Problem 3.1

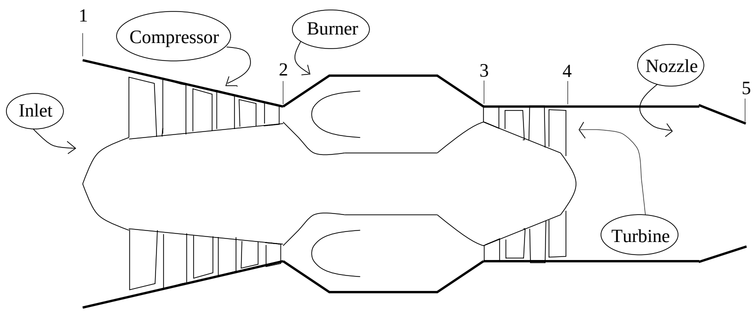

Figure 1: Turbojet without afterburner

Problem definition: Calculate:

- Propelling nozzle area
- Net thrust
- SFC

Solution: The International Standard Atmosphere (ISA) table at page 150 yields the ambient conditions (altitude = 7000 meter):

$$\begin{cases} T_a = 242.7 \text{ K} \\ P_a = 0.4165 \text{ bar} \end{cases}$$

The inlet: C.R.S. p. 106 gives:

$$T_{0a} = [\text{ adiabatic }] = T_{01} = T_a + \frac{C_a^2}{2c_p} = ... = 276.3 \text{ K}$$

and equation 3.10a produces:

$$P_{01} = P_a \left[1 + \eta_i \cdot \frac{C_a^2}{2c_p T_a}\right]^{\frac{\gamma_a}{\gamma_a - 1}} = ... = 0.642 \text{ bar}$$

The compressor: The pressure ratio was given $\Longrightarrow$

$$P_{02} = 8.0 \cdot P_{01} = 5.137 \text{ bar}$$

C.R.S. p. 60 gives:

$$T_{02} - T_{01} = T_{01}\left[\left(\frac{P_{02}}{P_{01}}^{\frac{\gamma_a - 1}{\eta_{\infty,c}\cdot\gamma_a}}\right) - 1\right]$$

$$\Longrightarrow$$

$$T_{02} = ... = 547.0 \text{ K}$$

The burner: The pressure drop and burner exit tempereture were given:

$$\begin{cases} P_{03} = 0.94 \cdot P_{02} = 4.83 \text{ bar} \\ T_{03} = 1200 \text{ K} \end{cases}$$

The turbine: The turbine/compressor work-exchange equality yields:

$$\frac{c_{p,a}}{\eta_m}(T_{02} - T_{01}) = c_{p,g}(T_{03} - T_{04})$$

$$\Longrightarrow$$

$$T_{04} = T_{03} - \frac{c_{p,a}}{\eta_m c_{p,g}}(T_{02} - T_{01}) = ... = 960.6 \text{ K}$$

P_{04} is obtained from 2.20:

$$T_{03} - T_{04} = T_{03}\left(1 - \frac{1}{\left(\frac{P_{03}}{P_{04}}\right)^{\frac{\eta_{\infty,t}\cdot\gamma_g}{\gamma_g - 1}}}\right)$$

$$\Longrightarrow$$

$$P_{04} = ... = 1.70 \text{ bar}$$

The nozzle: Does the nozzle operate unchoked? The choking pressure ratio for the given efficiency is (3.14):

$$\frac{P_{04}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_j}\frac{\gamma_g - 1}{\gamma_g + 1}\right]^{\frac{\gamma_g}{\gamma_g - 1}}} = 1.919$$

Here we have:

$$\frac{P_{04}}{P_a} = ... = 4.09$$

Thus, the nozzle is choked. C.R.S. p. 111 gives:

$$\begin{cases} T_5 = T_c = \frac{2}{\gamma_g+1} T_{04} = 823.5 \text{ K} \\ P_5 = P_{04} \frac{1}{\frac{P_{04}}{P_c}} = 0.887 \text{ bar} \end{cases}$$

The ideal gas law gives:

$$\rho_5 = \frac{P_5}{RT_5} = 0.375 \text{ kg/m}^3$$

The 1D continuity equation gives (together with $M_5 = \frac{C_5}{\sqrt{\gamma R T_5}}$ and $M = 1$):

$$\dot{m} = \rho_5 A_5 C_5$$

$$\Longrightarrow$$

$$A_5 = \frac{\dot{m}}{\rho_5 \sqrt{\gamma_g R T_5}} = ... = 0.0712 \text{ m}^2$$

since C_5 is equal to the speed of sound (the nozzle is choked).

The thrust is obtained from (C.R.S p.100, choked nozzle):

$$F = \dot{m}(C_5 - C_a) + A_5(P_5 - P_a) = ... = 7.870 \text{ kN}$$

The SFC is obtained from (f= fuel air ratio):

$$\text{SFC} = \frac{\dot{m}_f}{F} = \frac{\dot{m} \cdot f}{F} = \frac{f}{F_s}$$

We know that $T_{03} - T_{02} = 653$ K = combustion temperature rise. Fig 2.17 (C.R.S. p. 70) gives:

$$f_{theoretical} = 0.0179$$

$$\Longrightarrow$$

$$f_{real} = \frac{0.0179}{0.97} = 0.0184$$

The SFC is consequently:

$$\text{SFC} = \frac{15 \cdot 0.0184}{7870} = ... = 35.07 \text{mg /s N}$$