Oblique Shocks and Expansion Waves

Expansion Wave Relations

Niklas Andersson
Division of Fluid Dynamics
Department of Mechanics and Maritime Sciences
Chalmers University of Technology

The aim is to derive relations of temperature, pressure and density over an expansion wave. Total temperature upstream and downstream of the expansion wave is calculated as

$$\frac{T_{o_1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \tag{1}$$

$$\frac{T_{o_2}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 \tag{2}$$

The temperature ratio over the expansion wave may now be calculated as

$$\frac{T_2}{T_1} = \frac{T_2}{T_{o_2}} \frac{T_{o_1}}{T_1} = \frac{1 + \dfrac{\gamma - 1}{2} M_1^2}{1 + \dfrac{\gamma - 1}{2} M_2^2} = \frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2}$$

The expansion is isentropic and thus total temperature and both total pressure are unaffected by the expansion. Therefore, $T_{o_1} = T_{o_2}$ and thus

$$\frac{T_2}{T_1} = \frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} \tag{3}$$

The pressure and density ratios can be obtained from Eqn. 3 using the isentropic relations

$$\frac{p_2}{p_1} = \left[\frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} \right]^{\gamma/(\gamma - 1)} \tag{4}$$

$$\frac{\rho_2}{\rho_1} = \left[\frac{2 + (\gamma - 1)M_1^2}{2 + (\gamma - 1)M_2^2}\right]^{1/(\gamma-1)} \tag{5}$$