Exercise 2.1

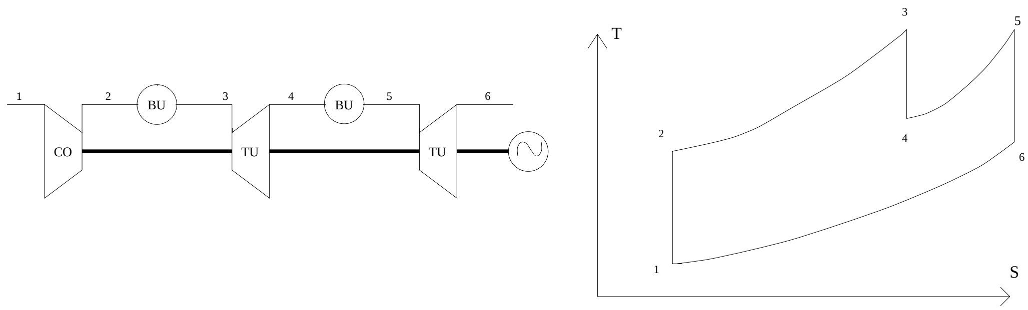

Figure 1: Ideal gas turbine cycle with reheat

Problem definition:

- Ideal cycle assumptions (C.R.S. p.45)
- Reheat to $T_5 = T_3$
- Pressure ratio equally distributed between turbines

Find the compressure pressure ratio r, that optimizes the specific work output W_s.

Solution: The specific work is found from

$$W_s = \underbrace{c_p(T_3 - T_4) + c_p(T_5 - T_6)}_{\text{Turbine work}} - \underbrace{c_p(T_2 - T_1)}_{\text{Compressor work}} \tag{1}$$

We have to derive W_s as a function of r!

Use that the pressure ratio is equally divided between the turbines, i.e.:

$$r = \frac{P_2}{P_1} = \frac{P_3}{P_4} \cdot \frac{P_5}{P_6} = \left(\frac{P_3}{P_4}\right)^2 = \left(\frac{P_5}{P_6}\right)^2 \tag{2}$$

and that

$$T_5 = T_3 = \text{ Fixed upper limit due to strength of turbine material} \tag{3}$$

Combine these two relations with 1 to get:

$$W_s = c_p T_3 \left(1 - \frac{T_4}{T_3}\right) + c_p T_5 \left(1 - \frac{T_6}{T_5}\right) - c_p T_1 \left(\frac{T_2}{T_1} - 1\right) =$$

$$= [\text{ Isentropic compression}] =$$

$$c_p T_3 \left(1 - \left(\frac{1}{\sqrt{r}}\right)^{\frac{\gamma-1}{\gamma}}\right) + c_p T_5 \left(1 - \left(\frac{1}{\sqrt{r}}\right)^{\frac{\gamma-1}{\gamma}}\right) - c_p T_1 \left(r^{\frac{\gamma-1}{\gamma}} - 1\right) =$$

$$= [T_3 = T_5] = c_p T_3 \left[2\left(1 - \left(\frac{1}{\sqrt{r}}\right)^{\frac{\gamma-1}{\gamma}}\right) - \frac{T_1}{T_3}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)\right] \tag{4}$$

Introduce:

$$\begin{cases} \frac{T_3}{T_1} = t \\ r^{\frac{\gamma-1}{\gamma}} = \alpha \end{cases}$$

$$\Longrightarrow$$

$$\frac{W_s}{c_p T_3} = 2\left(1 - \alpha^{-\frac{1}{2}}\right) - \frac{1}{t}(\alpha - 1) \tag{5}$$

Differentiate with respect to α:

$$\frac{d}{d\alpha}\left(\frac{W_s}{c_p T_3}\right) = -2(-\frac{1}{2})\alpha^{-\frac{3}{2}} - \frac{1}{t} = 0 \tag{6}$$

$$\Longrightarrow$$

$$\alpha = r^{\frac{\gamma-1}{\gamma}} = t^{\frac{2}{3}}$$

Learning advice

To get a better feel for this example try to

1. Derive the thermal efficiency of the ideal/Brayton cycle
2. Select optimal pressure ratio

Use section 2.1 C.R.S. as tutorial!