Backscatter from a scale-similarity model: embedded LES of channel flow, developing boundary layer flow and backstep flow [2]
Lars Davidson

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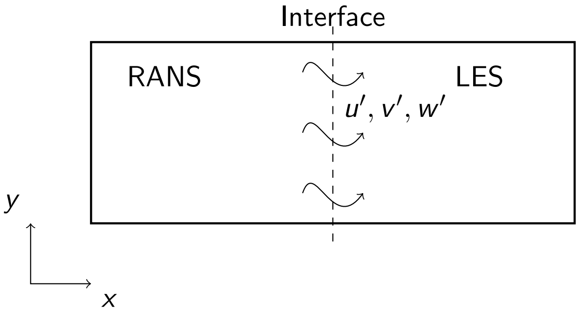
Embedded LES: Problem Formulation



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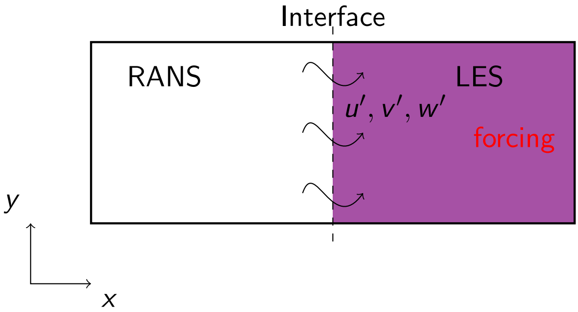

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- To promote transition from RANS to LES (reducing the gray area), additional forcing may be used in the LES region

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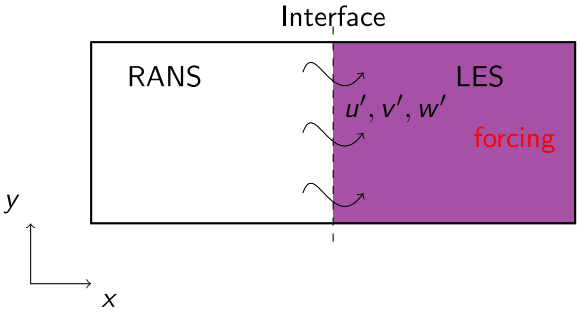

- At the interface between RANS and LES, turbulent fluctuations, u', v', w', are imposed to stimulate growth of resolved fluctuations
- To promote transition from RANS to LES (reducing the gray area), additional forcing may be used in the LES region
- In the present work, forcing is added using a scale-similarity model

Momentum Equation

The momentum equations for LES read

$$\frac{D\bar{u}_i}{Dt} + \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} = \frac{\partial}{\partial x_k}\left((\nu + \nu_{SGS})\frac{\partial \bar{u}_i}{\partial x_k}\right) - \frac{\partial \tau_{ik}}{\partial x_k}$$

where D/Dt denotes material derivative. The stress tensor, τ_{ik}, is obtained from the scale-similarity model

$$\tau_{ik} = \overline{\bar{u}_i \bar{u}_k} - \bar{\bar{u}}_i \, \bar{\bar{u}}_k$$

Turbulent Kinetic Energy Eq

- Let us take a closer look at the equation for the resolved, turbulent kinetic energy, $K = \langle \bar{u}'_i \bar{u}'_i \rangle / 2$, which reads ($\langle . \rangle$ denotes averaging in time)

$$\frac{DK}{Dt} + \langle \bar{u}'_k \bar{u}'_i \rangle \frac{\partial \langle \bar{u}_i \rangle}{\partial x_k} \quad + \quad \frac{1}{\rho} \frac{\partial \langle \bar{p}' \bar{u}'_i \rangle}{\partial x_i} + \frac{1}{2} \frac{\partial \langle \bar{u}'_k \bar{u}'_i \bar{u}'_i \rangle}{\partial x_k} = \nu \left\langle \frac{\partial^2 \bar{u}'_i}{\partial x_k \partial x_k} \bar{u}'_i \right\rangle \quad - \quad \left\langle \left(\frac{\partial \tau_{ik}}{\partial x_k} - \left\langle \frac{\partial \tau_{ik}}{\partial x_k} \right\rangle \right) \bar{u}'_i \right\rangle$$

- The second line is simply the $\bar{u}'_i$ eq. multiplied by $\bar{u}'_i$

Turbulent Kinetic Energy Eq (cont'd)

- The right side can be re-written as

$$\underbrace{\nu\left\langle\frac{\partial^2\bar{u}'_i}{\partial x_k\partial x_k}\bar{u}'_i\right\rangle}_{\varepsilon^{non}}-\left\langle\frac{\partial\tau_{ik}}{\partial x_k}\bar{u}'_i\right\rangle=$$

$$\nu\frac{\partial^2 K}{\partial x_k\partial x_k}-\underbrace{\nu\left\langle\frac{\partial\bar{u}'_i}{\partial x_k}\frac{\partial\bar{u}'_i}{\partial x_k}\right\rangle}_{\varepsilon}-\underbrace{\left\langle\frac{\partial\tau_{ik}}{\partial x_k}\bar{u}'_i\right\rangle}_{\varepsilon_{SGS}}$$

- The first term on the left side is the non-isotropic (i.e. the true) viscous dissipation, ε^{non}; this is predominately negative.

Turbulent Kinetic Energy Eq (cont'd)

- The right side can be re-written as

$$\underbrace{\nu\left\langle \frac{\partial^2 \bar{u}'_i}{\partial x_k \partial x_k}\bar{u}'_i\right\rangle}_{\varepsilon^{non}} - \left\langle \frac{\partial \tau_{ik}}{\partial x_k}\bar{u}'_i\right\rangle =$$

$$\nu\frac{\partial^2 K}{\partial x_k \partial x_k} - \underbrace{\nu\left\langle \frac{\partial \bar{u}'_i}{\partial x_k}\frac{\partial \bar{u}'_i}{\partial x_k}\right\rangle}_{\varepsilon} - \underbrace{\left\langle \frac{\partial \tau_{ik}}{\partial x_k}\bar{u}'_i\right\rangle}_{\varepsilon_{SGS}}$$

- The first term on the left side is the non-isotropic (i.e. the true) viscous dissipation, ε^{non}; this is predominately negative.
- The first term on the right side is the viscous diffusion

Turbulent Kinetic Energy Eq (cont'd)

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- The first term on the left side is the non-isotropic (i.e. the true) viscous dissipation, ε^{non}; this is predominately negative.
- The first term on the right side is the viscous diffusion
- the second term, ε, is the (isotropic) dissipation which is positive

Turbulent Kinetic Energy Eq (cont'd)

- The right side can be re-written as

$$\underbrace{\nu \left\langle \frac{\partial^2 \bar{u}_i'}{\partial x_k \partial x_k} \bar{u}_i' \right\rangle}_{\varepsilon^{non}} - \left\langle \frac{\partial \tau_{ik}}{\partial x_k} \bar{u}_i' \right\rangle =$$

$$\nu \frac{\partial^2 K}{\partial x_k \partial x_k} - \underbrace{\nu \left\langle \frac{\partial \bar{u}_i'}{\partial x_k} \frac{\partial \bar{u}_i'}{\partial x_k} \right\rangle}_{\varepsilon} - \underbrace{\left\langle \frac{\partial \tau_{ik}}{\partial x_k} \bar{u}_i' \right\rangle}_{\varepsilon_{SGS}}$$

- The first term on the left side is the non-isotropic (i.e. the true) viscous dissipation, ε^{non}; this is predominately negative.
- The first term on the right side is the viscous diffusion
- the second term, ε, is the (isotropic) dissipation which is positive
- The last term, ε_{SGS}, can be positive (forward scattering=dissipation) or negative (backward scattering=forcing).

Physical Interpretation

- The SGS term

$$\varepsilon_{SGS} = \left\langle \frac{\partial \tau_{ik}}{\partial x_k} \bar{u}_i' \right\rangle$$

consists of a net SGS force vector, T_i^{SGS}, (per unit mass), multiplied by a velocity fluctuation vector, $\bar{u}_i'$ i.e.

$$\varepsilon_{SGS} = \left\langle T_i^{SGS} \bar{u}_i' \right\rangle$$

- When the SGS vector, T_i^{SGS}, opposes the fluctuation, $\bar{u}_i'$, it is damping the fluctuation, i.e. it is dissipative

Select Forward or Backscatter

- We want to be able to make the term ε_{SGS} dissipative or forcing

$$\underbrace{\nu \left\langle \frac{\partial^2 \bar{u}'_i}{\partial x_k \partial x_k} \bar{u}'_i \right\rangle}_{\varepsilon^{non}} - \left\langle \frac{\partial \tau_{ik}}{\partial x_k} \bar{u}'_i \right\rangle =$$

$$\nu \frac{\partial^2 K}{\partial x_k \partial x_k} - \underbrace{\nu \left\langle \frac{\partial \bar{u}'_i}{\partial x_k} \frac{\partial \bar{u}'_i}{\partial x_k} \right\rangle}_{\varepsilon} - \underbrace{\left\langle \frac{\partial \tau_{ik}}{\partial x_k} \bar{u}'_i \right\rangle}_{\varepsilon_{SGS}}$$

- The viscous term in the mom. eq., $\nu \left\langle \frac{\partial^2 \bar{u}'_i}{\partial x_k \partial x_k} \right\rangle$, is dissipative
- If $-\frac{\partial \tau_{ik}}{\partial x_k}$ has the same sign as $\frac{\partial^2 \bar{u}'_i}{\partial x_k \partial x_k}$, then ε_{SGS} is dissipative
- Otherwise, it is a forcing term (backscatter)

Select Backscatter Events

- We want the SGS stress tensor to act as backscatter in the K equation.
- Hence we add $-\partial\tau_{ik}/\partial x_k$ to the momentum equation only when its sign is opposite to that of the viscous diffusion term. i.e. [1]

$$M_{ik} = \mathrm{sign}\left(\frac{\partial\tau_{ik}}{\partial x_k}\frac{\partial^2\bar{u}_i'}{\partial x_k\partial x_k}\right), \quad \widetilde{M}_{ik} = \max(M_{ik},0), \quad \left(\frac{\partial\tau_{ik}}{\partial x_k}\right)^- = -\widetilde{M}_{ik}\frac{\partial\tau_{ik}}{\partial x_k}$$

$$\frac{\partial^2 \bar{u}_i'}{\partial x_k \partial x_k} \text{ vs. } \frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_k}$$

$$M_{ik} = \mathrm{sign}\left(\frac{\partial \tau_{ik}}{\partial x_k}\frac{\partial^2 \bar{u}_i'}{\partial x_k \partial x_k}\right), \quad \widetilde{M}_{ik} = \max(M_{ik}, 0), \quad \left(\frac{\partial \tau_{ik}}{\partial x_k}\right)^- = -\widetilde{M}_{ik}\frac{\partial \tau_{ik}}{\partial x_k}$$

- $\bar{u}_i'$, is not known at run-time. It could be computed as $\bar{u}_i' = \bar{u}_i - \langle \bar{u}_i \rangle_{ra}$, where $\langle \bar{u}_i \rangle_{ra}$ denotes the running-time average of $\bar{u}_i$.
- It was shown in [1] that, for $y^+ \gtrsim 20$ in channel flow, the second derivative of $\bar{u}_i'$ is almost 100% correlated with that of $\bar{u}_i$
- Hence, in the present work, the relation at the top-left is replaced by

$$M_{ik} = \mathrm{sign}\left(\frac{\partial \tau_{ik}}{\partial x_k}\frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_k}\right)$$

Stability

- The forcing has a positive feedback, i.e. the more the momentum eq is destabilized, the larger the velocity gradients, the larger the forcing
- Hence, the forcing term has to be limited

$$\left|-\frac{\partial \tau_{ik}}{\partial x_k}\right| \leq \beta(\nu + \nu_{SGS})\left|\frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_k}\right|$$

The baseline value is $\beta = 2$.

PANS Low Reynolds Number Model [3]

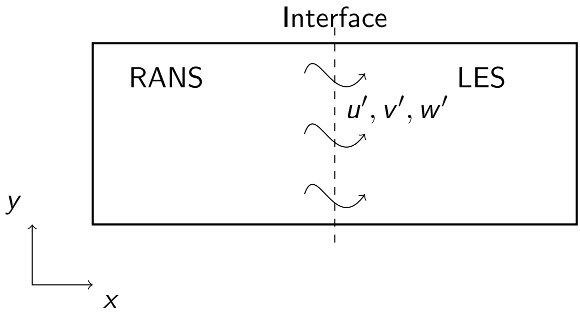
$$\frac{\partial k}{\partial t} + \frac{\partial (kU_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{ku}}\right)\frac{\partial k}{\partial x_j}\right] + (P - \varepsilon)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial (\varepsilon U_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} P \frac{\varepsilon}{k} - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

$$\nu_t = C_\mu f_\mu \frac{k^2}{\varepsilon}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1}), \sigma_{ku} \equiv \sigma_k \frac{f_k^2}{f_\varepsilon}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

- LRN Damping functions, f_2, f_μ as in [3]
- RANS region: $f_k = 1.0$
- LES region: i) $f_k = 0.4$ or ii) $f_k = \frac{1}{c_\mu^{1/2}}\left(\Delta / L_t\right)^{2/3}, L_t = (k_{res} + k)^{3/2}/\varepsilon$
- Option i and ii give same results. but Option ii unstable in backstep flow with forcing

Test Case I: Channel Flow



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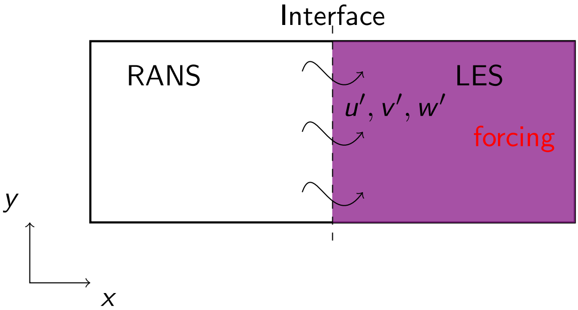

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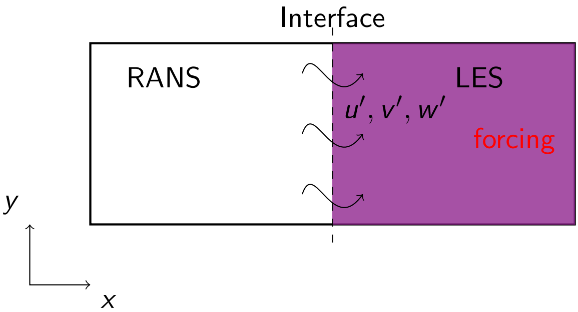
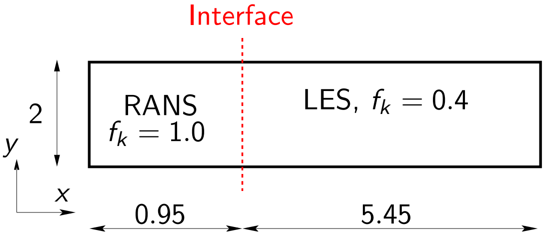


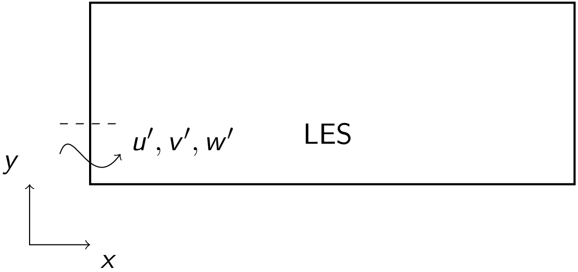


$Re_\tau = 950$ based on u_τ

$128 \times 80 \times 32$ (x, y, z) cells

$z_{max} = 1.6$

Test Case II: Boundary Layer Flow



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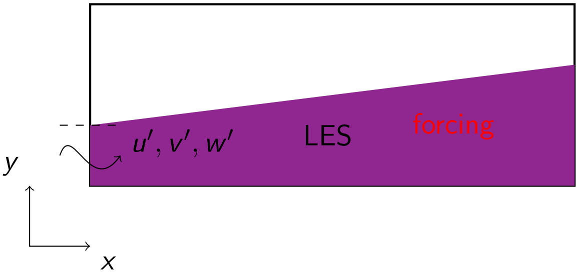

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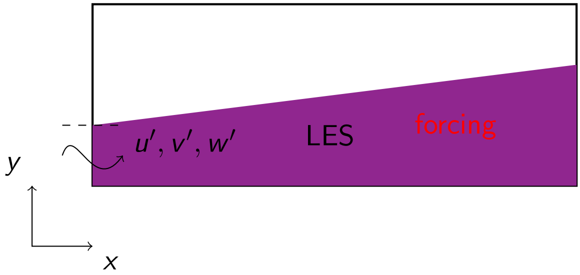
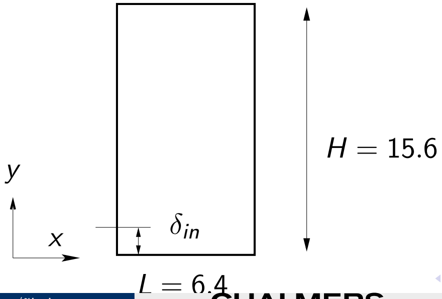
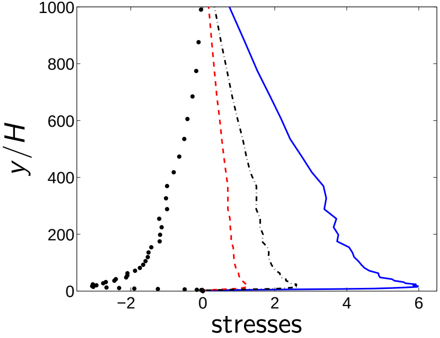
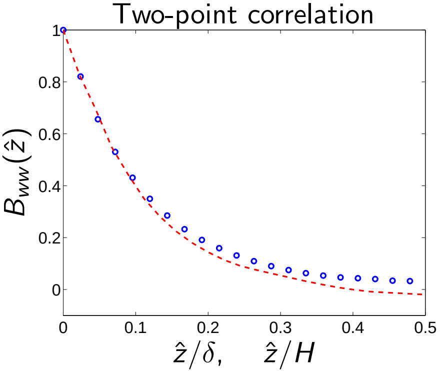




$\delta_{in} = 1, \; Z_{max} = 1.6$
$Re_\theta = 3\,600$
$Re_\delta = U_{free}\delta_{in}/\nu$
$\simeq 28\,000$

Inlet turb. fluctuation, 2-point correlations



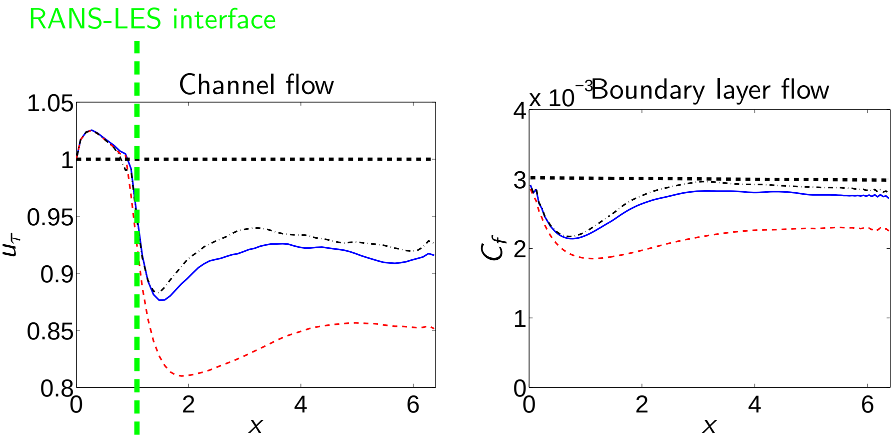


—— : u_{rms}^{+2}, - - - : v_{rms}^{+2}, - · - : w_{rms}^{+2}

· · · · : $\langle u'v' \rangle^{+}$

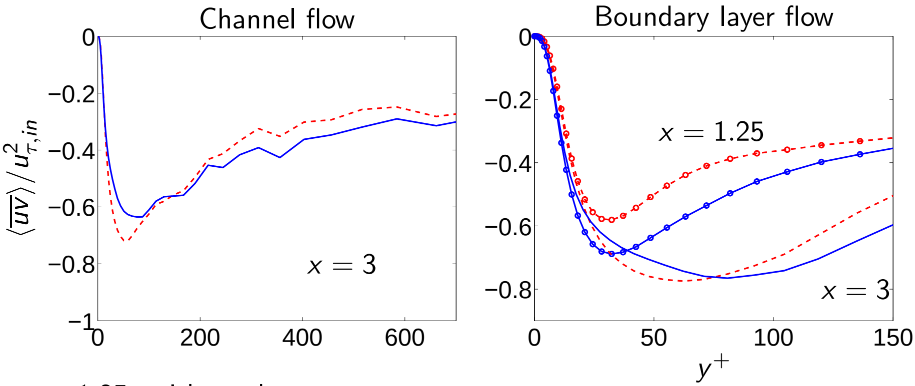
○: inlet; - - - : $x = 3\delta_{in}$

Results: Skin Friction



- backscatter
- no backscatter
- backscatter with $\beta = 3$ (limiter)
- :target value.

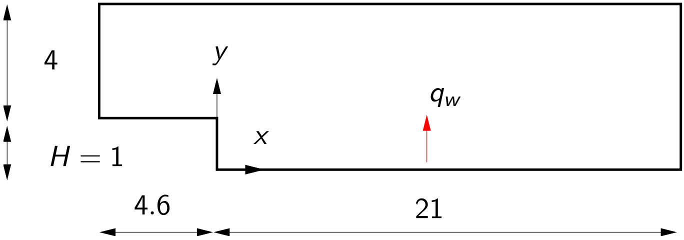
Results: Resolved Shear Stresses



$x = 1.25$: with markers
$x = 3$: without markers
—— backscatter
- - - no backscatter.

Backstep flow, computational domain

- $Re_H = 28\,000$, $336 \times 152 \times 64$ cells (x, y, z), $z_{max} = 1.6$



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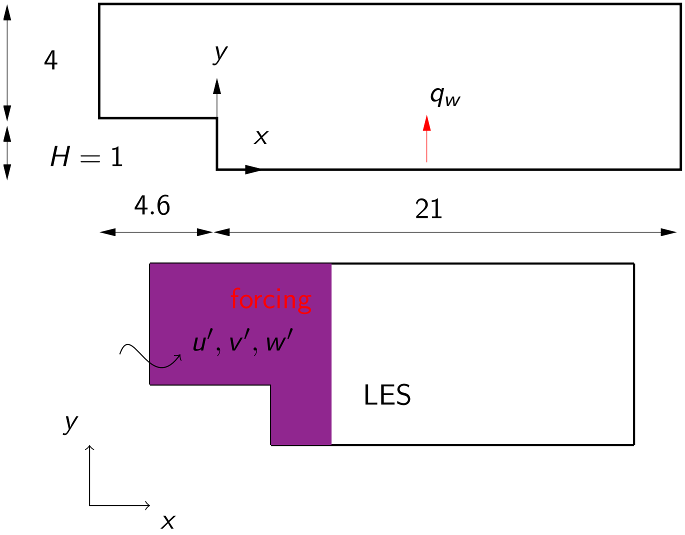
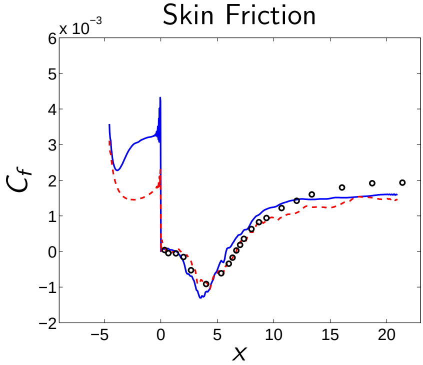
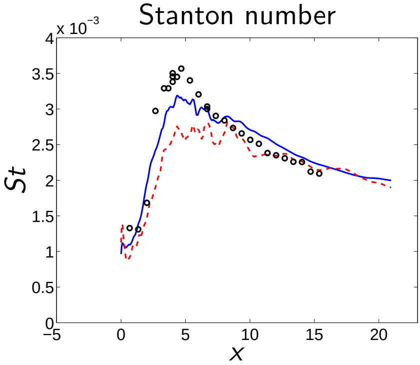


Skin friction and St number





—— backscatter

- - - no backscatter

○: Experiments by Vogel & Eaton [4]

Conclusions

- The gray area issue at RANS-LES interface has been addressed
- The stresses, τ_{ik}, from a scale-similarity model was used for forcing
- The forcing was achieved be selecting the instants when $-\dfrac{\partial \tau_{ik}}{\partial x_k}$ corresponds to backscatter
- It is found that the forcing indeed quickens the transition from RANS mode to LES mode
- The present approach can also be used for laminar-turbulent transition

Three-Day CFD Course at Chalmers

- Unsteady Simulations for Industrial Flows: LES, DES, hybrid LES-RANS and URANS
- 6-8 November 2013 at Chalmers, Gothenburg, Sweden
- Max 16 participants
- 50% lectures and 50% workshops in front of a PC
- Registration deadline: 18 October 2013
- For info, see http://www.tfd.chalmers.se/~lada/cfdkurs/cfdkurs.html

References I

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[3] Ma, J., Peng, S.-H., Davidson, L., and Wang, F.
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