Problem 7.1

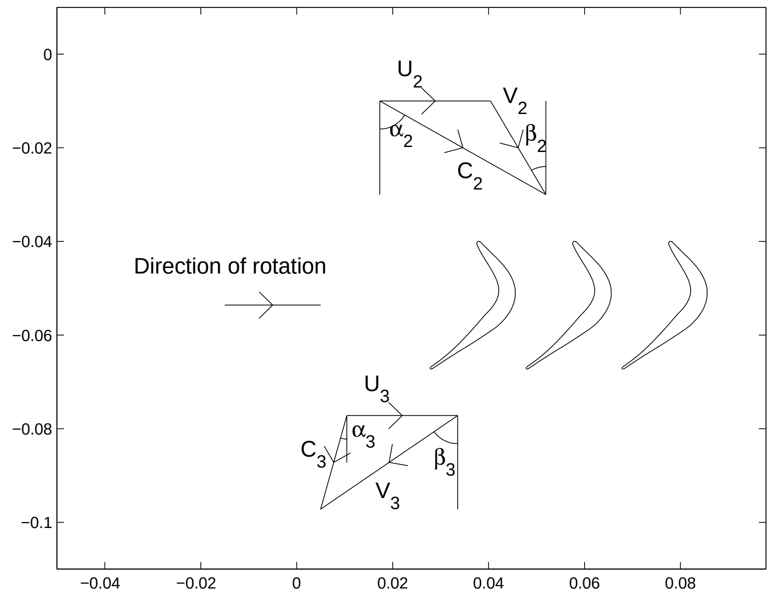

Figure 1: Rotor with air and blade angles

Problem definition: Calculate

- Rotor blade angles
- Degree of reaction
- Temperature drop coefficient
- Power output

Solution: Some trigonometric relations give:

$$C_2 = \frac{C_a}{cos(\alpha_2)} = 615.2\ m/s$$

$$C_{w2} = \sqrt{{C_2}^2 - {C_a}^2} = 557.5718\ m/s$$

$$\beta_2 = arctan(\frac{C_{w2} - U}{C_a}) = ... = 37.23$$

$$C_{w3} = C_a tan(\alpha_3) = 45.84\ m/s$$

$$\beta_3 = arctan(\frac{C_{w3} + U}{C_a}) = ... = 57.35$$

Since it was given that $C_1 = C_3$ and C_a is constant Equation 7.7 can be used to compute the degree of reaction:

$$\Lambda = \frac{C_a}{2U}(tan\beta_3 - tan\beta_2) = ... = 0.2892$$

The temperature drop coefficient is obtained from:

$$\Psi = \frac{2c_p \Delta T_{0,s}}{U^2} = \frac{2U\Delta C_w}{U^2} = ... = 3.3523$$

The power output is obtained from:

$$\text{Power output} = mU\Delta C_w = 4344.6$$

The throat area is obtained from the nozzle coefficient $\lambda_N = 0.05$ according to:

$$T_2 = [T_{02} = T_{01}] = T_{01} - \frac{{C_2}^2}{2c_p} = 835\ K$$

$$T_2' = T_2 - \lambda_N \frac{{C_2}^2}{2c_p} = 826.9\ K$$

The pressure ratio gives p_2 according to:

$$\frac{P_{01}}{P_2} = (\frac{T_{01}}{T_2'})^{\frac{\gamma}{\gamma-1}} = 2.140$$

$$P_2 = \frac{P_{01}}{\frac{P_{01}}{P_2}} = 1.869\ bar$$

For isentropic flow, the critical pressure ratio is $r_{crit} = 1.852$. Thus, the nozzle is choking. The throat conditions are then:

$$P_c = \frac{P_{01}}{r_{crit}} = 2.159$$

$$T_c = \frac{2}{\gamma + 1} T_{01} = 857.3\ K$$

$$\rho = \frac{P_c}{RT_c} = 0.8777\ kg/m^3$$

$$C_c = \sqrt{\gamma RT_c} = 572.7\ m/s$$

$$A_{throat} = \frac{m}{\rho_c C_c} = 0.03979\ m^2$$