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CALCULATION OF THE TURBULENT FLOW AND THE LOCAL AGE IN A NON-RECTANGULAR ROOM USING A FINITE VOLUME COMPUTER CODE WRITTEN IN GENERAL NON-ORTHOGONAL COORDINATES

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ABSTRACT

The flow a in non-rectangular room is calculated. A computer program written in general curvilinear coordinates [1] (which means that the coordinate lines can be of almost arbitrary form) is used. This type of program can be applied to rooms with complex geometries. The calculated results are compared with experimental data, and the agreement is good.

The local age, introduced by Sandberg and Sjöberg [2], is also calculated. It is valuable to calculate this ventilation parameter, as it provides useful information on how air and contaminants spread in a room. The physical meaning of the local age [2,3], is the amount of time that has elapsed since the air, passing point P, entered the room.

1. INTRODUCTION

Numerical simulation of air movements in ventilated rooms is carried out quite frequently by many researchers nowadays. Rooms usually have a simple geometry (horizontal floor and ceiling, vertical walls) where a grid can easily be specified so that the grid lines follow the boundaries.

When the geometry of a room is more complex in the sense that the boundaries are not orthogonal to each other (they are inclined, see Fig. 1, or curved), there are two possibilities:

i) to use a Cartesian grid and approximate the inclined boundaries with a "staircase", see Fig. 1a;

ii) to use a computer code which solves equations formulated in general non-orthogonal coordinates (BFC=Boundary-Fitted-Coordinates), which means that the grid lines can be of (almost) arbitrary form, see Fig. 1b

The latter way (ii) offers many advantages over the former (i):

1) boundaries can be precisely represented;
2) local grid refinement near the boundaries is possible;
3) numerical errors are reduced if (as often is the case) the grid can be made to approximate stream lines; and
4) surface-friction effects can be represented well.

In the present work a computer code which solves the equations formulated in general non-orthogonal coordinates [1] is used to calculate the turbulent flow in an auditorium, Fig. 1b. The standard k-ϵ turbulence model is used. The calculated flow field is compared with experiments carried out by Hanel and Köthnig [4], and the agreement is good.

The local age field, which was numerically simulated by Davidson and Olsson [3,5] recently, is also calculated.

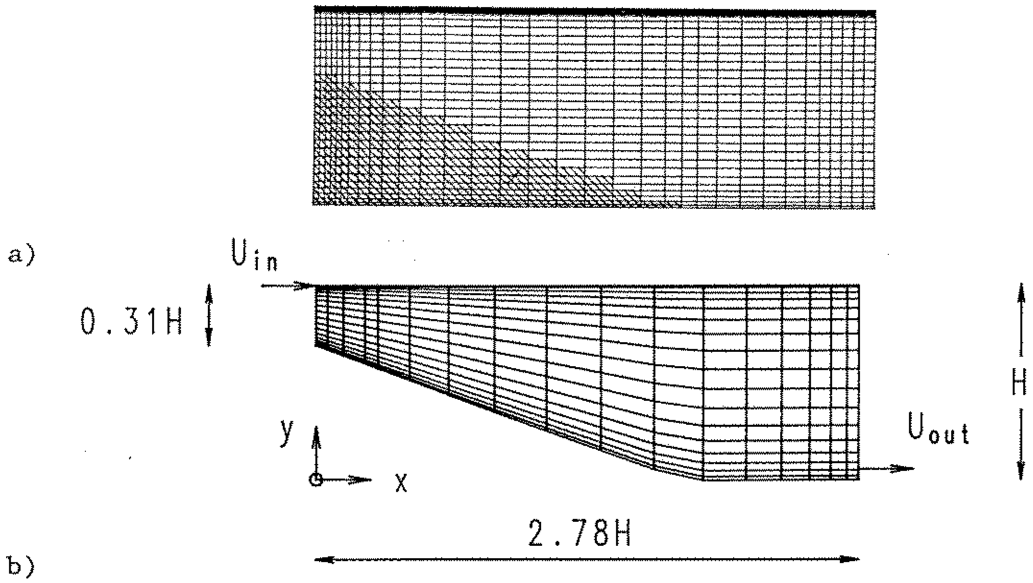

Figure 1. Configuration for the room with the grid included (schematically drawn). The height of the inlet, h = 0.009H; Reynolds number, $U_{in}h/\nu$ = 3700. a) Cartesian grid; the velocities are zero in the shaded region. b) BFC grid used in the present study.

2. FORMULATION

2.1 Momentum Equations

The momentum equations and the continuity equation are solved using the SIMPLEC algorithm [6]; readers not interested in the precise form of the momentum equations in general coordinates can go directly to Section 2.2.

The momentum equations for turbulent flow in general coordinates, using covariant components can be written [1]

$$(\rho g^{jk} v_i v_j)_{,k} = - \frac{\partial p}{\partial x^i} + (\mu_{eff}\, g^{jk} v_{i,j})_{,k} \tag{1}$$

where g^{jk} denotes the contravariant components of the metric tensor. Here the subscripts (i,j,k) denote covariant components and superscripts (i,j,k) denote contravariant components; this convention is used throughout the paper. The comma notation is used for denoting the covariant derivative. In [1] it was shown that if a local coordinate system is used so that the direction of the neighbouring velocities, v_{inb} (i = co-ordinate direction, nb = neighbour), are kept the same as that of the velocity (v_{1e} or v_{2n}) being solved (see Fig. 2), Eq. (1) can be integrated and rewritten so that

$$\int_A g^{jk} (\rho v_i v_j - \mu_{eff} \frac{\partial v_i}{\partial x^j})\, n_k dA + \int_A p\, n_i dA = 0 \tag{2}$$

where A denotes the bounding area of the control volume with the volume V, and $\vec{n}$ is its normal vector.

The neighbour velocities with a prime (see Fig. 2) denote velocity vectors projected on $\overrightarrow{PE}$. The velocity v'_{1ee}, for instance, is calculated as

$v'_{1ee} = \vec{v} \cdot \overrightarrow{PE}$

where PE ($=\vec{g}_{1e}$) is a covariant unit vector.

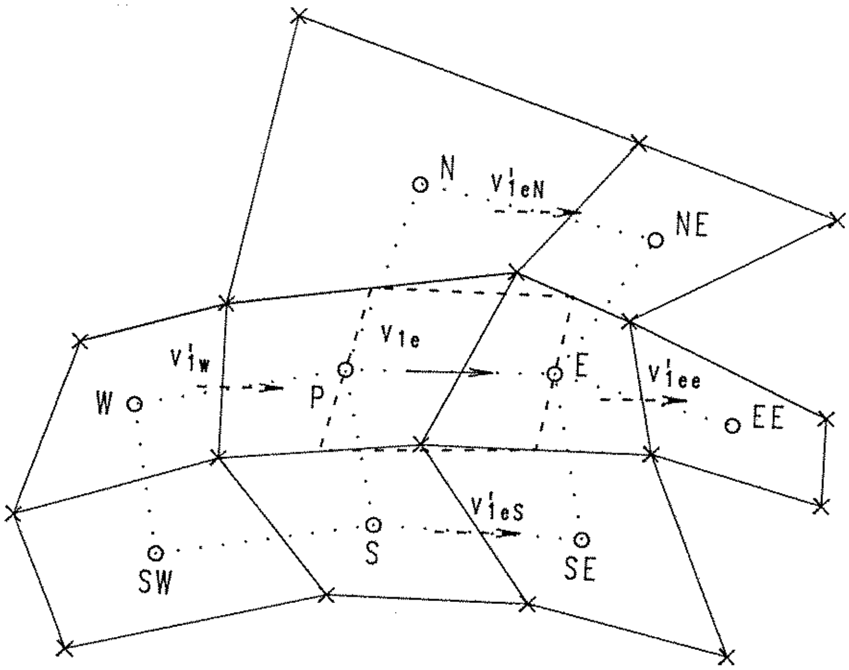

Figure 2. The grid (see Section 2.3). The dashed arrows show the neighbour velocity vectors projected on $\overrightarrow{PE}$, i.e. v'_{1ee}, v'_{1w}, v'_{1eN} and v'_{1eS}.

In most studies on deriving discretised equations for flow in complex geometries, the terms due to curvature, divergence and non-orthogonality of the grid have been included using Christoffel symbols and metric tensors. Since the number of these terms is rather large, this is very cumbersome and may also be inaccurate (there appear terms containing up to the third derivative of the grid coordinates). This is not the case with the present formulation.

2.2 The k-ϵ Turbulence Model

The standard k-ϵ turbulence model is implemented in the computer program [7]. The equations for k and ϵ can be written

$$\int_A g^{jk}(\rho v_j \phi - \frac{\mu_{eff}}{\sigma_\phi} \frac{\partial \phi}{\partial x^j}) \, n_k dA + \int_V b_\phi dV = 0 \qquad (3)$$

where $\phi = k$ (turbulent kinetic energy) or $\phi = \epsilon$ (dissipation of k), σ_ϕ is the turbulent Prandtl number, and b_ϕ denotes the general source term, which for the k and ϵ-equations takes the following forms:

$$b_k = \int_V (P_k - \rho\epsilon) \, dV; \quad b_\epsilon = \int_V \frac{\epsilon}{k} (C_{1\epsilon} P_k - C_{2\epsilon} \rho\epsilon) \, dV$$

where P_k is the production term (see [7] for more details).

The turbulent viscosity is calculated as

$$\mu_t = \rho C_\mu k^2/\epsilon$$

The constants in the turbulence model have been assigned their standard values [8]: $C_\mu=0.09$, $C_{1\epsilon}=1.44$, $C_{2\epsilon}=1.92$, $\sigma_k=1.0$, $\sigma_\epsilon=1.3$

2.3 The Grid

A grid is shown in Fig. 2. The crosses define the corners of the scalar control volumes, and the circles define the scalar nodes. The position of a scalar node is defined as the average of its four cell corners. The lines which connect these nodes (dotted lines in Fig. 2) define the direction of the covariant base vectors, $\vec{g}_i$.

The v_1-control volume is staggered in the positive x^1-direction; it is outlined with dashed lines in Fig. 2. Its east face, for example, is defined as being midway between the east faces of scalar control volumes P and E.

2.4 Boundary Conditions

The velocities at the inlet were prescribed according to experiments; then the turbulent quantities were estimated. Conventional wall-functions [8] were used for the velocities, k and ϵ at all of the walls. Impermeable boundaries (zero flux) were prescribed at the walls for the local age, τ_P. Zero stream-wise gradient was imposed for all variables at the outlet.

3. THE LOCAL AGE

The age of the air at a control volume within the room is the time that has elapsed since the air, passing this control volume, entered the room. In [3] the local age was calculated using the step-down method, i.e. the room was initially filled with contaminant, c_0 (= initial concentration), and the decay of the concentration was calculated. The local age was obtained from the formula

$$\tau_P = \frac{1}{c_0} \int_0^\infty c_P(t)\, dt \qquad (4)$$

where τ_P and c_P denote the local age and the concentration at control volume P, respectively, and t denotes time. A computationally much cheaper way of calculating the local age (which was also used in [5]) is, as shown by Sandberg [9], to solve the (steady) transport equation (3) with $\phi = \tau_P$ and $b_\phi = \rho$. The boundary condition was $\tau_P = 0$ at the inlet.

4. RESULTS

The configuration with the grid is shown in Fig. 1b. The calculated results are compared with experimental data from Hanel and Köthnig [4].

When the flow in ventilated rooms with small inlets is numerically simulated, it is not uncommon to prescribe the v_1-velocity at a line where x = constant, which means that fewer grid lines are needed in the inlet region. In the present calculation the v_1-velocity was prescribed at x/H = 0.1, using the formula for the velocity in a wall jet [10]

$$v_1/U_{in} = [\cosh(\frac{y}{\delta_{1/2}} - 0.14)]^2 \qquad (5)$$

where $\delta_{1/2}$ is the half-width of the wall jet, which was determined from the experiments [4]; Eq. (5) was used for grid lines which were nearer the ceiling than $\delta_{1/2}$.

In Fig. 3 the calculated velocity vectors are shown, and in Fig. 4 the calculated velocity profiles using two different grids are compared with experimental data, and the agreement is good.

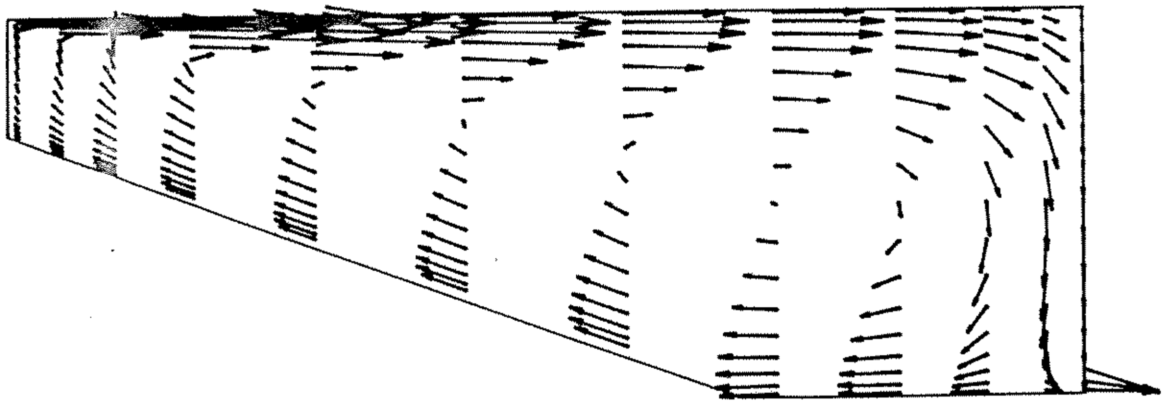

Figure 3. Calculated velocity vectors using the 33 x 33 node grid.

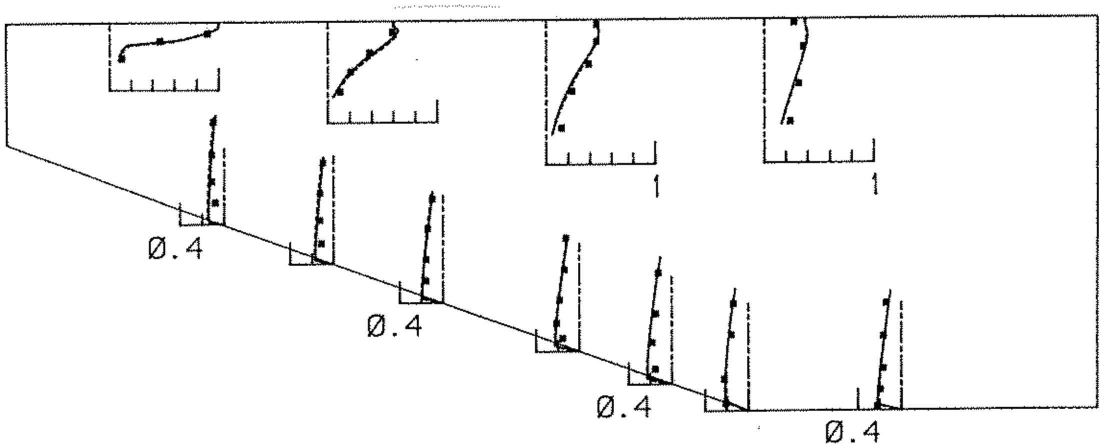

Figure 4. Velocity profiles of the absolute velocity, $|\vec{v}|/U_{in}$. Solid lines: 33 x 33 node grid; dotted lines: 53 x 62 node grid; markers: experiments [4].

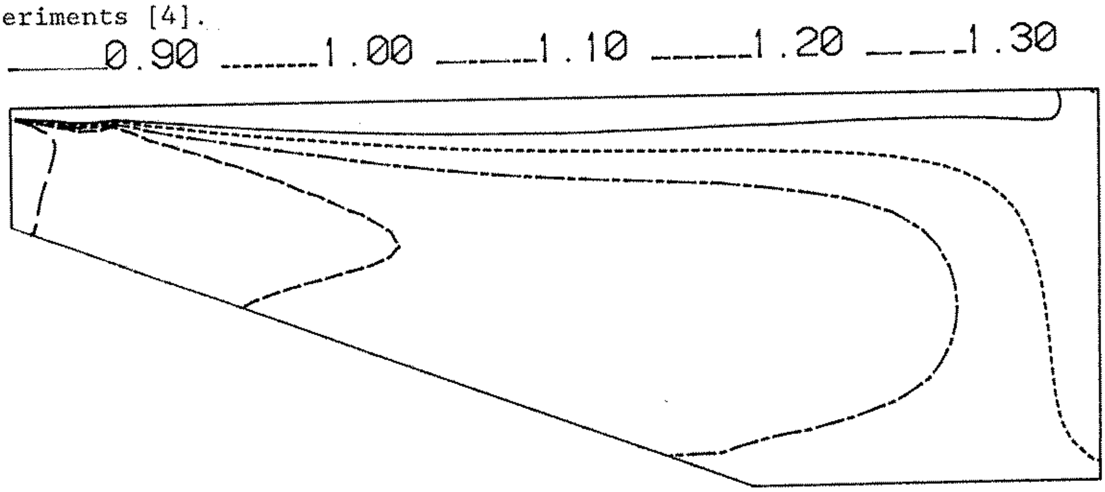

Figure 5. Contours of the calculated age using the 33 x 33 node grid. The numbers above the room denote the age scaled with the age at the exit (= the nominal time constant of the room).

The calculated local age field is presented in Fig. 5. The local age is, as expected, low near the ceiling (the air reaches this region quickly), and the age of the air increases gradually nearer the floor. The air is at its oldest below the inlet; this was to be expected as the air reaches this region last.

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