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CALCULATION OF THE FLOW IN A VENTILATED ROOM USING DIFFERENT FINITE-DIFFERENCE SCHEMES AND DIFFERENT TREATMENTS OF THE WALLS

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ABSTRACT

Three difference schemes have been compared for calculating the flow in a two-dimensional room : the standard hybrid upwind/central scheme, the skew-upwind scheme, and the QUICK scheme. The two latter schemes are generally considered to be more accurate than the first. In the present study, however, no large differences were found between the different schemes ; rather the opposite, in fact, as QUICK was shown to produce totally unrealistic results when a coarse grid was used.
Usually wall-functions are used when boundary conditions are prescribed at walls, which means that the boundary layers have not to be resolved (i.e. very few grid lines are placed in the boundary layer). An alternative is to resolve the boundary layer using a Low-Reynolds number k-ε model (LR-model). These two alternatives are compared in the present study, the LR-model giving slightly better predictions.

1. INTRODUCTION

Much work has been carried out on numerical simulation of the flow in ventilated rooms. In most (if not all) works the hybrid upwind/central difference scheme (HDS) or pure upwind difference scheme (UDS) has been used. The most common method of treating the wall-boundaries is, furthermore, the use of wall-functions.
The object of the present study is twofold : to investigate if a more accurate difference scheme such as QUICK [1] (quadratic upstream-weighted interpolation scheme) or SUDS [2] (skew-upwind differencing) predicts the flow better than HDS, and, to investigate if a Low-Reynolds k-ε model gives better predictions in a ventilated room than wall functions.

2. MATHEMATICAL MODEL

2.1. Governing equations

The equations for steady-state, incompressible flow can be written in Cartesian tensor notation as follows ($u_1 = u$, $u_2 = v$, $x_1 = x$, $x_2 = y$) :

ρ	: density		k	: turbulent kinetic energy
u, v, u_i	: velocity components		ε	: turbulent dissipation
p	: pressure			

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\rho \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_{eff}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]$$

The effective viscosity hypothesis has been used to represent the combined molecular and turbulent stresses, where :

$$\mu_{eff} = \mu + \mu_t$$

2.2. Turbulence model

The standard k-ε model, valid for high Reynolds number, can be extended so as to be valid for low Reynolds number as well, where viscous effects become important. The model by Jones and Launder [3] has been used in the present study, and it can be written as :

$$\frac{\partial}{\partial x_i}(\rho u_i k) = \frac{\partial}{\partial x_i}[(\mu + \frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial x_i}] + P_k - \rho\varepsilon - 2\mu(\frac{\partial k^{1/2}}{\partial x_k})^2$$

$$\frac{\partial}{\partial x_i}(\rho u_i \varepsilon) = \frac{\partial}{\partial x_i}[(\mu + \frac{\mu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_i}] + \frac{\varepsilon}{k}(c_{\varepsilon 1} P_k - c_{\varepsilon 2}\rho\varepsilon) + 2\mu\mu_t(\frac{\partial^2 u_i}{\partial x_k \partial x_k})^2/\rho$$

where :

$$P_k = \mu_t(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})\frac{\partial u_i}{\partial x_j},\ \mu_t = c_\mu \rho k^2/\varepsilon,\ \sigma_k = 1,\ \sigma_\varepsilon = 1.3,\ \sigma_t = 0.9,$$

$$c_{\varepsilon 1} = 1.44,\ c_{\varepsilon 2} = 1.92\ [1\text{-}0.3\ \exp(-R_t^2)],\ c_\mu = 0.09\ \exp[-\frac{3.4}{(1 + R_t/50)^2}],$$

$$R_t = \rho k^2/(\mu\varepsilon)$$

The boundary conditions k = ε = 0 at walls were used. In order to insure stability, a linearization of the source term $2\mu(\frac{\partial k^{1/2}}{\partial x_k})^2$ has been made : we wrote it as a(k)k ; a is updated at each iteration.
In the standard k-ε models the last term in the k-equation, as well as the last term in the ε-equation are zero (standard boundary conditions are used for k and ε) ; the functions $c_{\varepsilon 2}$ and c_μ are furthermore constants (= 1.92 and 0.09 respectively) [4].

3. THE FINITE-DIFFERENCE SCHEMES

Every transport equation in fluid dynamics contains convective terms ; the general transport equation for the variable ϕ contains the $\partial(\rho u \phi)/\partial x$ for example. In control-volume formulations the equations are discretized by first integrating them over a control volume. When the term above is integrated in the x-direction (see Fig. 1), we obtain :

$$(\rho u \phi)_e - (\rho u \phi)_w$$

The problem now arises : how to estimate ϕ (ϕ is stored in the nodes W, P and E) ?
Three different schemes, used in the present study, are briefly presented below.

3.1. Hybrid Upwind/Central Differencing Scheme (HDS)

This scheme approximates the convective terms by central difference scheme (CDS) if the Peclet number (= local Reynolds number) is below two, and by upwind differencing otherwise, i.e. (see Fig. 1) :

$$\phi_w = \phi_W, \text{ if } |P_w| \geqslant 2 \text{ and } u_w > 0 \;;\; \phi_w = \phi_P, \text{ if } |P_w| \geqslant 2 \text{ and } u_w < 0$$

$$\phi_w = \alpha\,\phi_P + (1-\alpha)\,\phi_W, \text{ if } |P_w| < 2$$

where α is an interpolation factor, which is equal to 0.5 if the face w lies midway between W and P.

3.2. Quadratic Upstream-Weighted Interpolation (QUICK)

This scheme of Leonard [1] utilizes a polynomial of second order fitted to three nodes, two nodes located upstream of the face, and one node located downstream ; the scheme is thus a form of upwind scheme. It combines the accuracy of CDS (second order accuracy) with the inherent stability of UDS (due to its upwind character). For a uniform grid the ϕ_w is approximated as :

$$\phi_w = 0.125\,\phi_{WW} + 0.75\,\phi_W + 0.375\,\phi_P, \quad u_w > 0$$

$$\phi_w = 0.375\,\phi_W + 0.75\,\phi_P - 0.125\,\phi_E, \quad u_w < 0$$

3.3. Skew-Upwind Difference Scheme (SUDS)

This is only a first-order scheme of Raithby [2], but it has the advantage of reducing one of the most important and frequent agencies of numerical diffusion for two-dimensional recirculating flows : flow-to-grid skewness. The basic idea of the scheme is to apply upwind differences in a vectorial rather that in a componential sense. If, for example, the angle between the velocity vector at face w and the x-axis (see Fig. 1) is 45° (i.e. $u_w = v_w > 0$) ϕ_w is approximated as :

$$\phi_w = \phi_W + \phi_{SW}$$

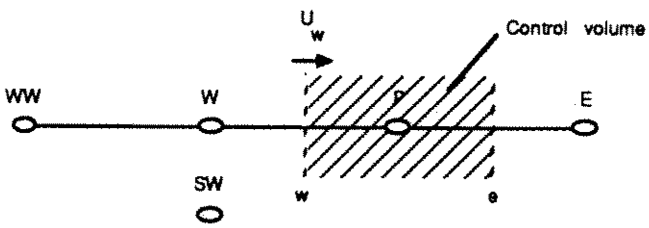

Figure 1. Control volume

4. RESULTS

The three difference schemes (HDS, QUICK and SUDS) are compared with either numerical or experimental data in two test cases : driven cavity, and the flow in a two-dimensional room. The first case was chosen in order to verify that the difference schemes had been correctly implemented.
In the last section the flow in a ventilated room is calculated using the Low-Reynolds k-ε model presented in chapter 2.
All calculations were carried out on a SUN workstation.

4.1. Comparison of three difference schemes

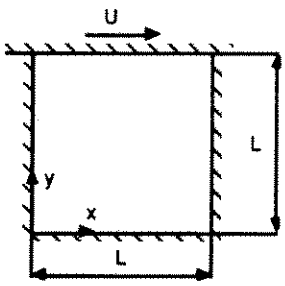

Figure 2. Configuration for the wall-driven square cavity

Driven Cavity Flow. The wall-driven square-cavity (see Fig. 2) has been examined by many workers, and accurate numerical data exist. The Reynolds number $Re = uL/\nu = 1000$ was chosen. The results are compared with data obtained by Schreiber and Keller [5].
The calculated u-profiles at $x/L = 0.5$ (using a 22 x 22 nodes grid) are shown in Fig. 3. As can be seen, QUICK is superior to SUDS and HDS, and the results predicted with QUICK are in close agreement with those in [4].

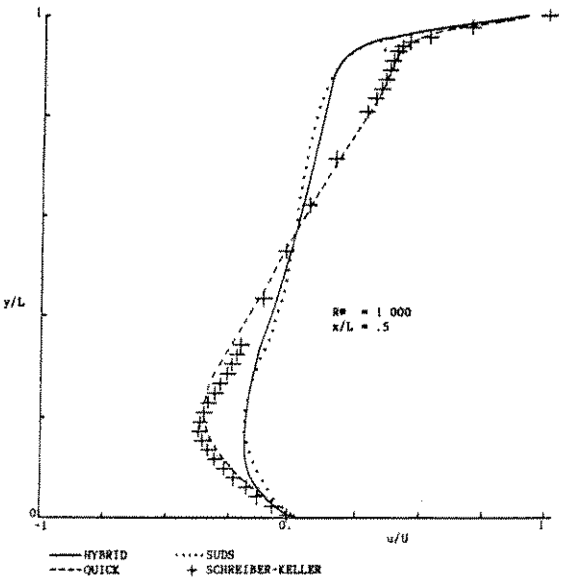

Figure 3. Vertical u-velocity profile (driven cavity)

A Two-dimensional Ventilated Room Using Wall Functions. The configuration of the room is shown in Fig. 4. Two computed velocity profiles are compared to experimental values [6] in Fig. 5 ; a 53 x 44 nodes grid was used. The three schemes all perform well.

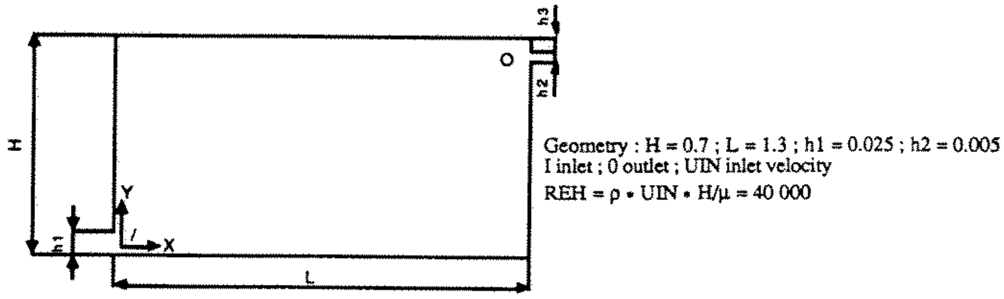

Figure 4. Description of the configuration

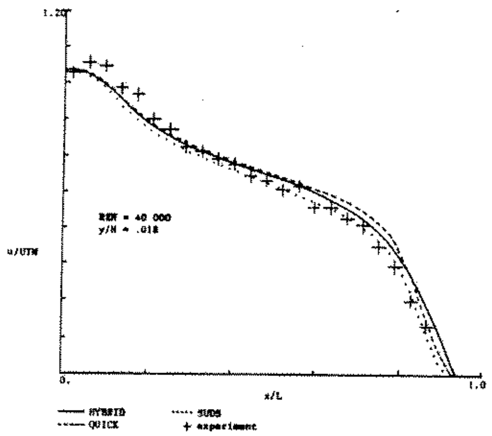

Figure 5a. Horizontal u-velocity profile (ventilated room)

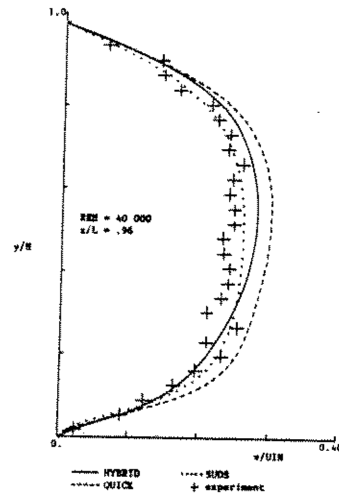

Figure 5b. Vertical v-velocity profile (ventilated room)

A coarser grid (28 x 24 nodes) was also tested. With this grid the results using HDS and SUDS were good, whereas the results predicted using QUICK were disastrous. This was due to severe under -and over- shoots near the outlet, where steep gradients prevail. The dimensions of the outlet is smaller than the inlet for this configuration ; this is the reason why steeper gradients occur near the outlet rather than, as normally is the case, near the inlet.

4.2. Low-Reynolds calculation of the flow of the ventilated room

A 65 x 76 grid was used with high resolution near walls. In particular, 17 grid points were put into the inlet in order to get a detailed analysis of the lower wall jet. We have no experimental data very close to the walls, nevertheless the Low-Reynolds model influences the core of the flow.
Two velocity profiles (computed with and without the Low-Reynolds model) are compared to experimental data [6] in Fig. 6. The Low-Reynolds model predictions are slightly better.

We now give an analysis of the lower wall jet to show that we recover some known features about wall jets and about Low-Reynolds predictions. It is convenient to use the variables :

$$u^+ = u/v_* \;;\; y^+ = y v_* \rho/\mu \;;\; k^+ = k/v_*^2,$$

where u is the velocity component parallel to the wall, v_* is the friction velocity, and y is the normal distance to the wall (see Fig. 4).
Since we are dealing with a cavity flow, these wall variables are x dependent.
In Fig. 7 we compare the Low-Reynolds prediction of u^+ (log y^+) to the universal law :

$$u^+ = 5.5 \log\ y^+ + 5.45 \quad \text{(in principle valid for } y+ > 30)$$

As noticed in [3] the slope of the computed u^+ function in the semi-logarithmic region ($15 \leq y^+ \leq 50$) is distinctly higher than the 5.5. Actually we do not reach here the universal regime.
We can however check that u^+ (y^+) starts linear :

$$u^+ = y^+ \qquad \text{for } y^+ < 5$$

as it should be.
Looking at k^+ (y^+) for several x/l values we can show that k^+ takes its maximum at $y^+ = 15$ (independently of x) which corresponds to the location of the maximum production of k (see Fig. 8). This fact has been verified experimentally for a lot of near wall flows see [7].

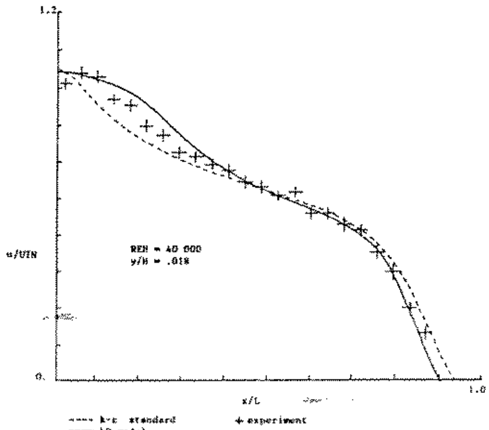

Figure 6a. Horizontal u-velocity profile (ventilated room)

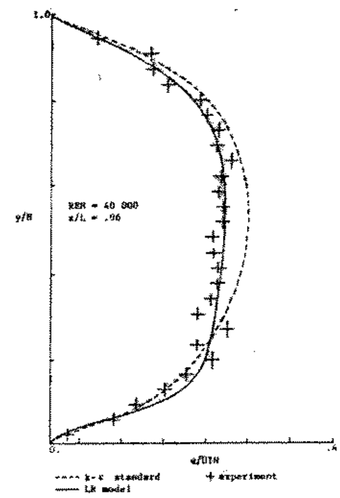

Figure 6b. Vertical v-velocity profile (ventilated room)

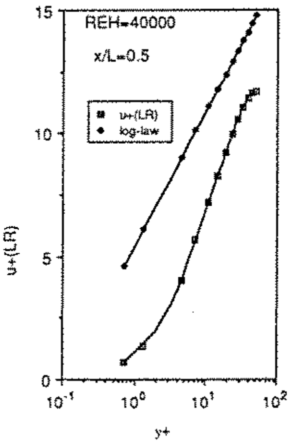

Figure 7. Calculated velocity profile and log-law

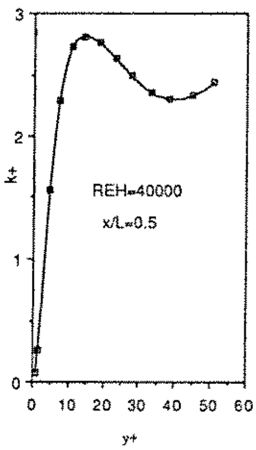

Figure 8. Calculated turbulent energy profile

5. CONCLUSION

The usual hybrid discretization scheme and the use of wall functions are very often considered as crude approximations producing some inaccuracy in numerical fluid flow simulations. We have shown in this paper that for standard ventilation flows, more envolved discretization schemes yields very similar predictions to those obtain with the hybrid scheme. Moreover in the case of coarse grid, the QUICK scheme produces erroneous results.
We have also shown that the Low-Reynolds k-ε turbulence model gives only slight improvements of the predictions in the core of the flow, eventhough wall functions have been applied in the standard k-ε model in regions where y^+ is moderate ($y^+ < 50$). In the future more complex configurations including obstacles will be considered to see whether our conclusions remain true.

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