Detached Eddy Simulations: Analysis of a limit on the dissipation term for reducing spectral energy transfer at cut-off

Lars Davidson*, Christophe Friess**

*Division of Fluid Dynamics
Dept. of Mechanics and Maritime Sciences
SE-412 96 Gothenburg, Sweden

**Aix-Marseille Université
CNRS, Centrale Marseille
M2P2 UMR 7340, 13451, Marseille, France

December 11, 2021

(ID)DES: PHYSICAL MEANING OF ψ

$$
\begin{aligned}
C^k &= P^k + D^k - \psi\varepsilon && k - \text{equation} \\
C^\varepsilon &= C_{\varepsilon 1}\frac{\varepsilon}{k}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon^2}{k} && \varepsilon - \text{equation} \\
\psi &= \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)
\end{aligned}
$$

Partition of Turbulent Kinetic Energy

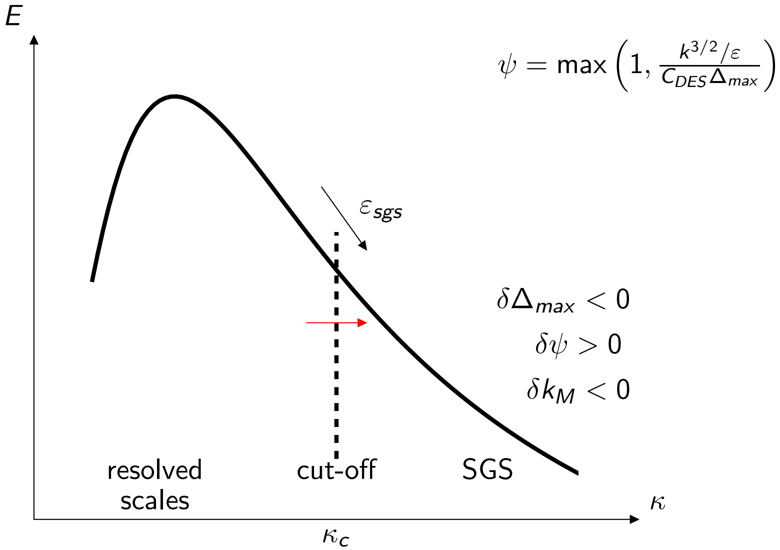


Figure: Energy spectrum.

Object of the Present Work

Object of the Present Work

- To control – limit – how the fast energy partition (i.e. the cut-off) changes in physical space

Object of the Present Work

- To control – limit – how the fast energy partition (i.e. the cut-off) changes in physical space
- i.e. limit $\dfrac{\partial \kappa_c}{\partial x_j}$

Object of the Present Work

- To control – limit – how the fast energy partition (i.e. the cut-off) changes in physical space
- i.e. limit $\dfrac{\partial \kappa_c}{\partial x_j}$
- by limiting $\psi = \max\left(1, \dfrac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$

Object of the Present Work

- To control – limit – how the fast energy partition (i.e. the cut-off) changes in physical space
- i.e. limit $\dfrac{\partial \kappa_c}{\partial x_j}$
- by limiting $\psi = \max\left(1, \dfrac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$
- i.e. limiting the dissipation term, $\psi\varepsilon$, in the k equation

Perturbation Analysis

Perturbation Analysis

- Along mean streamlines, k_M and ε_M are assumed to be in equilibrium. We get
- $$\begin{aligned} \frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0 \\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0 \end{aligned}$$

Perturbation Analysis

- Along mean streamlines, k_M and ε_M are assumed to be in equilibrium. We get
- $$\begin{aligned}\frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0\\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0\end{aligned}$$
- For infinitesimal perturbations we get

$$\begin{aligned}0 &= \delta P^k + \delta D^k - \varepsilon_M\delta\psi\\ 0 &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k\left(\frac{\delta P^k}{P^k} - \frac{\delta k_M}{k_M}\right) + C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M}\left(\frac{\delta k_M}{k_M}\right) + \delta D^\varepsilon\end{aligned}$$

Perturbation Analysis

- Along mean streamlines, k_M and ε_M are assumed to be in equilibrium. We get
- $$\begin{aligned} \frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0 \\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0 \end{aligned}$$
- For infinitesimal perturbations we get

$$\begin{aligned} 0 &= \delta P^k + \delta D^k - \varepsilon_M \delta\psi \\ 0 &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k\left(\frac{\delta P^k}{P^k} - \frac{\delta k_M}{k_M}\right) + C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M}\left(\frac{\delta k_M}{k_M}\right) + \delta D^\varepsilon \\ 0 &= \delta\varepsilon_M \end{aligned}$$

Perturbation Analysis

- Along mean streamlines, k_M and ε_M are assumed to be in equilibrium. We get
-
$$\begin{aligned}\frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0 \\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0\end{aligned}$$
- For infinitesimal perturbations we get

$$\begin{aligned}0 &= \delta P^k + \delta D^k - \varepsilon_M \delta\psi \\ 0 &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k\left(\frac{\delta P^k}{P^k} - \frac{\delta k_M}{k_M}\right) + C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M}\left(\frac{\delta k_M}{k_M}\right) + \delta D^\varepsilon \\ 0 &= \delta\varepsilon_M\end{aligned}$$

- 5 Eqns, 7 Unknowns: $\delta\varepsilon_M$, δP^k, $\delta\psi$, δk_M, $\delta\varepsilon_M$, $\underline{\delta D^k, \delta D^\varepsilon}$

Perturbation Analysis

Perturbation Analysis

- D^k, D^ε?

Perturbation Analysis

- D^k, D^ε?
- Boussinesq and neglecting quadratic perturbation terms gives

Perturbation Analysis

- D^k, D^ε?
- Boussinesq and neglecting quadratic perturbation terms gives
- $$\underline{D^k} + \delta D^k = \frac{C_\mu}{\sigma_k}\frac{\partial}{\partial x_j}\left[\underline{\frac{k_M^2}{\varepsilon_M}\frac{\partial k_M}{\partial x_j}} + \frac{k_M^2}{\varepsilon_M}\frac{\partial(\delta k_M)}{\partial x_j} + \frac{2(\delta k_M)k_M}{\varepsilon_M}\frac{\partial k_M}{\partial x_j}\right]$$

Perturbation Analysis

- D^k, D^ε?
- Boussinesq and neglecting quadratic perturbation terms gives
- $$\underline{D^k} + \delta D^k = \frac{C_\mu}{\sigma_k}\frac{\partial}{\partial x_j}\left[\underline{\frac{k_M^2}{\varepsilon_M}\frac{\partial k_M}{\partial x_j}} + \frac{k_M^2}{\varepsilon_M}\frac{\partial(\delta k_M)}{\partial x_j} + \frac{2(\delta k_M)k_M}{\varepsilon_M}\frac{\partial k_M}{\partial x_j}\right]$$

Following [5], we assume that $\delta k_M/k_M$ does not vary in space, i.e.

Perturbation Analysis

- D^k, D^ε?
- Boussinesq and neglecting quadratic perturbation terms gives
-

$$\underline{D^k} + \delta D^k = \frac{C_\mu}{\sigma_k}\frac{\partial}{\partial x_j}\left[\underline{\frac{k_M^2}{\varepsilon_M}\frac{\partial k_M}{\partial x_j}} + \frac{k_M^2}{\varepsilon_M}\frac{\partial(\delta k_M)}{\partial x_j} + \frac{2(\delta k_M)k_M}{\varepsilon_M}\frac{\partial k_M}{\partial x_j}\right]$$

Following [5], we assume that $\delta k_M/k_M$ does not vary in space, i.e.

$$\frac{\partial(\delta k_M/k_M)}{\partial x_j} = 0 \Rightarrow \quad \frac{\partial(\delta k_M)}{\partial x_j} = \frac{\delta k_M}{k_M}\frac{\partial k_M}{\partial x_j} \Rightarrow$$

Perturbation Analysis

- D^k, D^ε?
- Boussinesq and neglecting quadratic perturbation terms gives
-

$$\underline{D^k} + \delta D^k = \frac{C_\mu}{\sigma_k} \frac{\partial}{\partial x_j} \left[\underline{\frac{k_M^2}{\varepsilon_M} \frac{\partial k_M}{\partial x_j}} + \frac{k_M^2}{\varepsilon_M} \frac{\partial(\delta k_M)}{\partial x_j} + \frac{2(\delta k_M) k_M}{\varepsilon_M} \frac{\partial k_M}{\partial x_j} \right]$$

Following [5], we assume that $\delta k_M / k_M$ does not vary in space, i.e.

$$\frac{\partial(\delta k_M / k_M)}{\partial x_j} = 0 \Rightarrow \quad \frac{\partial(\delta k_M)}{\partial x_j} = \frac{\delta k_M}{k_M} \frac{\partial k_M}{\partial x_j} \Rightarrow$$

$$\frac{\delta D^k}{D^k} = \frac{3\delta k_M}{k_M} \quad \frac{\delta D^\varepsilon}{D^\varepsilon} = \frac{2\delta k_M}{k_M}$$

Perturbation Analysis: Final Step

Perturbation Analysis: Final Step

- 7 Eqns, 7 Unknowns:

Perturbation Analysis: Final Step

- 7 Eqns, 7 Unknowns: The perturbation analysis gives [4, 2]

Perturbation Analysis: Final Step

- 7 Eqns, 7 Unknowns: The perturbation analysis gives [4, 2]

$$\frac{\delta\psi}{\psi} = \frac{3\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right), \quad \psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$$

Perturbation Analysis: Final Step

- 7 Eqns, 7 Unknowns: The perturbation analysis gives [4, 2]

$$\frac{\delta\psi}{\psi} = \frac{3\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right), \quad \psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$$

- For $\psi < C_{\varepsilon 2}/C_{\varepsilon 1}$: an increase in ψ (a decrease in Δ_{max}) $\Rightarrow$ negative δk_M (i.e. less modeled and more resolved)

Perturbation Analysis: Final Step

- 7 Eqns, 7 Unknowns: The perturbation analysis gives [4, 2]

$$\frac{\delta\psi}{\psi} = \frac{3\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right), \quad \psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$$

- For $\psi < C_{\varepsilon 2}/C_{\varepsilon 1}$: an increase in ψ (a decrease in Δ_{max}) $\Rightarrow$ negative δk_M (i.e. less modeled and more resolved)
- But for $\psi > C_{\varepsilon 2}/C_{\varepsilon 1}$ this relation is reversed.

Perturbation Analysis: Final Step

- 7 Eqns, 7 Unknowns: The perturbation analysis gives [4, 2]

$$\frac{\delta\psi}{\psi} = \frac{3\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right), \quad \psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$$

- For $\psi < C_{\varepsilon 2}/C_{\varepsilon 1}$: an increase in ψ (a decrease in Δ_{max}) $\Rightarrow$ negative δk_M (i.e. less modeled and more resolved)
- But for $\psi > C_{\varepsilon 2}/C_{\varepsilon 1}$ this relation is reversed.
- Hence we introduce a limit: $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1}$

Perturbation Analysis: Final Step

- 7 Eqns, 7 Unknowns: The perturbation analysis gives [4, 2]

$$\frac{\delta\psi}{\psi} = \frac{3\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right), \quad \psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$$

- For $\psi < C_{\varepsilon 2}/C_{\varepsilon 1}$: an increase in ψ (a decrease in Δ_{max}) $\Rightarrow$ negative δk_M (i.e. less modeled and more resolved)
- But for $\psi > C_{\varepsilon 2}/C_{\varepsilon 1}$ this relation is reversed.
- Hence we introduce a limit: $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1} \equiv 1.9/1.5 = 1.27$

Partition of Turbulent Kinetic Energy

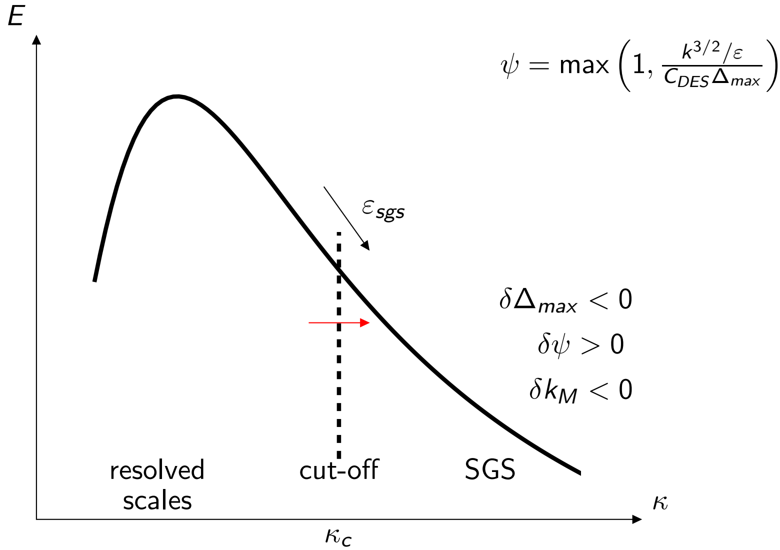


Figure: Energy spectrum.

Perturbation Analysis: Retaining Convection

Perturbation Analysis: Retaining Convection

- Above we set the convection to zero, i.e.

$$\begin{aligned}\frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0 \\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0\end{aligned}$$

Perturbation Analysis: Retaining Convection

- Above we set the convection to zero, i.e.

$$\begin{aligned} \frac{dk_M}{dt} &= P^k + D^k - \psi \varepsilon_M = 0 \\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1} \frac{\varepsilon_M}{k_M} P^k + D^\varepsilon - C_{\varepsilon 2} \frac{\varepsilon_M^2}{k_M} = 0 \end{aligned}$$

- Now let's keep it and neglect the diffusion ($D^k = D^\varepsilon = 0$), i.e.

$$P^k - C^k - \psi \varepsilon_M = 0, \quad C_{\varepsilon 1} \frac{\varepsilon_M}{k_M} P^k - C^\varepsilon - C_{\varepsilon 2} \frac{\varepsilon_M^2}{k_M} = 0$$

Perturbation Analysis: Retaining Convection

- Above we set the convection to zero, i.e.

$$\begin{aligned}\frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0\\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0\end{aligned}$$

- Now let's keep it and neglect the diffusion ($D^k = D^\varepsilon = 0$), i.e.

$$P^k - C^k - \psi\varepsilon_M = 0, \quad C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k - C^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0$$

- The perturbation analysis gives [2]

$$\frac{\delta\psi}{\psi} = \frac{\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right) \qquad D^k = D^\varepsilon = 0$$

Perturbation Analysis: Retaining Convection

- Above we set the convection to zero, i.e.

$$\begin{aligned}\frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0\\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0\end{aligned}$$

- Now let's keep it and neglect the diffusion ($D^k = D^\varepsilon = 0$), i.e.

$$P^k - C^k - \psi\varepsilon_M = 0, \quad C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k - C^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0$$

- The perturbation analysis gives [2]

$$\begin{aligned}\frac{\delta\psi}{\psi} &= \frac{\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right) && D^k = D^\varepsilon = 0\\ \frac{\delta\psi}{\psi} &= 3\frac{\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right) && C^k = C^\varepsilon = 0\end{aligned}$$

Perturbation Analysis: Retaining Convection

- Above we set the convection to zero, i.e.

$$\begin{aligned}\frac{dk_M}{dt} &= P^k + D^k - \psi\varepsilon_M = 0\\ \frac{d\varepsilon_M}{dt} &= C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0\end{aligned}$$

- Now let's keep it and neglect the diffusion ($D^k = D^\varepsilon = 0$), i.e.

$$P^k - C^k - \psi\varepsilon_M = 0, \quad C_{\varepsilon 1}\frac{\varepsilon_M}{k_M}P^k - C^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon_M^2}{k_M} = 0$$

- The perturbation analysis gives [2]

$$\begin{aligned}\frac{\delta\psi}{\psi} &= \frac{\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right) & D^k &= D^\varepsilon = 0\\ \frac{\delta\psi}{\psi} &= 3\frac{\delta k_M}{k_M}\left(1 - \frac{C_{\varepsilon 2}}{C_{\varepsilon 1}\psi}\right) & C^k &= C^\varepsilon = 0\end{aligned}$$

- Same limit: $\psi \le C_{\varepsilon 2}/C_{\varepsilon 1}$

The Turbulence Model

The Turbulence Model

- The AKN low-Reynolds number of Abe *et al.* [1] is used as underlying RANS model

$$
\begin{aligned}
\frac{\partial k}{\partial t} + \frac{\partial \bar{u}_j k}{\partial x_j} &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \psi\varepsilon \\
\frac{\partial \varepsilon}{\partial t} + \frac{\partial \bar{u}_j \varepsilon}{\partial x_j} &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} f_1 P_k \frac{\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \\
C_{\varepsilon 1} &= 1.5, \quad C_{\varepsilon 2} = 1.9, \quad C_\mu = 0.09 \\
\nu_t &= C_\mu f_\mu \frac{k^2}{\varepsilon}, \quad \sigma_k = 1.4, \quad \sigma_\varepsilon = 1.4
\end{aligned}
$$

The Turbulence Model

- The AKN low-Reynolds number of Abe *et al.* [1] is used as underlying RANS model

$$
\begin{aligned}
\frac{\partial k}{\partial t} + \frac{\partial \bar{u}_j k}{\partial x_j} &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \psi\varepsilon \\
\frac{\partial \varepsilon}{\partial t} + \frac{\partial \bar{u}_j \varepsilon}{\partial x_j} &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} f_1 P_k \frac{\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \\
C_{\varepsilon 1} &= 1.5, \quad C_{\varepsilon 2} = 1.9, \quad C_\mu = 0.09 \\
\nu_t &= C_\mu f_\mu \frac{k^2}{\varepsilon}, \quad \sigma_k = 1.4, \quad \sigma_\varepsilon = 1.4
\end{aligned}
$$

- together with IDDES of Shur *et. al* [10]
- With $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1}$ is called IDDES-PC (Partition Control)

The Numerical Method

The Numerical Method

- The Python finite volume code pyCALC-LES [3] is used.

The Numerical Method

- The Python finite volume code pyCALC-LES [3] is used.
- Fully vectorized (i.e. no `for` loops).

The Numerical Method

- The Python finite volume code pyCALC-LES [3] is used.
- Fully vectorized (i.e. no `for` loops).
- Fractional step. For velocities, second-order central differencing in space and Crank-Nicolson in time.

The Numerical Method

- The Python finite volume code pyCALC-LES [3] is used.
- Fully vectorized (i.e. no `for` loops).
- Fractional step. For velocities, second-order central differencing in space and Crank-Nicolson in time.
- For k and ε, hybrid central/upwind scheme

The Numerical Method

- The Python finite volume code pyCALC-LES [3] is used.
- Fully vectorized (i.e. no `for` loops).
- Fractional step. For velocities, second-order central differencing in space and Crank-Nicolson in time.
- For k and ε, hybrid central/upwind scheme
- The discretized equations are solved with Python sparse matrix solvers.

The Numerical Method

- The Python finite volume code pyCALC-LES [3] is used.
- Fully vectorized (i.e. no `for` loops).
- Fractional step. For velocities, second-order central differencing in space and Crank-Nicolson in time.
- For k and ε, hybrid central/upwind scheme
- The discretized equations are solved with Python sparse matrix solvers.
- The pyAMG solver [9] is used for the pressure Poisson equation (very efficient)

The Numerical Method

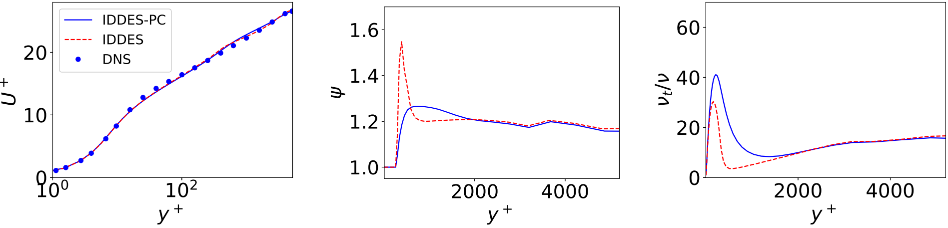
- The Python finite volume code pyCALC-LES [3] is used.
- Fully vectorized (i.e. no `for` loops).
- Fractional step. For velocities, second-order central differencing in space and Crank-Nicolson in time.
- For k and ε, hybrid central/upwind scheme
- The discretized equations are solved with Python sparse matrix solvers.
- The pyAMG solver [9] is used for the pressure Poisson equation (very efficient)

Channel flow, periodic b.c.

- $Re_\tau = u_\tau h/\nu = 5\,200$
- The mesh has $32 \times 96 \times 32$ $(x,\ y,\ z)$ cells $\Rightarrow$ $(\Delta x^+, \Delta z^+) = (800, 400)$.

Channel flow, periodic b.c.

- $Re_\tau = u_\tau h/\nu = 5\,200$
- The mesh has $32 \times 96 \times 32$ $(x,\ y,\ z)$ cells $\Rightarrow$ $(\Delta x^+, \Delta z^+) = (800, 400)$.



(A) Mean velocity. (B) ψ (C) Turbulent viscosity

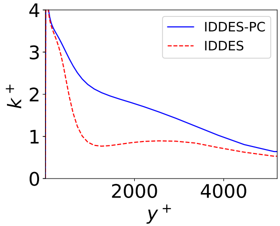
Figure: : IDDES-PC model; : IDDES; Markers: DNS [8]

Channel flow inlet-outlet

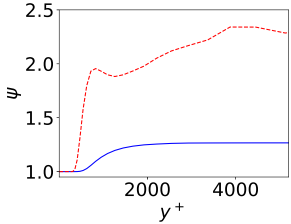
- RANS inlet values on k and ε.

Channel flow inlet-outlet

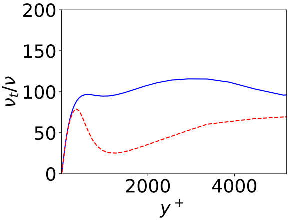
- RANS inlet values on k and ε.



(A) Modeled k.



(B) ψ.



(C) Turbulent viscosity.

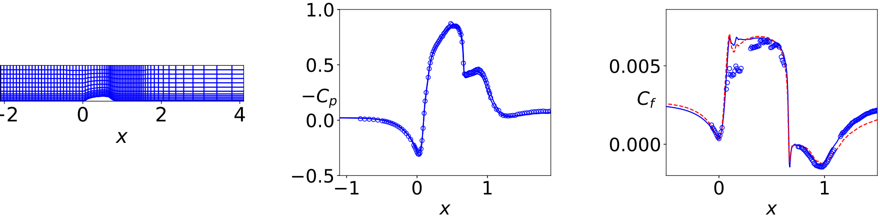
Figure: Profiles at $x = \delta$, i.e. one half-channel width.

HUMP FLOW

- The Reynolds number is $Re_c = 936\,000$
- The spanwise extent is $z_{max} = 0.2$.
- The mesh has $582 \times 128 \times 32$ cells $(x,\ y,\ z)$

HUMP FLOW

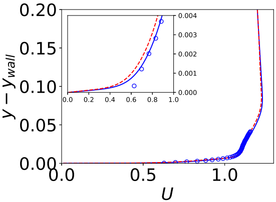
- The Reynolds number is $Re_c = 936\,000$
- The spanwise extent is $z_{max} = 0.2$.
- The mesh has $582 \times 128 \times 32$ cells $(x,\ y,\ z)$



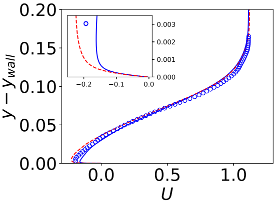
(A) Grid. (B) Pressure coefficient. (C) Skin friction

FIGURE: ━━ : IDDES-PC model; ━ ━ . IDDES. Markers: experiments [7, 6]

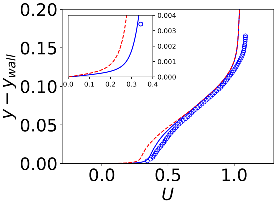
HUMP FLOW. VELOCITIES.



(A) $x = 0.65$.



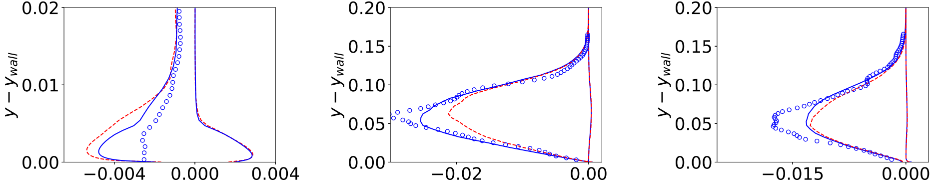
(B) $x = 1.0$.



(C) $x = 1.30$.

FIGURE: —— : IDDES-PC model; - - . IDDES. Markers: experiments [7, 6]

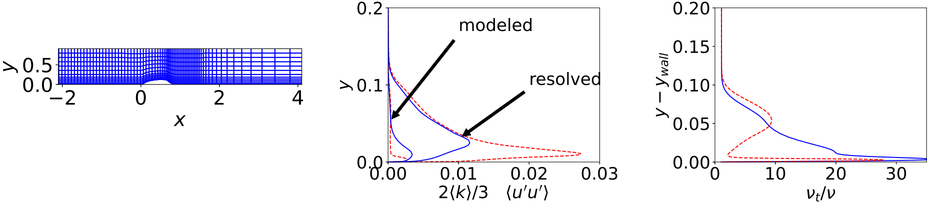
hump flow. Shear stresses.



(A) $x = 0.65$. (B) $x = 1.0$. (C) $x = 1.30$.

Figure: ——— : IDDES-PC model; – – . IDDES. Markers: experiments [7, 6]

HUMP FLOW. TURBULENT VISCOSITY AND $\langle u'u' \rangle$.



(A) Grid. (B) Streamwise fluctuations. (C) Turbulent viscosity.

FIGURE: $x = 0$. —— : IDDES-PC model; – – . IDDES.

Conclusions

- A limiter on $\psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$ is proposed which reads

Conclusions

- A limiter on $\psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$ is proposed which reads
- $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1}$

Conclusions

- A limiter on $\psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$ is proposed which reads
- $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1} \equiv 1.9/1.5 = 1.27$

Conclusions

- A limiter on $\psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$ is proposed which reads
- $\psi \le C_{\varepsilon 2}/C_{\varepsilon 1} \equiv 1.9/1.5 = 1.27$
- The limiter is derived using perturbation analysis which assumes

$$\frac{\partial(\delta k_M)}{\partial x_j} = \frac{\delta k_M}{k_M}\frac{\partial k_M}{\partial x_j}$$

Conclusions

- A limiter on $\psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$ is proposed which reads
- $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1} \equiv 1.9/1.5 = 1.27$
- The limiter is derived using perturbation analysis which assumes

$$\frac{\partial(\delta k_M)}{\partial x_j} = \frac{\delta k_M}{k_M}\frac{\partial k_M}{\partial x_j}$$

- The limiter is shown to reduce $\frac{\partial(\delta k_M)}{\partial x_j}$

Conclusions

- A limiter on $\psi = \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)$ is proposed which reads
- $\psi \leq C_{\varepsilon 2}/C_{\varepsilon 1} \equiv 1.9/1.5 = 1.27$
- The limiter is derived using perturbation analysis which assumes

$$\frac{\partial(\delta k_M)}{\partial x_j} = \frac{\delta k_M}{k_M}\frac{\partial k_M}{\partial x_j}$$

- The limiter is shown to reduce $\frac{\partial(\delta k_M)}{\partial x_j}$
- Good results are obtained (better than standard IDDES for the hump flow)

References I

[1] K. Abe, T. Kondoh, and Y. Nagano. A new turbulence model for predicting fluid flow and heat transfer in separating and reattaching flows - 1. Flow field calculations. *Int. J. Heat Mass Transfer*, 37(1):139–151, 1994.

[2] L. Davidson. Fluid mechanics, turbulent flow and turbulence modeling. eBook, Division of Fluid Dynamics, Dept. of Mechanics and Maritime Sciences, Chalmers University of Technology, Gothenburg, 2021.

[3] L. Davidson. pyCALC-LES: a Python code for DNS, LES and Hybrid LES-RANS. Division of Fluid Dynamics, Dept. of Mechanics and Maritime Sciences, Chalmers University of Technology, Gothenburg, 2021.

[4] C. Friess and L. Davidson. A formulation of PANS able to mimic IDDES. *International Journal of Heat and Fluid Flow*, 86(108666), 2020.

References II

[5] C. Friess, R. Manceau, and T.B. Gatski. Toward an equivalence criterion for hybrid RANS/LES methods. *International Journal of Heat and Fluid Flow*, 122:233–246, 2015.

[6] D. Greenblatt, K. B. Paschal, C.-S. Yao, and J. Harris. A separation control CFD validation test case Part 1: Zero efflux oscillatory blowing. AIAA-2005-0485, 2005.

[7] D. Greenblatt, K. B. Paschal, C.-S. Yao, J. Harris, N. W. Schaeffler, and A. E. Washburn. A separation control CFD validation test case. Part 1: Baseline & steady suction. AIAA-2004-2220, 2004.

[8] M. Lee and R. D. Moser. Direct numerical simulation of turbulent channel flow up to $Re_\tau \approx 5200$. *Journal of Fluid Mechanics*, 774:395–415, 2015.

[9] L. N. Olson and J. B. Schroder. PyAMG: Algebraic multigrid solvers in Python v4.0, 2018. Release 4.0.

References III

[10] M. L. Shur, P. R. Spalart, M. Kh. Strelets, and A. K. Travin. A hybrid RANS-LES approach with delayed-DES and wall-modelled LES capabilities. *International Journal of Heat and Fluid Flow*, 29:1638–1649, 2008.