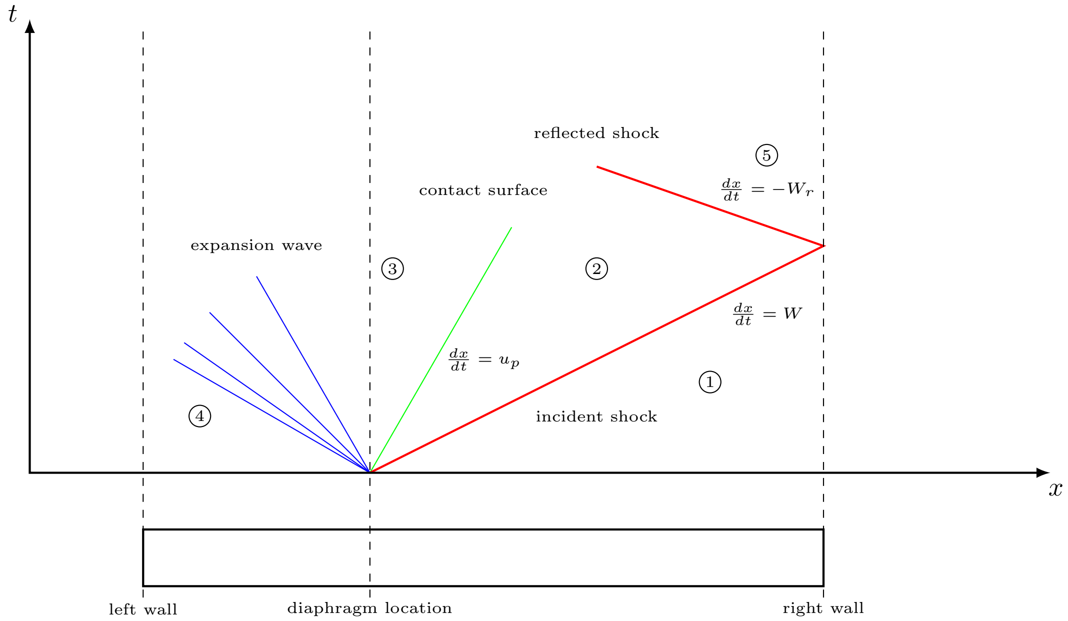
Unsteady Wave Motion

Shock Reflection

Niklas Andersson
Division of Fluid Dynamics
Department of Mechanics and Maritime Sciences
Chalmers University of Technology

Shock Reflection

When the incident shock wave reaches the wall, a shock propagating in the opposite direction is generated with a shock strength such that the velocity of the induced flow behind the incident shock is reduced to zero. The flow can not go through the wall and thus the velocity must be zero in the vicinity of the wall. The properties of the incident shock wave are directly related to the pressure ratio over the shock wave. Therefore, it would be convenient to have a relation between the reflected shock wave and incident shock wave.



Figur 1: shock reflection at solid wall (located at x=4.0)

The Incident Shock Wave

The pressure ratio over the incident shock in Fig. 1 can be obtained as

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_s^2 - 1\right) \tag{1}$$

where M_s is the wave Mach number, which is calculated as

$$M_s = \frac{W}{a_1} \tag{2}$$

In Eqn. 2, W is the speed with which the incident shock wave travels into region 1 and a_1 is the speed of sound in region 1 (see Fig. 1).
Solving Eqn. 1 for M_s, we get

$$M_s = \sqrt{\frac{\gamma + 1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \tag{3}$$

Anderson derives the relations for calculation of the ratio T_2/T_1

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{\dfrac{\gamma + 1}{\gamma - 1} + \dfrac{p_2}{p_1}}{1 + \dfrac{\gamma + 1}{\gamma - 1}\dfrac{p_2}{p_1}}\right) \tag{4}$$

From Eqn. 4 it is easy to get the corresponding relation for ρ_2/ρ_1

$$\frac{\rho_2}{\rho_1} = \frac{1 + \dfrac{\gamma + 1}{\gamma - 1}\dfrac{p_2}{p_1}}{\dfrac{\gamma + 1}{\gamma - 1} + \dfrac{p_2}{p_1}} \tag{5}$$

Anderson also shows how to obtain the induced velocity, u_p, behind the incident shock wave, *i.e.* the velocity in region 2 (see Fig. 1).

$$u_p = W\left(1 - \frac{\rho_1}{\rho_2}\right) = M_s a_1\left(1 - \frac{\rho_1}{\rho_2}\right) \tag{6}$$

The Reflected Shock Wave

The pressure ratio over the reflected shock can be obtained from Eqn. 1 by analogy

$$\frac{p_5}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_r^2 - 1\right) \tag{7}$$

where M_r is the Mach number of the reflected shock wave defined as

$$M_r = \frac{W_r + u_p}{a_2} \tag{8}$$

where W_r is the speed of the reflected shock wave and a_2 is the speed of sound in region 2 (see Fig. 1).

Solving Eqn. 7 for M_r gives

$$M_r = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_5}{p_2} - 1\right) + 1} \tag{9}$$

The ratios T_5/T_2 and ρ_5/ρ_2 can be obtained from Eqns. 4 and 5 by analogy

$$\frac{T_5}{T_2} = \frac{p_5}{p_2}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_5}{p_2}}{1 + \frac{\gamma+1}{\gamma-1}\frac{p_5}{p_2}}\right) \tag{10}$$

$$\frac{\rho_5}{\rho_2} = \frac{1 + \frac{\gamma+1}{\gamma-1}\frac{p_5}{p_2}}{\frac{\gamma+1}{\gamma-1} + \frac{p_5}{p_2}} \tag{11}$$

The velocity in region 2 which is the same as the induced flow velocity behind the incident shock wave can be obtained as

$$u_p = W_r\left(\frac{\rho_5}{\rho_2} - 1\right) = M_r a_2\left(1 - \frac{\rho_2}{\rho_5}\right) \tag{12}$$

Reflected Shock Relation

With the relations for the incident shock wave and reflected shock wave defined, we now have the tools to derive a relation between the incident and reflected shock waves. The induced flow velocity u_p calculated using the relation obtained for the incident shock wave must of course be the same as when calculated using reflected wave properties, *i.e.* the result of Eqn. 6 is identical to that of Eqn. 12

$$M_r a_2 \left(1 - \frac{\rho_2}{\rho_5}\right) = M_s a_1 \left(1 - \frac{\rho_1}{\rho_2}\right) \tag{13}$$

rewriting gives

$$M_r \left(1 - \frac{\rho_2}{\rho_5}\right) = M_s \left(1 - \frac{\rho_1}{\rho_2}\right) \frac{a_1}{a_2} \tag{14}$$

Assuming calorically perfect gas gives $a = \sqrt{\gamma R T}$ and thus

$$M_r \left(1 - \frac{\rho_2}{\rho_5}\right) = M_s \left(1 - \frac{\rho_1}{\rho_2}\right) \sqrt{\frac{T_1}{T_2}} \tag{15}$$

Let's first look at the term on the left hand side of Eqn. 15

$$M_r \left(1 - \frac{\rho_2}{\rho_5}\right)$$

Using the ρ_5/ρ_2 and p_2/p_5 from Eqns. 11 and 7 and simplifying gives

$$M_r \left(1 - \frac{\rho_2}{\rho_5}\right) = \left(\frac{2}{\gamma + 1}\right) \left(\frac{M_r^2 - 1}{M_r}\right) \tag{16}$$

Using the same approach on the corresponding term for the incident shock wave on the right hand side of Eqn. 15 gives

$$M_s \left(1 - \frac{\rho_1}{\rho_2}\right) = \left(\frac{2}{\gamma + 1}\right) \left(\frac{M_s^2 - 1}{M_s}\right) \tag{17}$$

Now, inserting 16 and 17 in Eqn. 15 gives

$$\left(\frac{2}{\gamma+1}\right)\left(\frac{M_r^2-1}{M_r}\right) = \left(\frac{2}{\gamma+1}\right)\left(\frac{M_s^2-1}{M_s}\right)\sqrt{\frac{T_1}{T_2}} \tag{18}$$

Simplifying and inverting gives

$$\left(\frac{M_r}{M_r^2-1}\right) = \left(\frac{M_s}{M_s^2-1}\right)\sqrt{\frac{T_2}{T_1}} \tag{19}$$

The rightmost term in Eqn. 19 ($\sqrt{T_2/T_1}$) needs to be rewritten. Inserting 1 in 4 and expanding all terms gives

$$\frac{T_2}{T_1} = \frac{2(\gamma+1)+(\gamma+1)(\gamma-1)M_s^2+4\gamma(M_s^2-1)+2\gamma(\gamma-1)M_s^2(M_s^2-1)}{(\gamma+1)^2 M_s^2} =$$

$$= \frac{2(\gamma+1)+(\gamma+1)(\gamma-1)M_s^2+4\gamma(M_s^2-1)}{(\gamma+1)^2 M_s^2} + \frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\gamma =$$

$$= \frac{2(\gamma+1)+(\gamma+1)(\gamma-1)M_s^2+4\gamma(M_s^2-1)-(2(\gamma-1)(M_s^2-1))}{(\gamma+1)^2 M_s^2} +$$
$$\frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right)$$

Finally we end up with the following relation

$$\frac{T_2}{T_1} = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right) \tag{20}$$

The temperature ratio over the incident shock wave is now totally defined by the incident Mach number M_s and the ratio of specific heats γ. With 20 in 19 we get the sought relation between the reflected and incident Mach numbers.

$$\left(\frac{M_r}{M_r^2-1}\right) = \left(\frac{M_s}{M_s^2-1}\right)\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right)} \tag{21}$$

It should be noted that Eqn. 21 is valid for calorically perfect gases only.