Problem 3.2

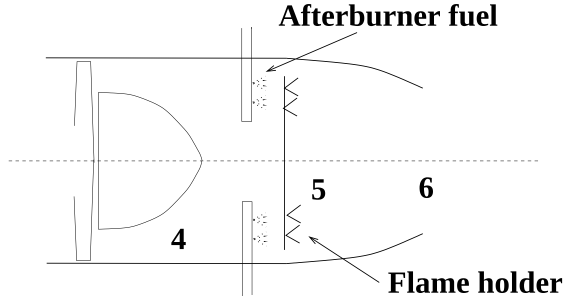

Figure 1: Propelling nozzle with afterburner

Problem definition: Calculate:

- Percentage increase in nozzle area required
- Percentage increase in net thrust

Solution: Is the nozzle still choked? P_{05} is now:

$$P_{05} = P_{04}(1 - 0.03) = 1.657 \text{ bar}$$

resulting in a pressure ratio over the nozzle of $\frac{P_{05}}{P_a} = 4.03$, which is still larger than the choking pressure ratio determined to 1.919 (Problem 3.1).
We have derived the equation for the compressible continuity equation:

$$\frac{m\sqrt{RT_0}}{AP_0} = \sqrt{\gamma}M\left(1 + \frac{\gamma - 1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} = \mathrm{X}(M, \gamma) = [Choked] = \mathrm{X}(\gamma)$$

Since γ is assumed not to change between the two cases we obtain:

$$\frac{A_{hot}}{A_{cold}} = \frac{\frac{\sqrt{T_{05,hot}}}{P_{05,hot}}}{\frac{\sqrt{T_{04,cold}}}{P_{04,cold}}} = \frac{1}{0.97}\frac{\sqrt{2000}}{\sqrt{960.6}} = 1.4875$$

Thus a 48.75% increase in area will be required. The net thrust is determined from:

$$F = \dot{m}(C_6 - C_a) + A_6(P_6 - P_a)$$

The new area is $A_6 = 1.4875A_5$. P_6 is $P_6 = \frac{P_{05}}{1.919} = 0.864\text{bar}$. C_6 is the speed of sound at T_6, i.e. $\sqrt{\gamma RT_6}$ where T_6 is determined from

$$\frac{T_{06}}{T_6} = \frac{\gamma + 1}{2}$$

which gives $T_6 = \frac{2000}{\frac{\gamma+1}{2}} = 1714.3\text{K}$. Plugging in the data in the thrust expression yields:

$$F = 12951.7\ N$$

Thus an increase in thrust with $\frac{12951.7 - 7870}{7870}$, i.e. 64.6% is noted.