Problem 4.1

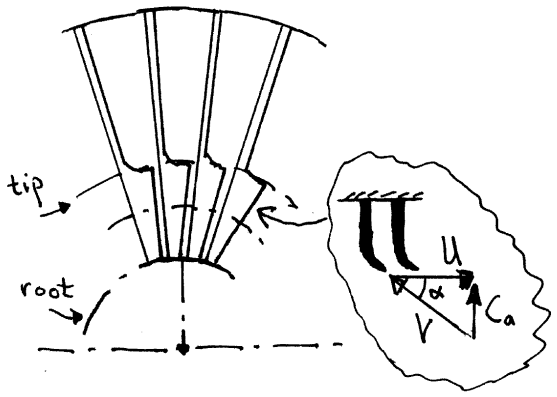

Figure 1: Impeller with impeller eye

Problem definition: Calculate

- Blade inlet angle at root and tip
- Mach number at tip

Solution: The 1D continuity equation reads (derived during Lecture 5 - continuity in stagnation property form):

$$\frac{\dot{m}\sqrt{RT_0}}{AP_0} = \sqrt{\gamma}M\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{1}$$

where only the Mach number is unknown! Plugging the given data into 1 yields:

$$\frac{\dot{m}\sqrt{RT_0}}{AP_0} = 0.4006 \tag{2}$$

(where it was used that $A = \pi(r_{tip}^2 - r_{root}^2)$)

$$\text{Iterate in M} \Longrightarrow$$

$$M = 0.367$$

The Mach number yields the static properties using the relations at page 448 in C.R.S, i.e. (also derived during Lecture 5 - Mach number relations for stagnation

properties):

$$\frac{T_0}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$
$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

using the Mach number determined above produces:

$$T = 280.5 \text{ K}$$
$$P = 0.911 \text{ bar}$$
$$\rho = \frac{P}{RT} = 1.132 \text{ kg/m}^3$$

Continuity gives:

$$C_a = 123 \text{ m/s}$$

The blade velocities are:

$$U_{root} = 2\pi r_{root} \cdot N = ... = 110 \text{ m/s}$$
$$U_{tip} = ... = 254 \text{ m/s}$$

The blade angles are:

$$\alpha_{root} = \arctan\left(\frac{C_a}{U_{root}}\right) = 48.19^\circ$$
$$\alpha_{tip} = \arctan\left(\frac{C_a}{U_{tip}}\right) = 25.80^\circ$$

The tip Mach number is:

$$M_{tip} = \frac{\sqrt{C_a^2 + U_{tip}^2}}{\sqrt{\gamma RT}} = ... = 0.84$$