet (Fig. 7(*a*)), as desired.

A more detailed investigation was conducted by analyzing the budget of the ω-equation to clarify the contribution of each term. Figure 9 illustrates the budget of the ω-equation in both Wilcox's and the modified models in the wall-jet at sections $x = 2H$ and $y = H - h/2$, respectively. The modification makes each term redistribution. The near-wall change in the turbulent cross-diffusion term across the wall-jet is consistent with the above analysis. Figure 9(*a*) shows that the contribution of this term is limited mainly in the inner region of the wall-jet where the large gradients exist for both k and ω. Compared with Wilcox's model, all the terms in the modified model have been reduced near the wall surface, particularly the generation and destruction terms.

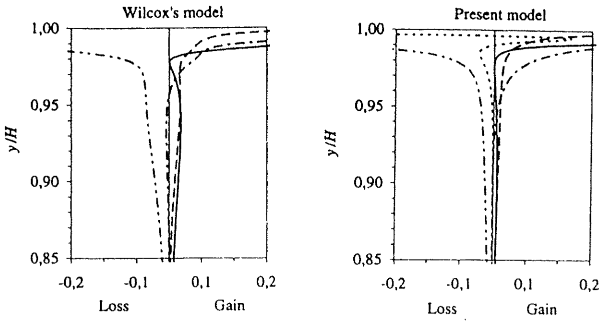

Fig. 9(*a*) Terms in the budget of ω-equation at $x = 2H$

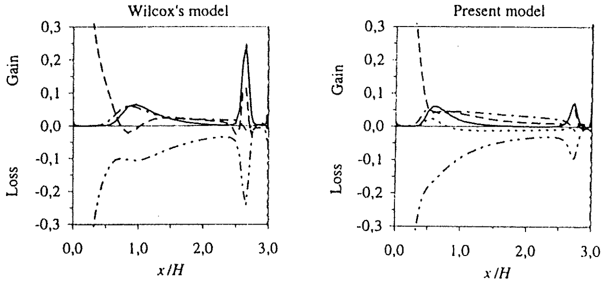

Fig. 9(*b*) Terms in the budget of ω-equation at $y = H - h/2$

Fig. 9 Comparison of the near-wall budget of the ω-equation in Wilcox's model and present model. — — — convection, ——— production, — . . . — destruction, — . — . diffusion, ... turbulent cross-diffusion

4 Conclusions

A modified form of Wilcox's two-equation LRN k-ω model is proposed for predicting internal recirculating flows. The modifications include adding a turbulent cross-diffusion term in the ω-equation, and re-establishing the closure constants and damping functions. The modified model reproduces correct near-wall asymptotic behaviors, and leads to improvements in the prediction of recirculating flows.

The turbulent cross-diffusion term in the ω-equation plays a role in the near-wall region. By altering the specific dissipation rate close to the wall, this term affects the prediction of the near-wall turbulence level, and thus of the eddy viscosity and momentum. The role played by this term depends on the sign of the gradients for both k and ω, and is usually limited in the region close to the wall. In general, this term reduces ω and enhances k near the wall, since the gradients there are often of opposite sign. The addition of this term helps to improve the model performance for the wall-jet inducing recirculation in an enclosure, where the original model underpredicts the near-wall turbulent kinetic energy.

In addition, it is well known that the k-ϵ model returns too high turbulent viscosity due to over-estimated turbulent length scale for flows with adverse pressure gradient (Launder, 1992), whereas Wilcox's standard k-ω model performs well (Wilcox, 1988). The present model has shown a reasonable ability to simulate the separated recirculating flows considered. The addition of the turbulent cross-diffusion term shows negligible influences on the turbulent length scale near the reattachment zone. On the other hand, this term is able to reduce the model's sensitivity to the freestream value of ω when solving free shear flows. Nonetheless, it is necessary to further validate this model

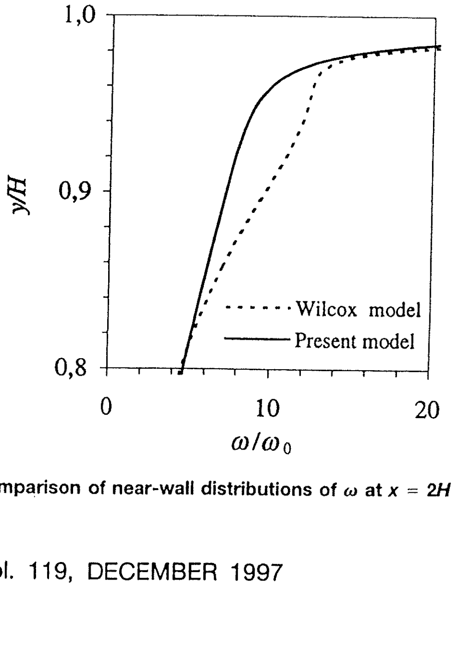

Fig. 8 Comparison of near-wall distributions of ω at $x = 2H$ ($\omega_0 = U_0/H$)

in predictions of adverse-pressure-gradient boundary layer flows in future work.

The calculation of channel flow shows that the new damping functions are able to properly reflect the near-wall low-Reynolds-number effect for attached turbulent flows. For the separated and recirculating flows considered, the damping functions, together with the other modifications, show a satisfactory ability to account for an equilibrium between damping the near-wall turbulence and enhancing the near-wall turbulent kinetic energy underpredicted by Wilcox's original model. The present model appears to be an effective LRN two-equation closure for predicting recirculating flows.

With the new damping functions, moreover, the mechanism describing transitions is preserved in the modified model. It can therefore be used for simulating transitions in boundary layers, as with Wilcox's LRN model.

In future work, this model will be further applied to 3D low-velocity and buoyancy-influenced recirculating flows of engineering interest, such as ventilation flows, where the existing LRN k-ϵ models fail to produce reasonable predictions. The model also needs to be validated for transition simulations of boundary layer flows.

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References

Abe, K., Kondoh, T., and Nagano, Y., 1994, "A New Turbulence Model for Predicting Fluid Flow and Heat Transfer in Separating and Reattaching Flows—I: Flow Field Calculations," *International Journal of Heat and Mass Transfer*, Vol. 37, pp. 139–151.

Abid, R., Rumsey, C., and Gatski, T., 1995, "Prediction of Non-equilibrium Turbulent Flows with Explicit Algebraic Stress Models," *AIAA Journal*, Vol. 33, pp. 2026–2031.

Davidson, L., 1989, "Ventilation by Displacement in a Three-Dimensional Room: A Numerical Study," *Building and Environment*, Vol. 24, pp. 263–272.

Davidson, L., and Farhanieh, B., 1992, "CALC-BFC: A Finite-Volume Code Employing Collocated Variable Arrangement and Cartesian Velocity Components for Computation of Fluid Flow and Heat Transfer in Complex Three-Dimensional Geometries,'' Report 92/4, Dept. of Thermo and Fluid Dynamics, Chalmers University of Technology, Gothenburg.

Huang, P. G., and Bradshaw, P., 1995, ''Law of the Wall for Turbulent Flows in Pressure Gradients,'' *AIAA Journal,* Vol. 33, pp. 624–632.

Kim, J., 1990, ''The Collaborative Testing of Turbulence Models (Organized by Bradshaw et al.),'' Data Disk No. 4.

Lam, C. K. G., and Bremhorst, K., 1981, ''A Modified Form of the k-ϵ Model for Predicting Wall Turbulence,'' ASME JOURNAL OF FLUIDS ENGINEERING, Vol. 103, pp. 456–460.

Larsson, J., 1996, ''Numerical Simulation of Turbine Blade Heat Transfer,'' Lic. Thesis, Report 96/4, Dept. of Thermo and Fluid Dynamics, Chalmers University of Technology, Gothenburg.

Launder, B. E., and Spalding, D. B., 1974, ''The Numerical Computation of Turbulent Flows,'' *Computer Methods in Applied Mechanics and Engineering,* Vol. 3, pp. 269–289.

Launder, B. E., 1992, ''On the Modeling of Turbulent Industrial Flows,'' *Proceedings of Computational Methods in Applied Sciences,* Ch. Hirsch et al., eds., Elsevier, pp. 91–102.

Le, H., Moin, P., and Kim, J., 1993, ''Direct Numerical Simulation of Turbulent Flow Over a Backward-Facing Step,'' *Proceedings of 9th Symposium on Turbulent Shear Flows,* Kyoto, Japan, 13.2.1–13.2.6.

Leonard, B. P., 1979, ''A Stable and Accurate Convective Modeling Procedure Based on Quadratic Upstream Interpolation,'' *Computer Methods in Applied Mechanics and Engineering,* Vol. 19, pp. 59–98.

Liu, F., and Zheng, X., 1994, ''Staggered Finite Volume Scheme for Solving Cascade Flow with a k-ω Turbulence Model,'' *AIAA Journal,* Vol. 32, pp. 1589–1596.

Mansour, N. N., Kim, J., and Moin, P., 1989, ''Near-Wall k-ϵ Turbulence Modeling,'' *AIAA Journal,* Vol. 27, pp. 1068–1073.

Menter, F. R., 1992, ''Improved Two-Equation k-ω Turbulence Models for Aerodynamic Flows,'' NASA TM-103975.

Menter, F. R., 1994, ''Two-Equation Eddy Viscosity Turbulence Models for Engineering Applications,'' *AIAA Journal,* Vol. 32, pp. 1598–1604.

Patankar, S. V., 1980, *Numerical Heat Transfer and Fluid Flow,* McGraw-Hill, Washington.

Patel, V. C., Rodi, W., and Scheuerer, G., 1984, ''Turbulence Models for Near-Wall and Low Reynolds Number Flows: A Review,'' *AIAA Journal,* Vol. 23, pp. 1308–1319.

Patel, V. C., and Yoon, J. Y., 1995, ''Application of Turbulence Models to Separated Flow Over Rough Surfaces,'' ASME JOURNAL OF FLUIDS ENGINEERING, Vol. 117, pp. 234–241.

Peng, S-H., Davidson, L., and Holmberg, S., 1996a, ''A Modified Low-Reynolds-Number k-ω Model for Internal Flows in Ventilated Enclosures,'' Report 96/23, Dept. of Thermo and Fluid Dynamics, Chalmers University of Technology, Gothenburg.

Peng, S-H., Davidson, L., and Holmberg, S., 1996b, ''The Two-Equation Turbulence k-ω Model Applied to Recirculating Ventilation Flows,'' Report 96/13, Dept. of Thermo and Fluid Dynamics, Chalmers University of Technology, Gothenburg.

Restivo, A., 1979, ''Turbulent Flow in Ventilation Rooms,'' Ph.D. Thesis, University of London.

Rhie, C. M., and Chow, W. L., 1983, ''Numerical Study of the Turbulent Flow Past an Airfoil With Trailing Edge Separation,'' *AIAA Journal,* Vol. 21, pp. 1525–1532.

Savill, A. M., 1995, ''One-Point Closures Applied to Transition,'' in ERCOFTAC Series Vol. 2 *Turbulence and Transition Modeling,* pp. 233–268, M. Hallbäck et al., eds., Kluwer Academic Publishers.

Shih, T. H., and Lumley, J. L., 1993, ''Kolmogorov Behavior of Near-Wall Turbulence and Its Application in Turbulence Modelling,'' *International Journal of Computational Fluid Dynamics,* Vol. 1, pp. 43–56.

Skovgaard, M., 1991, ''Turbulent Flow in Rooms Ventilated by the Mixing Principle: Comparisons between Computational Fluid Dynamics and Full-Scale Experiments,'' Ph.D. thesis, Department of Building Technology and Structural Engineering, Aalborg University, Aalborg.

Speziale, C. G., Abid, R., and Anderson, E. C., 1992, ''Critical Evaluation of Two-Equation Models for Near-Wall Turbulence,'' *AIAA Journal,* Vol. 30, pp. 324–331.

Wilcox, D. C., 1988, ''Reassessment of the Scale-Determining Equation for Advanced Turbulence Models,'' *AIAA Journal,* Vol. 26, pp. 1299–1310.

Wilcox, D. C., 1993, Turbulence Modeling for CFD, DCW Industries, Inc. CA.

Wilcox, D. C., 1994, ''Simulation of Transition with a Two-Equation Turbulence Model,'' *AIAA Journal,* Vol. 32, pp. 247–255.

Yang, Z., and Shih, T. H., 1993, ''New Time Scale Based k-ϵ Model for Near-Wall Turbulence,'' *AIAA Journal,* Vol. 31, pp. 1192–1198.

Yap, C., 1987, ''Turbulent Heat and Momentum Transfer in Recirculating and Impinging Flows,'' Ph.D. thesis, Faculty of Technology, University of Manchester.