CALCULATION OF SOME PARABOLIC AND ELLIPTIC FLOWS USING A NEW ONE-EQUATION TURBULENCE MODEL

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ABSTRACT

A new one-equation turbulence model has been developed and tested. The turbulent viscosity is obtained as the product of the squareroot of the turbulent kinetic energy and the turbulent length scale. The turbulent length scale is obtained from an equation, which is based on an analogy of von Karman's equation, and involves the turbulent kinetic energy and the normal distance from the (nearest) wall. The model is tested in three parabolic flows - the fully developed flow in a plane channel, the flat-plate boundary layer and the plane wall jet in stagnant surroundings. Four elliptical flows - in three two-dimensional rooms (two isothermally and one buoyantly ventilated) and one three-dimensional isothermally ventilated room - have also served as test cases for the model; the agreement with experimental data is shown to be satisfying. The CPU time was reduced by up to 60 percent compared with the k-ϵ model.

1. THE TURBULENCE MODEL

More details on this work is to be found in Davidson [1].

Several proposals for an algebraic formulation of a turbulent length scale have been presented; for a review see Rodi [2]. The approach followed by Bobyleva et al. [3] is based on an analogy of von Karman's formula

$$L_t = -\kappa \frac{\partial U/\partial y}{\partial^2 U/\partial y^2} \qquad (1)$$

where L_t is the turbulent length scale, y the co-ordinate normal to the wall, U the mean velocity parallel to the wall,

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and κ the von Karman constant. A variable Ψ with the same dimension as the velocity gradient $\partial U/\partial y$ is formed as

$$\Psi = k^{1/2}/L_t$$

(k denotes the turbulent kinetic energy) which, in analogy with Eq. (1), yields

$$L_t = -\kappa \frac{\Psi}{\partial \Psi/\partial y}$$

This equation can be rewritten as

$$\Psi^{-2} \frac{\partial \Psi}{\partial y} = -\kappa k^{-1/2}$$

which can be integrated so that

$$L_t = \kappa k^{1/2} \int k^{-1/2} dy + f(x) \qquad (2)$$

where $f(x)$ is an integration function. Since L_t is a turbulent length scale the integral in Eq. (2) can not be taken from the wall through the viscous sublayer. The lower limit of the integral is taken to be in the inertial sublayer, approximately $30 < y^+ < 100$, where $L_t = \kappa y$ is the appropriate boundary condition for all x, so that

$$L_t = \kappa k^{1/2}(y) \int_{y_p}^{y} k^{-1/2}(\upsilon) d\upsilon + \kappa y_p \qquad (3)$$

It may be noted that Eq. (3) is easily extended so as to be valid in the viscous sublayer as well; this is done by replacing κy_p with the well-known van Driest's formula $\kappa y_p [1-\exp(-y^+/26)]$.

It was found that the length scale equation in Eq.(3) is not appropriate for parabolic near wall flows. Three modifications of Eq. (3) were tested in [ ] and one of these is used in the present work; this modified length scale equation has the form

$$L_t = \min \left(\kappa k^{1/2}(y) \int_{y_p}^{y} k^{-1/2}(\upsilon) d\upsilon + \kappa y_p, 0.09\delta\right) \qquad (4)$$

(herafter denoted by KL1). δ is the width of the layer in the case of the flat-plate boundary layer and the plane wall jet; in the case of the plane channel it denotes the half-width of the channel. The usual eddy-viscosity relation

$$\nu_t = C'_\mu k^{1/2} L_t \qquad (5)$$

is used ($C'_\mu = 0.5477$). The standard k-equation together with Eqs. (4) and (5) form the one-equation turbulence model. The dissipation term in the k-equation is calculated as $C_D k^{3/2}/L_t$ (C_D=0.1643).

This model has two advantages:

i) It is physically sound; the turbulent length scale is calculated from turbulent quantities, and

ii) when y is in the vertical direction the buoyancy effect is automatically accounted for through the buoyancy term in the k-equation.

In calculations of elliptic flows (two- or three-dimensional) there are two or three co-ordinate directions which may be relevant when calculating L_t in Eq. (4). $L_{t,i}$ is formulated so that

$$L_{t,i} = \min\left(\kappa\, k^{1/2} \int_{n_{i,p}}^{n_i} k^{-1/2} dn_i + \kappa n_{i,p}, 0.09\Delta_i\right) \qquad (6)$$

where n_i is the normal co-ordinate from wall 'i', Δ_i the length of the room in n_i-direction, and subscript p denotes a point within the inertial sublayer. Four (two-dimensional calculations) or six (three-dimensional calculations) different $L_{t,i}$ can be calculated and L_t is taken as the minimum of these, i.e.

$$L_t = \min(L_{t,i}),\ i = -1,\ 1,\ -2,\ 2\ (,-3,\ 3) \qquad (7)$$

It suffices, actually, to calculate $L_{t,i}$ for the two (three) nearest walls.

2. SOLUTION PROCEDURE

The PHOENICS computer program developed by Spalding and his group [4], [5], has been used for the parabolic calculations. A computer program by Davidson and Hedberg [6] which is a derivative of TEACH-T, developed by Gosman and his group [7], has been used for the two- and three-dimensional elliptical calculations. These programs solve equations of the type

$$\frac{\partial}{\partial \tau}(\rho\phi) + \frac{\partial}{\partial x_i}\left(\rho U_i \phi - \Gamma_\phi \frac{\partial \phi}{\partial x_i}\right) = S_\phi \qquad (8)$$

by expressing them in finite difference form. ϕ denotes the variable solved for, S_ϕ is its source, and Γ_ϕ its exchange coefficient. The finite difference equations are solved by a procedure which is based on the SIMPLE procedure introduced by Caretto et al [8]. The four main features are: staggered grids for the velocities; formulation of the difference equations in implicit, conservative form, using hybrid upwind/central differencing; rewriting of the continuity equation into an equation for the pressure correction; and iterative solution of the equations.

In the present calculations the dependent variable in Eq. (8) takes the following forms: U, V, W, t, k, ϵ and 1 (continuity equation). The parabolic calculations were carried out using the method described by Patankar and Spalding [9], available as an option in PHOENICS. The standard k-ϵ model was used [2]. In the calculation where the temperature was involved, the Boussinesque approximation was used for the gravity source in the W-equation and the standard buoyancy source was used in the k-equation.

Standard wall functions [2] were employed for treating the flow adjacent to the walls. Constant profiles were set at the inlet for all cases except for the flat-plate boundary layer calculations, where profiles, according to experimental data, were set at an x-station, where the flow was considered to be fully turbulent. The exit velocity was calculated from mass balance and zero streamwise gradient was imposed on the remaining variables.

3. RESULTS

3.1. Parabolic calculations

In Fig. 1 the predicted profiles of the mean velocity and the turbulent viscosity for the plane channel are presented. The Reynolds number based on the bulk velocity and the half-width of the channel was 32000. The mean velocity is well predicted for both turbulence models. The predicted turbulent viscosity is also well in agreement with experimental data for y's up to 0.4δ; for larger y's the magnitude of ν_t is unimportant since the velocity gradient is very small, and, consequently, so also the shear stress ($=-\nu_t \partial U/\partial y$).

The predicted mean velocity and the turbulent shear stress profiles for the flat-plate boundary layer are shown in Fig. 2. The mean velocity profile is well in agreement with experimental data, whereas the turbulent shear stress is predicted slightly too high nearer the free stream for both turbulence models. The predicted displacement thickness is well in agreement with experimental data (discrepancies less than 3% for both models [1]) by Wieghardt and Tillman [12].

The predicted profiles of the mean velocity and the turbulent shear stress for the wall jet are shown in Fig. 3. The Reynolds number based on the height of the inlet and the inlet velocity was 18 000. The predicted profiles are in close agreement with experimental data for both models. The spreading rate of the wall jet, however, is better predicted with the KL1 model than with the k-ϵ model; $d\delta_{1/2}/dx$ ($\delta_{1/2}$ denotes the y-value where the velocity is half of the maximum velocity across the layer) was 0.094 and 0.084 for the k-ϵ model and the KL1 model, respectively. Ljuboja and Rodi [14] obtained 0.106 and Malin [15] 0.094 in their calculations using the standard k-ϵ model.

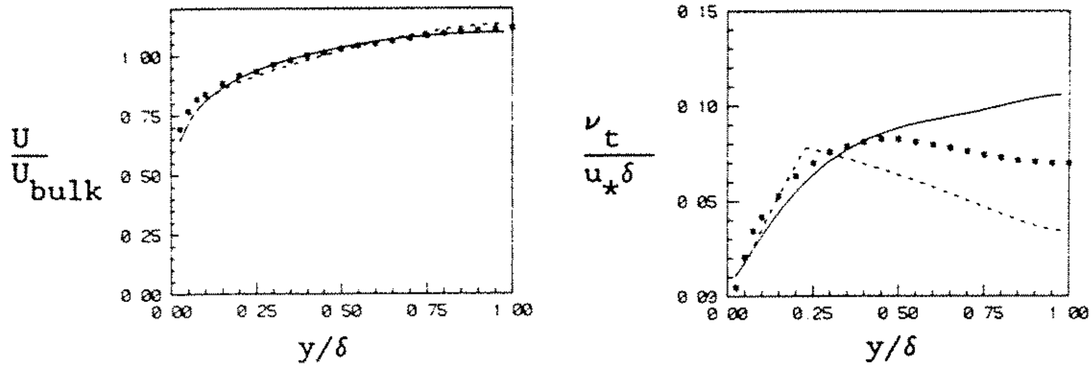


Figure 1. Plane channel. —— k-ϵ. - - - KL1. * Expts. [10].

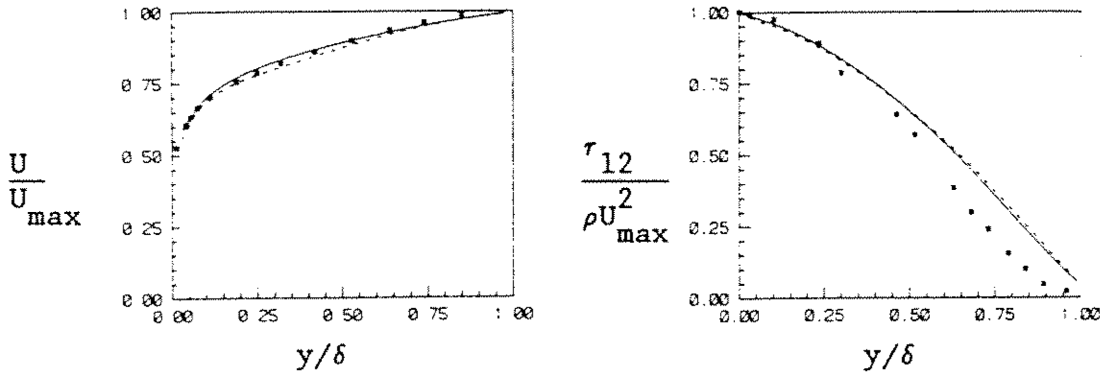


Figure 2. Flat-plate. —— k-ϵ. - - - KL1. * Expts. [11].

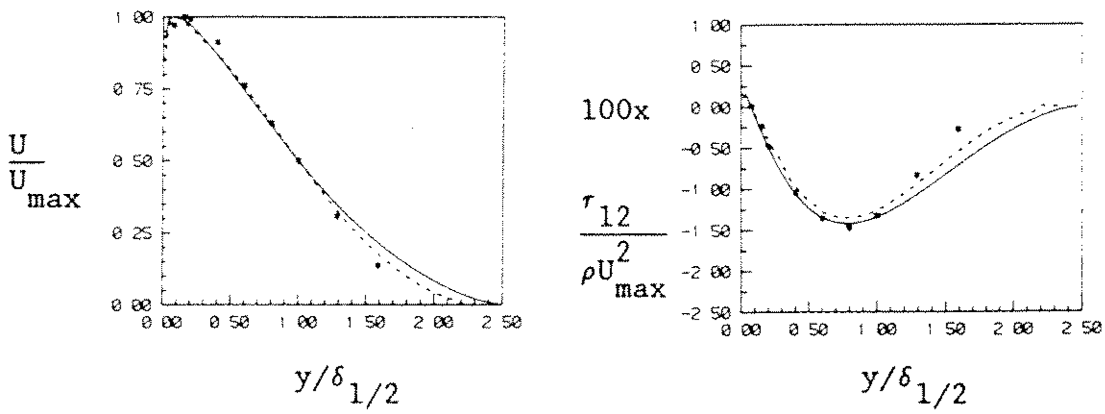


Figure 3. Wall jet. —— k-ϵ. - - - KL1. * Expts. [13].

These values should be compared with the experimental values $d\delta_{1/2}/dx$=0.071-0.075 [16].

50 nodes in the y-direction and forward steps of the order of one tenth of the layer were used for the flat-plate boundary layer and the wall jet; the corresponding figures for the plane channel were 30 and 0.3, respectively. The required CPU time for the parabolic calculations was reduced by approximately 15% when the KL1 model was used compared with when the k-ϵ model was used.

3.2. Elliptical calculations

The flow in three two-dimensional - two isothermally (denoted by Case Å and R2D) and one buoyantly (Case H), ventilated rooms, Fig. 4 - and one three-dimensional isothermally ventilated room (Case R3D), Fig. 5, have been calculated. In all cases except Case Å the inlet is situated

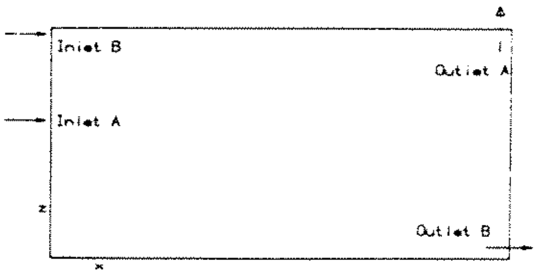


Figure 4. Flow configurations. A: Case Å; B: cases R2D and H.

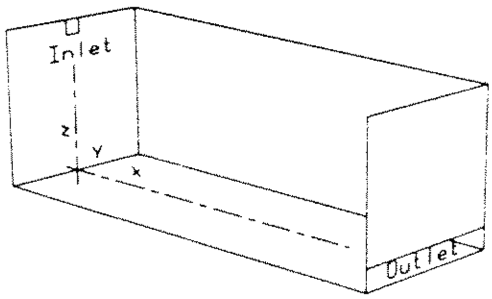


Figure 5. Flow configuration. Case R3D.

adjacent to the ceiling while in Case Å the inlet is displaced from the ceiling ($z_{in}/H=0.6$). Geometrical data and the Reynolds number based on the inlet velocity and the height of the inlet are presented in Table 1.

Grid refinements carried out for the two-dimensional calculations showed that the results were practically grid independent; no grid refinements were carried out for Case R3D since the grid was considered to be fine enough. Size of grids, CPU-time and number of iterations required to obtain converged solutions are summerized in Table 2. All calculations were performed on a VAX-750 machine.

a) Case R2D: Profiles of the U-velocity are shown in Fig. 6. At x/H=1 the predicted profile using the k-ϵ model is in closer agreement with experimental data than that predicted with the KL1 model. At x/H=2 the predicted profiles are very similar except near the ceiling where the profile predicted by the k-ϵ model appears to be better; three-dimensional effects were, however, observed in the experiments in this region.

Case	H [m]	h/H	L/H	B/H	b/B	Re	Expts. by
R2D	0.089	0.056	3	-	-	5000	Restivo [17]
Å	2	0.003	2	-	-	1180	Åkesson [18]
H	0.36	0.025	2.78	-	-	1340	Hanel [19]
R3D	0.089	0.1	3	1	0.1	5000	Restivo [17]

H (h), B (b) and L denote height, width and length, respectively, of a room (inlet).

Table 1. Configuration for the experimental investigations

Case	Grid size	CPU-time k-ϵ model [min]	CPU-time KL1 model [min]	Number of iter. k-ϵ model	Number of iter. KL1 model
R2D	35x20	13	6	274	133
Å	42x35	127	52	1200	590
H	35x27	25	17	330	260
R3D	18x15x19	134	137	290	320

Table 2. Grids, CPU-time and number of iterations

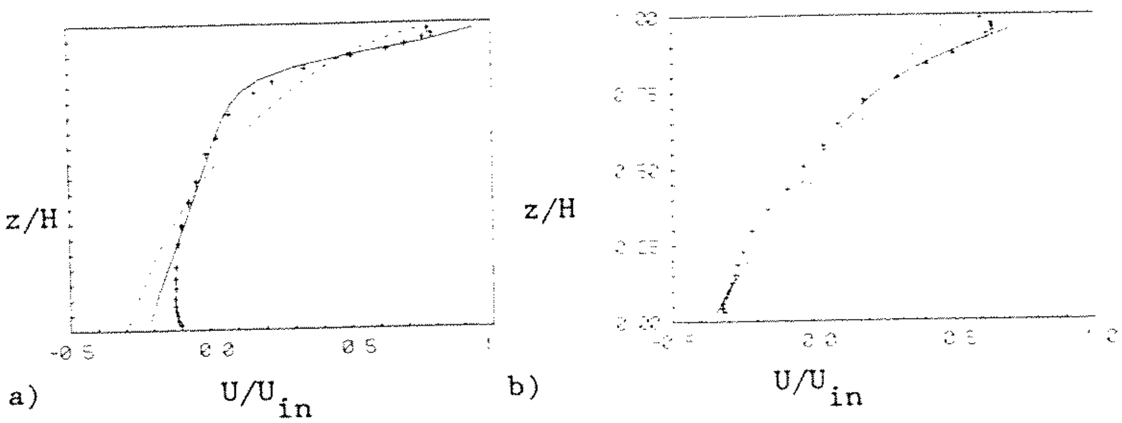
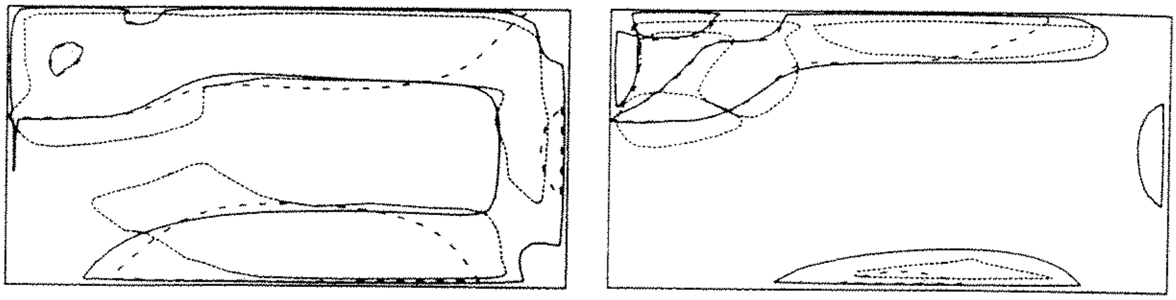


Figure 6. U-velocity profiles. —— k-ϵ. – – – KL1. + Expts. [17]. a) x/H=1; b) x/H=2. Case R2D.

b) Case Å: The turbulent viscosity had to be prescribed in the jet region (according to the theory of turbulent free jets) when the KL1 model was used; the reason for this is that the KL1 model is not suitable for predicting free jet flows. Velocity contours are compared with experimental data in Fig. 7; the contours are equally well in agreement with experimental data for the two models. The point where the jet reattaches to the upper wall was predicted to $x_R/H=0.5$ for both models; the corresponding experimental value is 0.6. The required CPU time (see Table 2) may seem rather high compared with the other cases. The flow pattern is, however, more complex than in Cases R2D and H and the number of nodes is also considerably higher. For a 35x28 nodes grid the CPU time was 20 minutes for the k-ϵ model and 11 minutes for the KL1 model.



a) b)

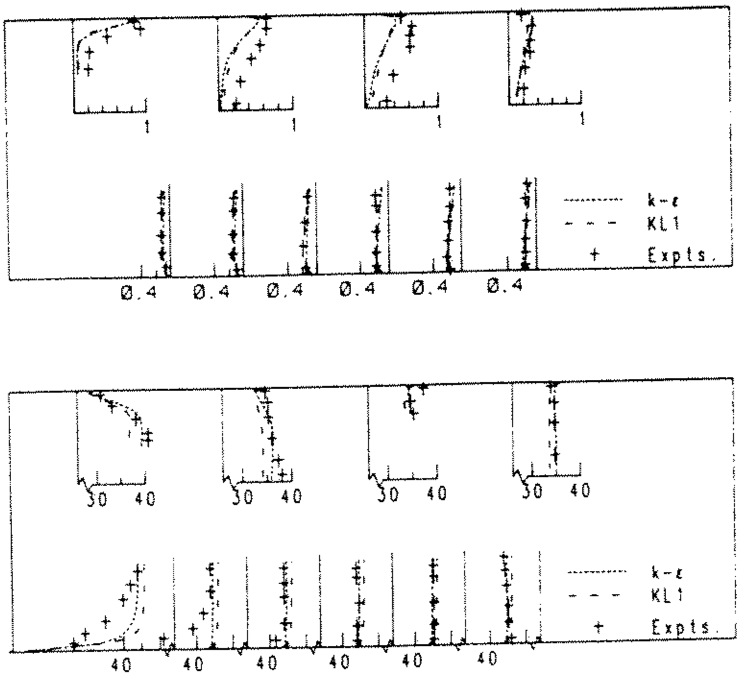
Figure 7. Velocity contours scaled with U_{in}; a) 0.05, b) 0.08. —— k-ϵ. – – – KL1. ..... Expts. [18]. Case A.

c) Case H: This is a buoyantly ventilated room. The inlet air has a temperature of 25^{o}C. Heat is supplied through the floor for $0.2 \le x/L \le 0.4$. The temperature is known at all walls from experiments, which makes it allowable to take no account of the radiated heat. The temperature (denoted by t) at the walls was set as follows.

$x=0$: $t=42$; $x=L$: $t=37$; $z=0$, $0 \le x/L < 0.2$: $t=42.5$; $z=0$, $0.2 \le x/L \le 0.4$: $t=112$; $z=0$, $0.4 < x/L \le 1$: $t=37$; $z=H$: $t=38$.

The velocity and the temperature profiles are shown in Fig. 8 (the profiles are plotted in the room at the positions where they occur). The profiles predicted by the two turbulence models are about equally well in agreement with experimental data.

d) Case R3D: This is a three-dimensional isothermally ventilated room, Fig. 5, for which the plane $y=0$ is a symmetry plane; the flow is thus calculated in one half of the room only, and, since $0.09\Delta_y$ is the maximum size of a turbulent eddy in the y-direction, Δ_y was set to B/2. The velocity profiles in Fig. 9, particularly those in the symmetry plane nearer the end wall, are slightly less well predicted by the KL1 model. The predictions with both models are considered to be accurate enough for engineering purposes. The CPU time required for obtaining a converged solution was, suprisingly enough, less with the k-ϵ model than that required with the KL1 model for this case.



a)

b)

Figure 8. a) Profiles of absolute velocity scaled with U_{in}. b) Profiles of temperature (in ^{o}C). ..... k-ϵ. - - - KL1. + Expts. [19].

4. CONCLUSIONS

A new one-equation turbulence model has been presented. It has been shown that the model gives as good agreement with experimental data as - or, for the wall jet, better than - the standard k-ϵ model for the parabolic calculations. The model was shown to be able to predict the flow in ventilated rooms accurately enough for engineering purposes. The CPU time required was reduced with up to 60% compared to the standard k-ϵ model; this reduction was mainly due to the need of fewer iterations.

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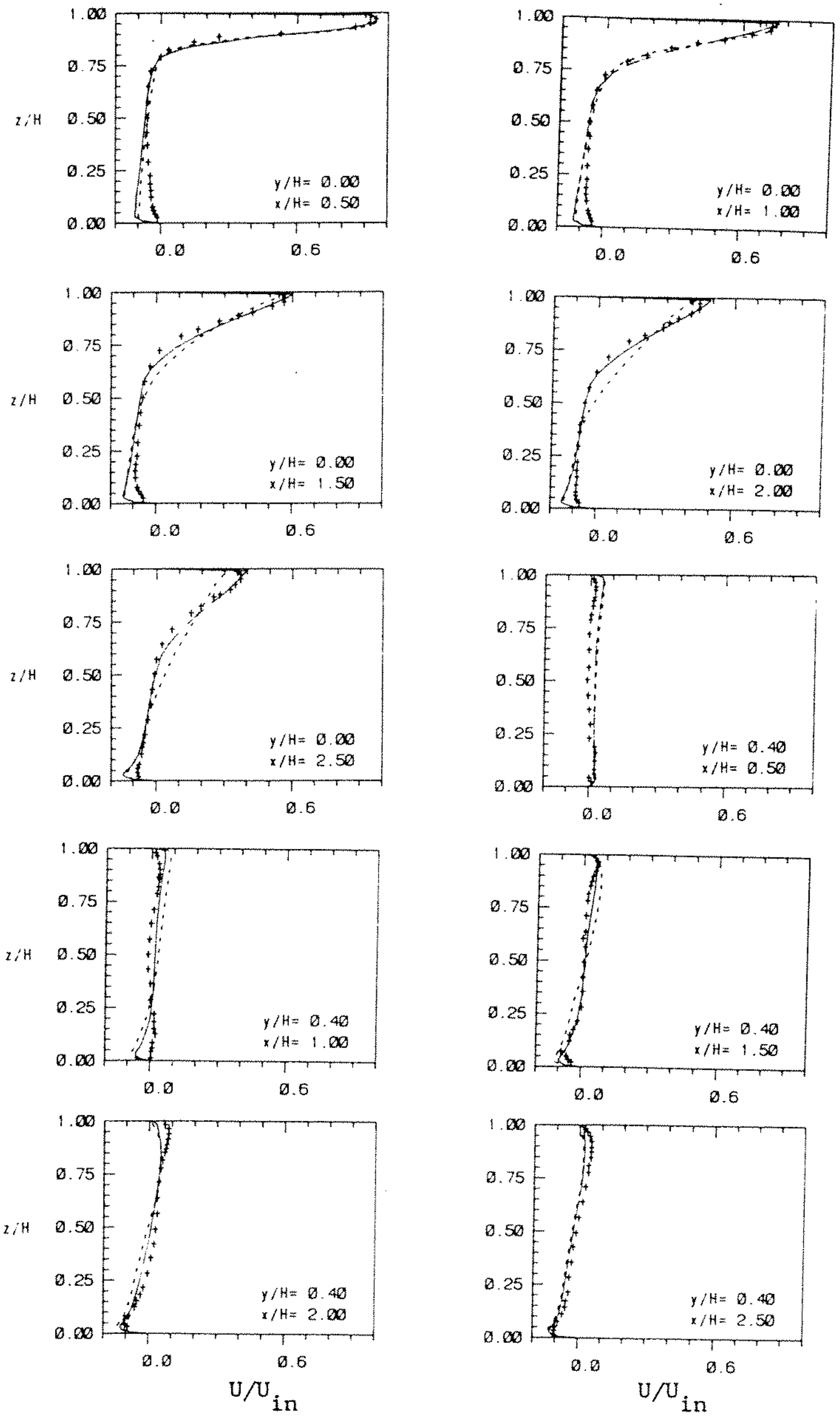

Figure 9. Profiles of the U-velocity. Case R3D. —— k-ε. — — — KL1. + Expts. [17]

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