# Problem 5.1

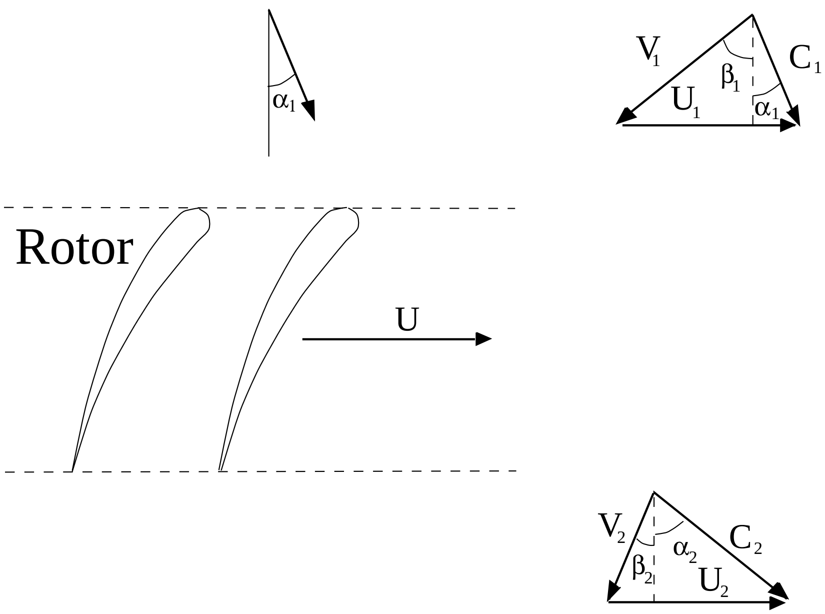


Figure 1: Rotor with air and blade angles

**Problem definition: Calculate**

- Air angles at root mean and tip
- Degree of reaction at root and tip

**Solution:** The stage temperature rise is (C.R.S. 194):

$$T_{02} - T_{01} = \frac{\lambda U C_a}{c_p}(\tan\beta_1 - \tan\beta_2) \tag{1}$$

The degree of reaction is (C.R.S. 196):

$$\Lambda = \frac{\text{Static enthalpy rise in rotor}}{\text{Static enthalpy rise in stage}} = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2) \tag{2}$$

Evaluated at the mean radius 1 and 2 gives:

$$\tan\beta_1 - \tan\beta_2 = 0.7204$$
$$\tan\beta_1 + \tan\beta_2 = 1.3333$$

$$\Longleftrightarrow$$

$$\beta_1 = 45.8^\circ$$
$$\beta_2 = 17.0^\circ$$

$\Lambda = 0.50$ designs are symmetrical (C.R.S. 197), i.e.

$$\alpha_2 = \beta_1 = 45.8^\circ$$
$$\alpha_1 = \beta_2 = 17.0^\circ$$

Since this is a free vortex design we must have $C_w r$ = constant. At the mean radius we have:

$$\Delta T_0 = \frac{\lambda U}{c_p} C_a \left( tan\alpha_2 - tan\alpha_1 \right) = \frac{\lambda U}{c_p} \left( C_{w2} - C_{w1} \right) = \frac{\lambda U}{c_p} \Delta C_w$$

which gives:

$$\frac{c_p \Delta T_0}{\lambda U} = \frac{1005 \cdot 20}{0.93 \cdot 200} = \Delta C_w = ... = 108.1 \text{ m/s} \tag{3}$$

where $C_{w1}$ is:

$$C_{w1} = C_a \tan\alpha_1 = 150 \cdot \tan(17.0) = 45.97 \tag{4}$$

3 and 4 give:

$$C_{w2} = ... = 154.0 \text{ m/s}$$

The free vortex condition for the leading edge gives:

$$C_{w1,t} = \frac{C_{w1,m} r_m}{r_t} = \frac{C_{w1,m} U_m}{U_t} = 36.77 \text{ m/s}$$
$$C_{w1,r} = ... = 61.29 \text{ m/s}$$

The free vortex condition for the trailing edge gives:

$$C_{w2,t} = \frac{C_{w2,m} r_m}{r_t} = 123.23 \text{ m/s}$$
$$C_{w2,r} = ... = 205.38 \text{ m/s}$$

At the leading edge we have:

$$\tan\beta_1 = \frac{U - C_{w1}}{C_a} \tag{5}$$

Applied to the root and tip 5 gives:

$$\beta_{1,t} = ... = 54.87^\circ$$
$$\beta_{1,r} = ... = 30.60^\circ$$

For the trailing edge we have:

$$\tan\beta_2 = \frac{U - C_{w2}}{C_a} \tag{6}$$

Applied to the root and tip 6 gives:

$$\beta_{2,t} = ... = 40.20^\circ$$
$$\beta_{2,r} = ... = -20.26^\circ$$

The air angles are obtained from (C.R.S. 187):

$$\frac{U}{C_a} = tan\alpha_1 + tan\beta_1$$
$$\frac{U}{C_a} = tan\alpha_2 + tan\beta_2$$

which yields (for the leading edge root and tip):

$$\alpha_{1,r} = 22.23^\circ$$
$$\alpha_{1,t} = 13.78^\circ$$

and (for the trailing edge root and tip):

$$\alpha_{2,r} = 53.86^\circ$$
$$\alpha_{2,t} = 39.40^\circ$$

Finally, the degree of reaction is determined by 2:

$$\Lambda_r = \frac{150}{2 \cdot 150}(\tan(30.60) + \tan(-20.26)) = ... = 0.1111$$

$$\Lambda_t = \frac{150}{2 \cdot 250}(\tan(54.87) + \tan(40.20)) = ... = 0.6800$$