Compressible Flow - TME085

Chapter 1

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Compressible Flow

"Compressible flow (gas dynamics) is a branch of fluid mechanics that deals with flows having significant changes in fluid density"

Wikipedia

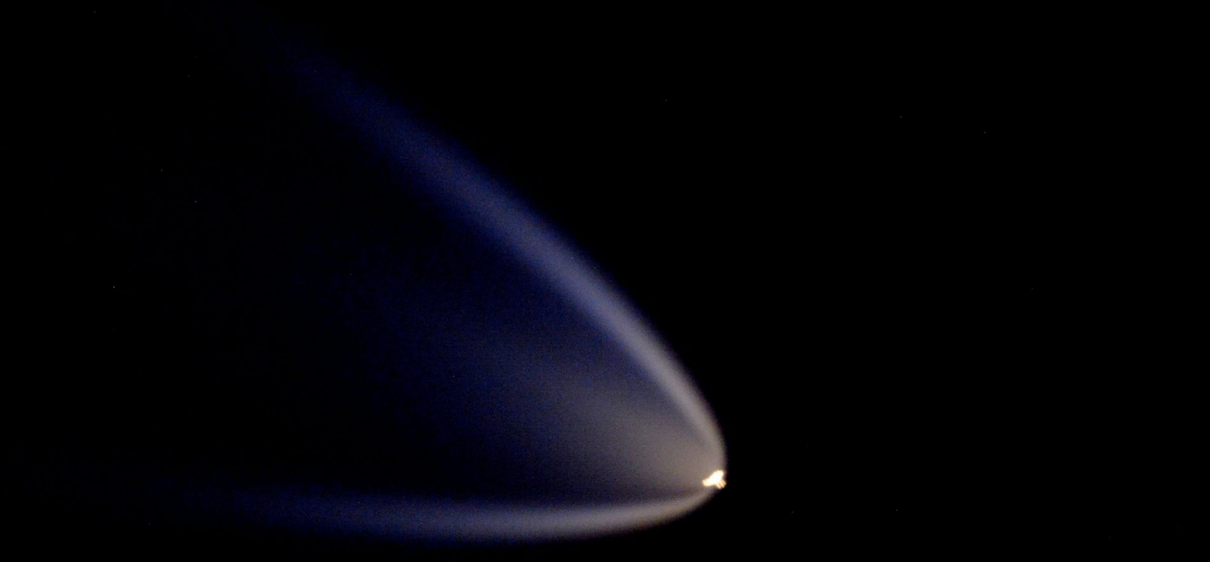
Gas Dynamics

"... the study of motion of gases and its effects on physical systems ..."

"... based on the principles of fluid mechanics and thermodynamics ..."

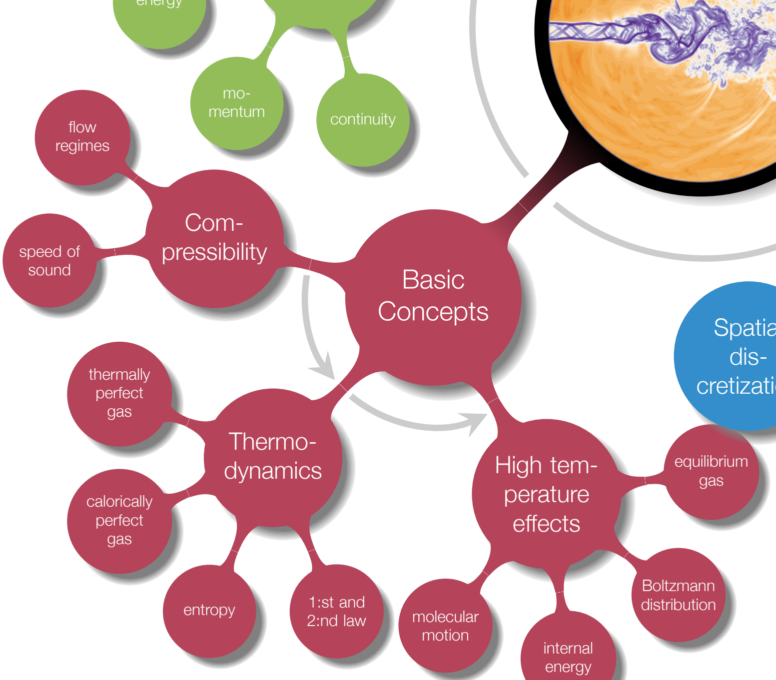
"... gases flowing around or within physical objects at speeds comparable to the speed of sound ..."

Wikipedia



Chapter 1 - Introduction

Overview



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Basic
Concepts
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Thermo-
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calorically
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entropy
1:st and
2:nd law
High tem-
perature
effects
equilibrium
gas
Boltzmann
distribution
molecular
motion
internal
energy

Learning Outcomes

1 Define the concept of compressibility for flows

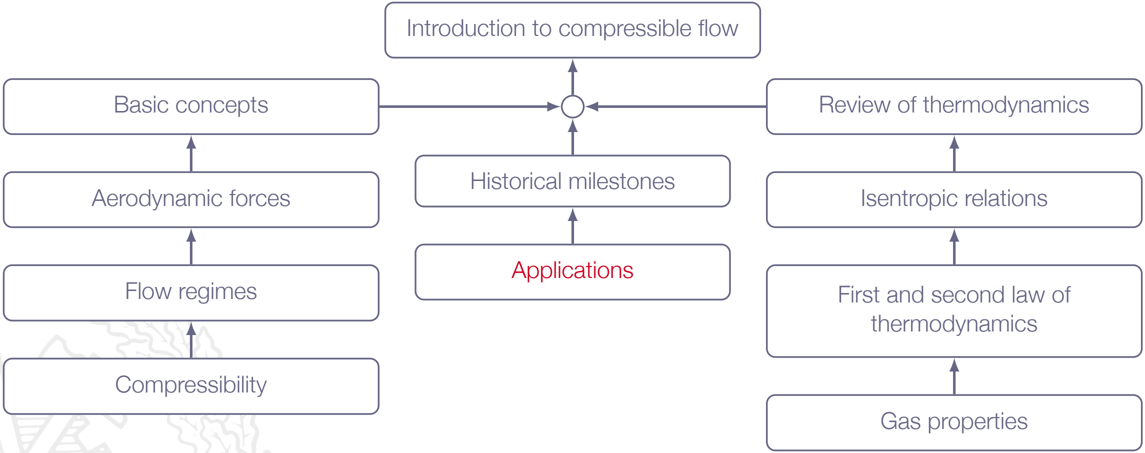
2 Explain how to find out if a given flow is subject to significant compressibility effects

3 Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

6 Define the special cases of calorically perfect gas, thermally perfect gas and real gas and explain the implication of each of these special cases

in this lecture we will find out what compressibility means and do a brief review of thermodynamics

Roadmap - Introduction to Compressible Flow



Applications - Classical

- Treatment of calorically perfect gas
- Exact solutions of inviscid flow in 1D
- Shock-expansion theory for steady-state 2D flow
- Approximate closed form solutions to linearized equations in 2D and 3D
- Method of Characteristics (MOC) in 2D and axi-symmetric inviscid supersonic flows

Applications - Modern

- Computational Fluid Dynamics (CFD)
- Complex geometries (including moving boundaries)
- Complex flow features (compression shocks, expansion waves, contact discontinuities)
- Viscous effects
- Turbulence modeling
- High temperature effects (molecular vibration, dissociation, ionization)
- Chemically reacting flow (equilibrium & non-equilibrium reactions)

Applications - Examples

Turbo-machinery flows:

- Gas turbines, steam turbines, compressors
- Aero engines (turbojets, turbofans, turboprops)

Aeroacoustics:

- Flow induced noise (jets, wakes, moving surfaces)
- Sound propagation in high speed flows

External flows:

- Aircraft (airplanes, helicopters)
- Space launchers (rockets, re-entry vehicles)

Internall flows:

- Nozzle flows
- Inlet flows, diffusers
- Gas pipelines (natural gas, bio gas)

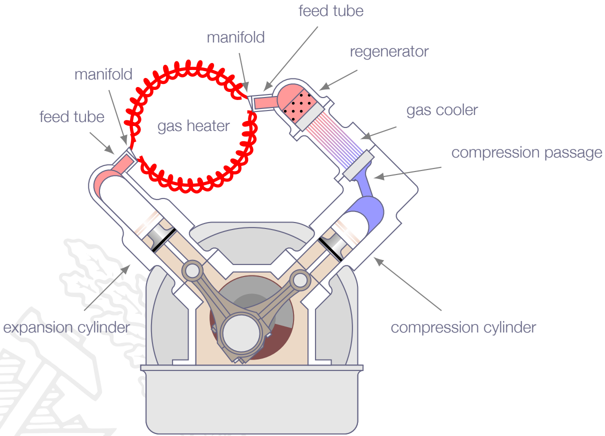
Free-shear flows:

- High speed jets

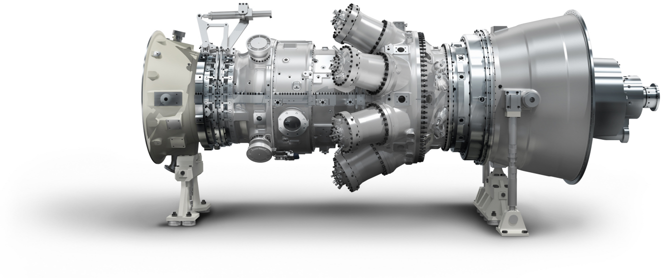
Combustion:

- Internal combustion engines (valve flow, in-cylinder flow, exhaust pipe flow, mufflers)
- Combustion induced noise (turbulent combustion)
- Combustion instabilities

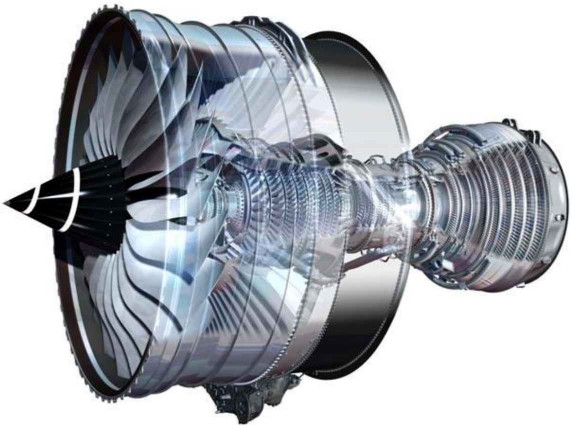
Applications - Stirling Engine



Applications - Siemens GT750



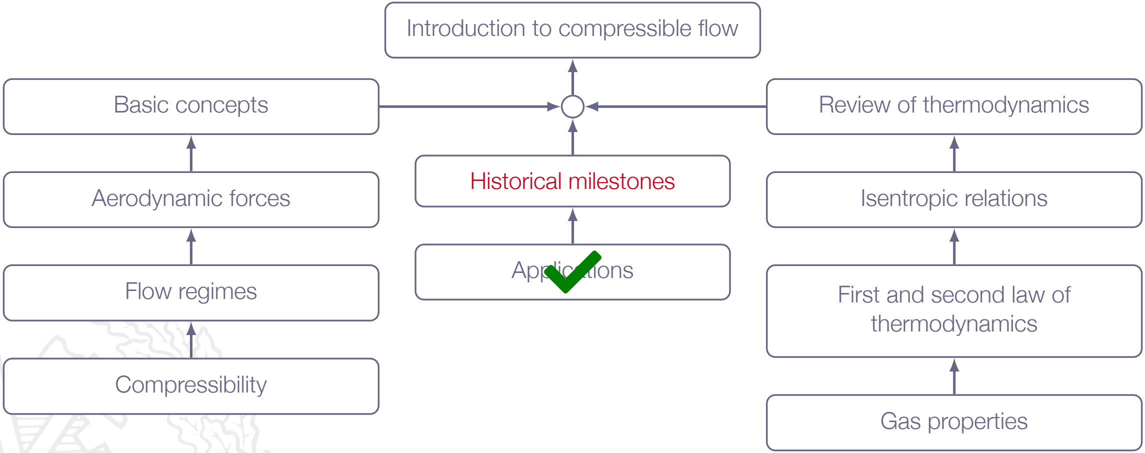
Applications - Rolls-Royce Trent XWB



Applications - Airbus A380

Applications - Vulcain Nozzle

Roadmap - Introduction to Compressible Flow

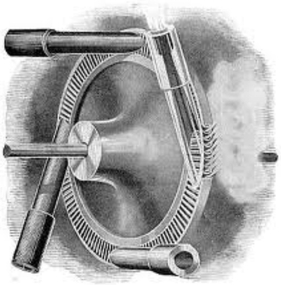


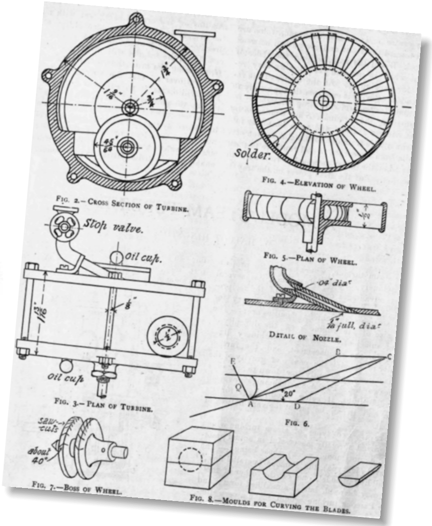
Historical Milestones

1893 C.G.P. de Laval, first steam turbine with supersonic nozzles (convergent-divergent). At this time, the significance was not fully understood, but it worked!

1947 Charles Yeager, flew first supersonic aircraft (XS-1), M 1.06

Historical Milestones - C.G.P. de Laval (1893)

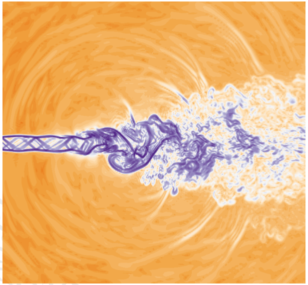


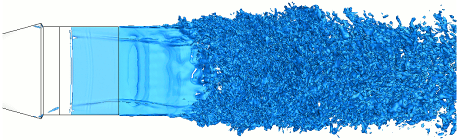


Historical Milestones - Charles Yeager (1947)

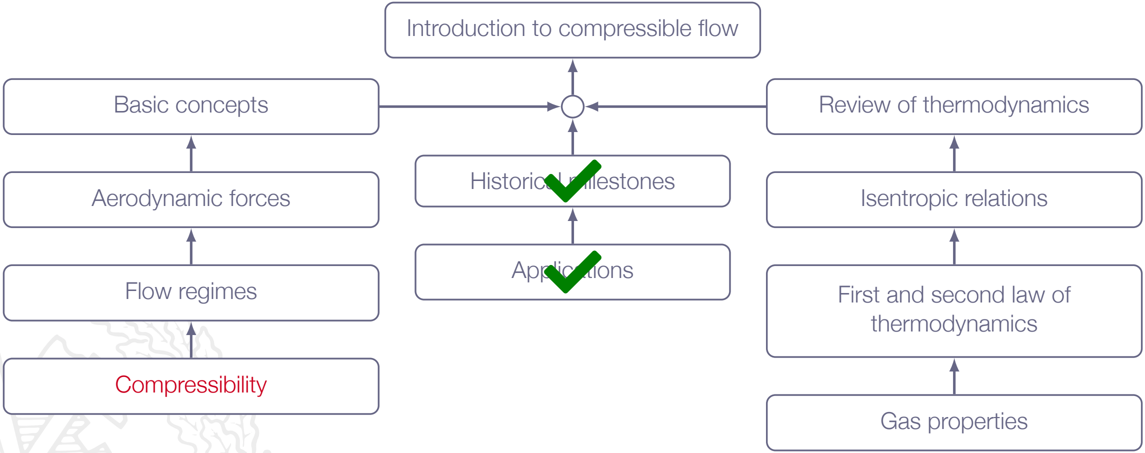
Modern Compressible Flow

Screeching rectangular supersonic jet





Roadmap - Introduction to Compressible Flow



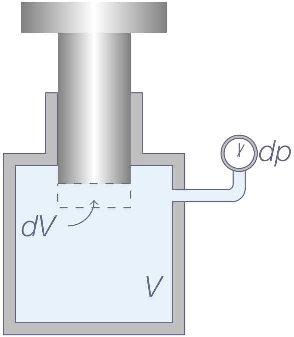
Chapter 1.2
Compressibility

Compressibility

$$\tau = -\frac{1}{\nu}\frac{\partial \nu}{\partial p}, \; (\nu = \frac{1}{\rho})$$

Not really precise!

Is T held constant during the compression or not?



Compressibility

Two fundamental cases:

Constant temperature

- Heat is cooled off to keep T constant inside the cylinder
- The piston is moved slowly

Adiabatic process

- Thermal insulation prevents heat exchange
- The piston is moved fairly rapidly (*gives negligible flow losses*)

Compressibility

Isothermal process:

$$\tau_T = -\frac{1}{\nu}\left(\frac{\partial \nu}{\partial p}\right)_T$$

Adiabatic reversible (*isentropic*) process:

$$\tau_S = -\frac{1}{\nu}\left(\frac{\partial \nu}{\partial p}\right)_S$$

Air at normal conditions:	$\tau_T \approx 1.0 \times 10^{-5}$	$[m^2/N]$
Water at normal conditions:	$\tau_T \approx 5.0 \times 10^{-10}$	$[m^2/N]$

Compressibility

$$\tau = -\frac{1}{\nu}\frac{\partial \nu}{\partial p} \text{ where } \nu = \frac{1}{\rho} \text{ and thus}$$

$$\tau = -\rho\frac{\partial}{\partial p}\left(\frac{1}{\rho}\right) = -\rho\left(-\frac{1}{\rho^2}\right)\frac{\partial \rho}{\partial p} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

$$\tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T$$

$$\tau_S = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_S$$

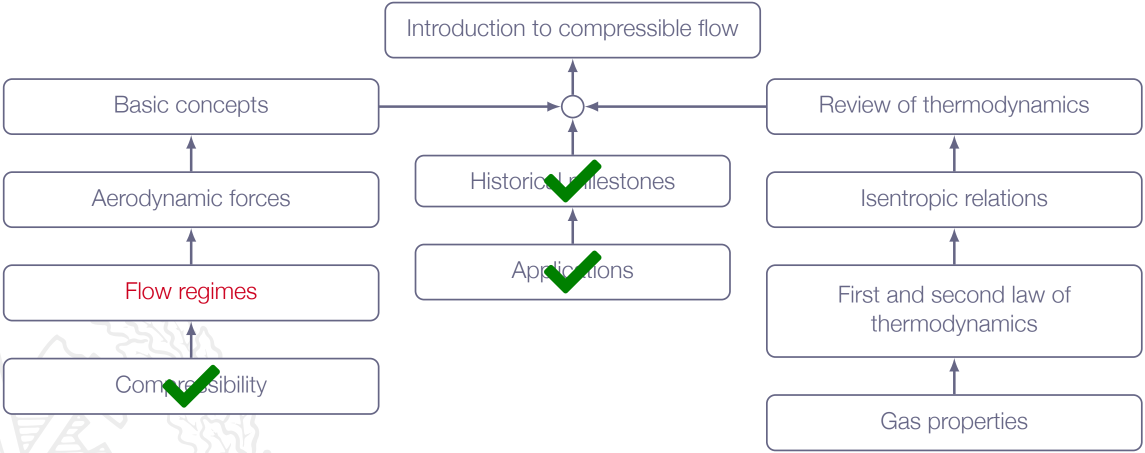
Compressibility

Definition of compressible flow:

If p changes with amount Δp over a characteristic length scale of the flow, such that the corresponding change in density, given by $\Delta\rho \sim \rho\tau\Delta$ p, is too large to be neglected, the flow is compressible (*typically, if* $\Delta\rho/\rho > 0.05$)

Note! Bernoulli´s equation is restricted to incompressible flow, *i.e.* it is not valid for compressible flow!

Roadmap - Introduction to Compressible Flow



Chapter 1.3
Flow Regimes

Flow Regimes

The freestream Mach number is defined as

$$M_\infty = \frac{U_\infty}{a_\infty}$$

where U_∞ is the freestream flow speed and a_∞ is the speed of sound at freestream conditions

Flow Regimes

Assume incompressible flow and estimate the maximum pressure difference using

$$\Delta p \approx \frac{1}{2}\rho_\infty U_\infty^2$$

For air at normal conditions we have

$$\tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T = \frac{1}{\rho R T} = \frac{1}{p}$$

(ideal gas law for perfect gas $p = \rho R T$)

Flow Regimes

Using the relations on previous slide we get

$$\frac{\Delta\rho}{\rho} \approx \tau_T \Delta p \approx \frac{1}{p_\infty}\frac{1}{2}\rho_\infty U_\infty^2 = \frac{\frac{1}{2}\rho_\infty U_\infty^2}{\rho_\infty R T_\infty}$$

for a calorically perfect gas we have $a = \sqrt{\gamma R T}$

which gives us $\dfrac{\Delta\rho}{\rho} \approx \dfrac{\gamma U_\infty^2}{2a_\infty^2}$

now, using the definition of Mach number we get:

$$\boxed{\frac{\Delta\rho}{\rho} \approx \frac{\gamma}{2} M_\infty^2}$$

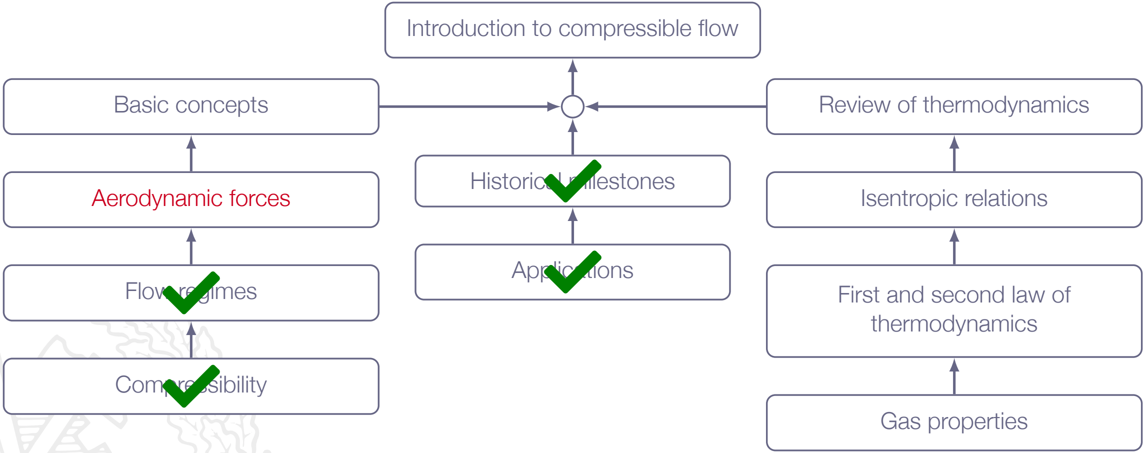
Flow Regimes

Incompressible	$M_\infty < 0.1$
Subsonic	$M_\infty < 1$ and $M < 1$ everywhere
Transonic	case 1: $M_\infty < 1$ and $M > 1$ locally case 2: $M_\infty > 1$ and $M < 1$ locally
Supersonic	$M_\infty > 1$ and $M > 1$ everywhere
Hypersonic	supersonic flow with high-temperature effects

Compressible

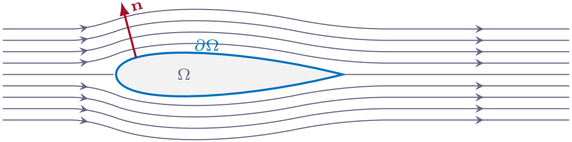
Local Mach number M is based on local flow speed, $U = |\mathbf{U}|$, and local speed of sound, a

Roadmap - Introduction to Compressible Flow



Chapter 1.5
Aerodynamic Forces

Aerodynamic Forces



Ω	region occupied by body
$\partial\Omega$	surface of body
$\mathbf{n}$	outward facing unit normal vector

Aerodynamic Forces

Overall forces on the body du to the flow

$$\mathbf{F} = \oiint (-p\mathbf{n} + \tau \cdot \mathbf{n}) dS$$

where p is static pressure and τ is a stress tensor

Aerodynamic Forces

Drag is the component of $\mathbf{F}$ which is parallel with the freestream direction:

$$D = D_p + D_f$$

where D_p is drag due to pressure and D_f is drag due to friction

Lift is the component of $\mathbf{F}$ which is normal to the free stream direction:

$$L = L_p + L_f$$

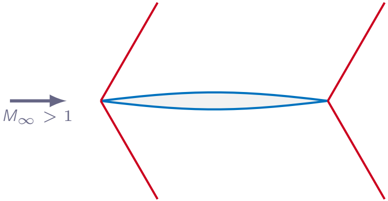
(L_f is usually negligible)

Aerodynamic Forces

Inviscid flow around slender body (*attached flow*)

- subsonic flow: $D = 0$
- transonic or supersonic flow: $D > 0$

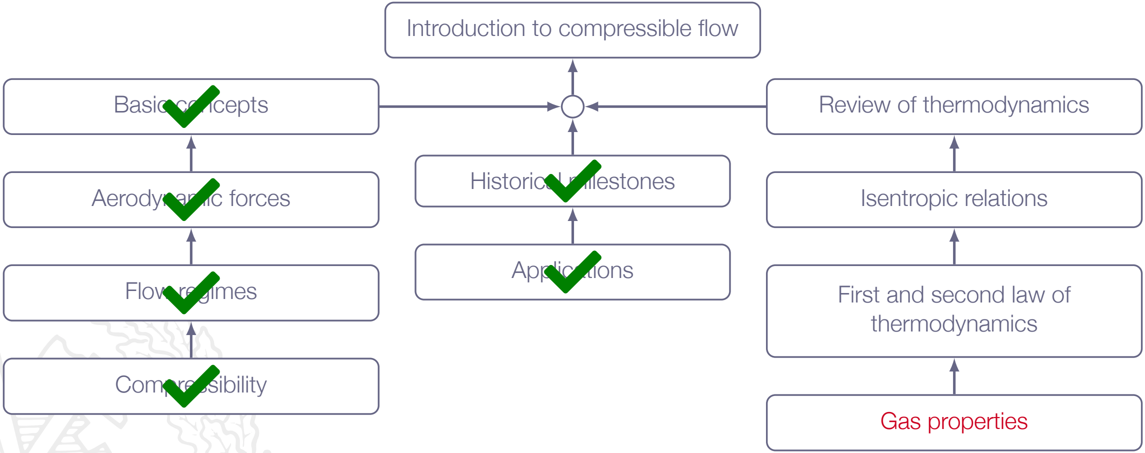
Explanation: Wave drag



Aerodynamic Forces

- Wave drag is an inviscid phenomena, connected to the formation of compression shocks and entropy increase
- Viscous effects are present in all Mach regimes
- At transonic and supersonic conditions a particular phenomena named ”shock/boundary-layer interaction” may appear
  - shocks trigger flow separation
  - usually leads to unsteady flow

Roadmap - Introduction to Compressible Flow



Chapter 1.4
Review of Thermodynamics

Thermodynamic Review

Compressible flow:

” strong interaction between flow and thermodynamics ... ”

Perfect Gas

All intermolecular forces negligible

Only elastic collitions between molecules

$p\nu = RT$ or $\dfrac{p}{\rho} = RT$

where R is the gas constant $[R] = J/kgK$

Also, $R = R_{univ}/M$ where M is the molecular weight of gas molecules (in $kg/kmol$) and $R_{univ} = 8314\, J/kmol\ K$

Internal Energy and Enthalpy

Internal energy e ($[e] = J/kg$)

Enthalpy h ($[h] = J/kg$)

$h = e + p\nu = e + \dfrac{p}{\rho}$ (*valid for all gases*)

For any gas in thermodynamic equilibrium, e and h are functions of only two thermodynamic variables (*any two variables may be selected*) *e.g.*

$e = e(T, \rho)$ or $h = h(T, p)$

Internal Energy and Enthalpy

Special cases:

Thermally perfect gas:

$e = e(T)$ and $h = h(T)$

OK assumption for air at near atmospheric conditions and $100K < T < 2500K$

Calorically perfect gas:

$e = C_v T$ and $h = C_p T$ (C_v and C_p are constants)

OK assumption for air at near atmospheric pressure and $100K < T < 1000K$

Specific Heat

For thermally perfect (and calorically perfect) gas

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p, \quad C_v = \left(\frac{\partial e}{\partial T}\right)_v$$

since $h = e + p/\rho = e + RT$ we obtain:

$$C_p = C_v + R$$

The ratio of specific heats, γ, is defined as:

$$\gamma \equiv \frac{C_p}{C_v}$$

Specific Heat

Important!

calorically perfect gas:

C_v, C_p, and γ are constants

thermally perfect gas:

C_v, C_p, and γ will depend on temperature

Specific Heat

$$C_p - C_v = R$$

$$C_p - C_v = R$$

Specific Heat

$$C_p - C_v = R$$

divide by C_v

$$C_p - C_v = R$$

divide by C_p

Specific Heat

$$C_p - C_v = R$$

divide by C_v

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_p - C_v = R$$

divide by C_p

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

Specific Heat

$$C_p - C_v = R$$

divide by C_v

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p - C_v = R$$

divide by C_p

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

Specific Heat

$$C_p - C_v = R$$

divide by C_v

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p - C_v = R$$

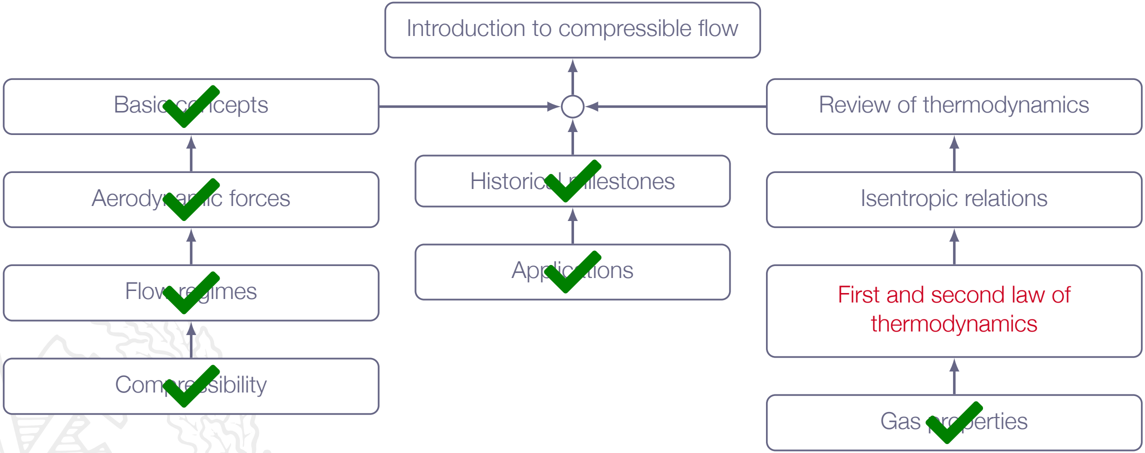
divide by C_p

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

valid for both thermally perfect and calorically perfect gas!

Roadmap - Introduction to Compressible Flow



First Law of Thermodynamics

A fixed mass of gas, separated from its surroundings by an imaginary flexible boundary, is defined as a "system". This system obeys the relation

$$de = \delta q - \delta w$$

where

de is a change in internal energy of system

δq is heat added to the system

δw is work done by the system (on its surroundings)

Note! de only depends on starting point and end point of the process while δq and δw depend on the actual process also

First Law of Thermodynamics

Examples:

Adiabatic process:

$$\delta q = 0.$$

Reversible process:

no dissipative phenomena (*no flow losses*)

Isentropic process:

a process which is both adiabatic and reversible

First Law of Thermodynamics

Reversible process:

$$\delta w = p d\nu = p d(1/\rho)$$

$$de = \delta q - p d(1/\rho)$$

Adiabatic & reversible process:

$$\delta q = 0.$$

$$de = -p d(1/\rho)$$

Entropy

Entropy s is a property of all gases, uniquely defined by any two thermodynamic variables, *e.g.*

$s = s(p, T)$ or $s = s(\rho, T)$ or $s = s(\rho, p)$ or $s = s(e, h)$ or ...

Second Law of Thermodynamics

Concept of entropy s:

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir} \text{ where } ds_{ir} > 0. \text{ and thus}$$

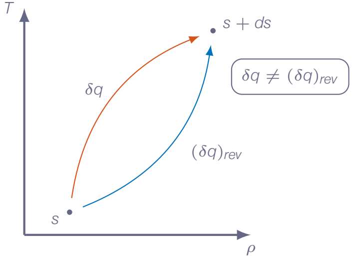
$$ds \geq \frac{\delta q}{T}$$

Second Law of Thermodynamics

Concept of entropy s:

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir} \text{ where } ds_{ir} > 0. \text{ and thus}$$

$$\boxed{ds \geq \frac{\delta q}{T}}$$



Second Law of Thermodynamics

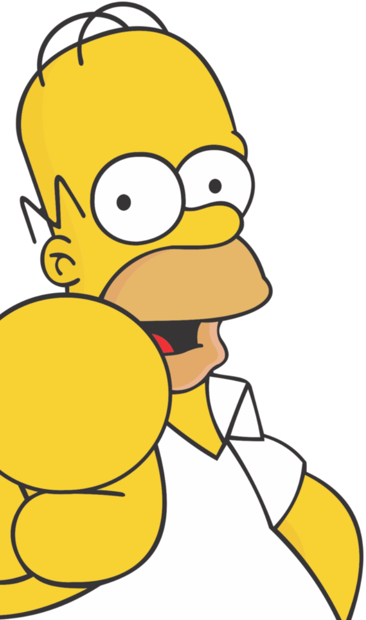
In general:

$$ds \geq \frac{\delta q}{T}$$

For adiabatic processes:

$$ds \geq 0.$$

Second Law of Thermodynamics



"In this house, we obey the laws of thermodynamics!"

Homer Simpson, after Lisa constructs a perpetual motion machine whose energy increases with time

Calculation of Entropy

For reversible processes ($\delta w = pd(1/\rho)$ and $\delta q = Tds$):

$$de = Tds - pd\left(\frac{1}{\rho}\right) \Leftrightarrow Tds = de + pd\left(\frac{1}{\rho}\right)$$

from before we have $h = e + p/\rho \Rightarrow$

$$dh = de + pd\left(\frac{1}{\rho}\right) + \left(\frac{1}{\rho}\right)dp \Leftrightarrow de = dh - pd\left(\frac{1}{\rho}\right) - \left(\frac{1}{\rho}\right)dp$$

Calculation of Entropy

For thermally perfect gases, $p = \rho RT$ and $dh = C_p dT \Rightarrow ds = C_p \frac{dT}{T} - R\frac{dp}{p}$

Integration from starting point (1) to end point (2) gives:

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - R \ln\left(\frac{p_2}{p_1}\right)$$

and for calorically perfect gases

$$\boxed{s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)}$$

Calculation of Entropy

If we instead use $de = C_v dT$ we get

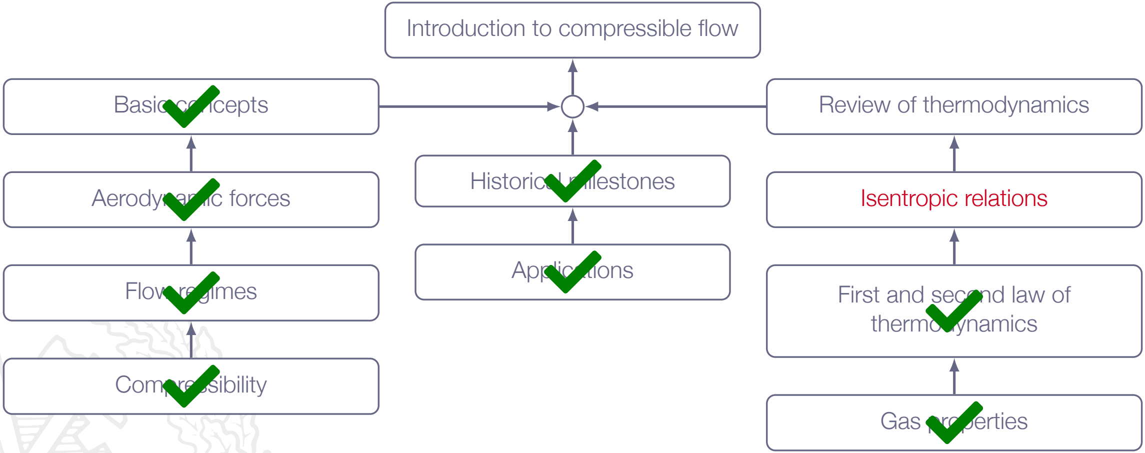
for thermally perfect gases

$$s_2 - s_1 = \int_1^2 C_v \frac{dT}{T} - R \ln \left(\frac{\rho_2}{\rho_1} \right)$$

and for calorically perfect gases

$$\boxed{s_2 - s_1 = C_v \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{\rho_2}{\rho_1} \right)}$$

Roadmap - Introduction to Compressible Flow



Isentropic Relations

For calorically perfect gases, we have

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

For adiabatic reversible processes:

$$ds = 0. \Rightarrow s_1 = s_2 \Rightarrow C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = 0 \Rightarrow$$

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{C_p}{R}\ln\left(\frac{T_2}{T_1}\right)$$

Isentropic Relations

with $\dfrac{C_p}{R} = \dfrac{C_p}{C_p - C_v} = \dfrac{\gamma}{\gamma - 1} \Rightarrow$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

Isentropic Relations

Alternatively, using $s_2 - s_1 = 0 = C_v \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{\rho_2}{\rho_1}\right) \Rightarrow$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

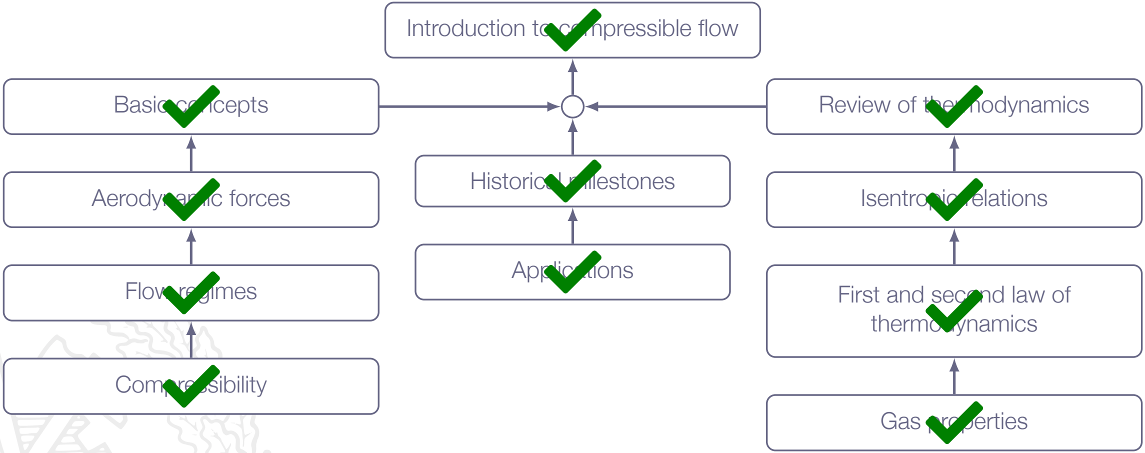
Isentropic Relations - Summary

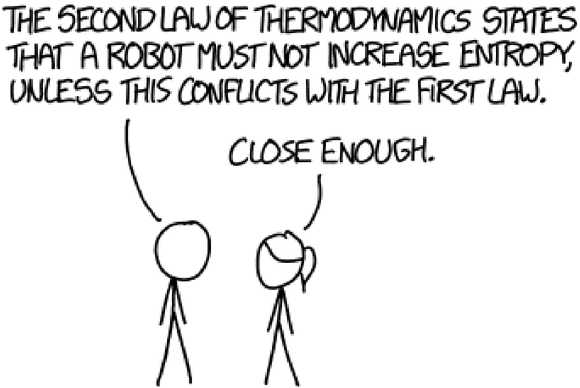
For an isentropic process and a calorically perfect gas we have

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

A.K.A. the isentropic relations

Roadmap - Introduction to Compressible Flow




THE SECOND LAW OF THERMODYNAMICS STATES THAT A ROBOT MUST NOT INCREASE ENTROPY, UNLESS THIS CONFLICTS WITH THE FIRST LAW.
CLOSE ENOUGH.