F MTF256, Assignment: Analysis of turbulent flow in a channel

In this exercise you will use data from a Direct Numerical Simulation (DNS) for fully developed channel flow. In DNS the unsteady, three-dimensional Navier-Stokes equations are solved numerically. The Re number based on the friction velocity and the half channel width is $Re_\tau = u_\tau h/\nu = 500$ ($h = \rho = u_\tau = 1$ so that $\nu = 1/Re_\tau$).

A $96 \times 96 \times 96$ mesh has been used. The streamwise, wall-normal and spanwise directions are denoted by x (x_1), y (x_2) and z (x_3) respectively. The cell size in x and z directions are $\Delta x = 0.0654$ and $\Delta z = 0.0164$. Periodic boundary conditions were applied in the x and z direction (homogeneous directions). All data have been made non-dimensional by u_τ and ρ.

You can do the assignment on your own or in a group of two. You should write a report where you analyze the results following the heading H1–H13. It is recommended (but the not required) that you use LATEX(an example of how to write in LATEXis available on the course www page). It is available on Linux. On Windows you can use, for example, Lyx (`www.lyx.org`) or MikTex (`www.miktex.org`) which are both free to donwload.

F.1 Time history

At the course home page `http://www.tfd.chalmers.se/˜lada/turbulent_flow/` you find a file `u_v_time_4nodes.dat` with the time history of v_1 and v_2. The file has eight columns of v_1 and v_2 at four nodes: $x_2/\delta = 0.0039$, $x_2/\delta = 0.0176$, $x_2/\delta = 0.107$ and $x_2/\delta = 0.47$. With $u_\tau = 1$ and $\nu = 1/Re_\tau = 1/500$ this correspond to $x_2^+ = 1.95$, $x_2^+ = 8.8$, $x_2^+ = 53.5$ and $x_2^+ = 235$. The sampling time step is $\Delta t = 0.0033$ (every second time step). The four points are located in the viscous sublayer, the buffer layer and in the logarithmic layer, see Fig. 6.2 at p. 49.

Use the Matlab program `pl_time.m` which loads and plots the time history of v_1. Start Matlab and run the program `pl_time`. Recall that the velocities have been scaled with the friction velocity u_τ, and thus what you see is really v_1/u_τ. The time history of v_1 at $x_2/\delta = 0.0176$ and $x_2/\delta = 0.107$ are shown. Study the time history of the blue line ($x_2/\delta = 0.0176$) more in detail. Make a zoom between, for example, $t = 10$ and $t = 11$ and $v_{1,min} = 3$ and $v_{1,min} = 21$. This is conveniently done with the command

```
axis([10 11 3 21])
```

In order to see the value at each sampling time step, change the plot command to

```
plot(t,u2,'b-',t,u2,'bo')
```

Use this technique to zoom, to look at the details of the time history. Alternatively, you can use the zoom buttons above the figure.

Plot v_1 for all four nodes. How does the time variation of v_1 vary for different positions? Why? Plot also v_2 at the four different positions. What is the differences between v_1 and v_2?

Plot v_1 and v_2 versus time, $t^+ = tu_\tau/\nu$ (viscous units). How fast are the variations of v_1 and v_2 in viscous time units?

F.2 Time averaging

Compute the average of the v_1 velocity at node 2. Add the following code (before the plotting section)

```
umean=mean(u2)
```

Here the number of samples is $n = 5000$ (the entire u2 array). Find out how many samples must be used to get a correct mean value. Start by trying with 100 samples as

```
umean_100=mean(u2(1:100))
```

What is the maximum and minimum value of v_1? Compare those to the mean.

Do the same exercise for the other three nodes.

Compute and plot also the instantaneous fluctuations; v_1' at node 1, for example, is computed as

```
u1_mean=mean(u1);
u1_fluct=u1-u1_mean;
```

F.3 Histogram/probability density

Equation 7.2 defines the *first* moment. Compute and plot the probability density for the instantaneous velocity $u_1 \ldots v_4$ and for their fluctuations; use the Matlab command hist (note that the probability density function of v_1 is different from that of v_1'). Normalize the probability functions, so that (here shown for v_1)

$$\int_{-\infty}^{\infty} f_{v1}(v)dv = 1 \tag{F.1}$$

To carry out the integration you can, for example, use the Matlab command trapz (type help trapz in the Matlab command window). Investigate how a probability density changes as the number of bins changes (to find out what a "bin" is, type help hist in the Matlab command window)

Consider the probability density functions of the fluctuations. The second moment corresponds to the variance of the fluctuations (or the square of the RMS, see Eq. F.5), i.e.

$$\overline{v'^2} = \int_{-\infty}^{\infty} v'^2 f(v')dv'$$

As in Eq. 7.4, $\overline{v'^2}$ is usually computed by integrating in time.

A probability density function is symmetric if positive values are as frequent and large as the negative values. What about $f_{u'1} \ldots f_{v'4}$, do they seem to be symmetric? Instead of "looking" at the probability density functions, we should use a definition of the degree of symmetry, which is the *skewness*. It is defined as

$$\overline{v'^3} = \int_{-\infty}^{\infty} v'^3 f(v')dv'$$

and is commonly normalized by v_{rms}^3, so that the skewness, $S_{v'}$, of v' is defined as

$$S_{v_1'} = \frac{1}{v_{1,rms}^3} \int_{-\infty}^{\infty} v_1'^3 f_{v'1}(v)dv$$

Again, the integration can also be carried out in time, i.e. $S_{v_1'} = \overline{v_1'^3}/v_{1,rms}^3$. Compute S for $f_{u'1} \dots f_{v'4}$. Note that you must use a normalized f (see Eq, F.1).

There is yet another statistical quantity which sometimes is used for describing turbulent fluctuations, namely the *flatness*. The variance (the square of RMS) tells us how large the fluctuations are in average, but it does not tell us if there are few very large fluctuations or frequent very small ones. The flatness gives this information, and it is defined computed from v'^4 and normalized by v_{rms}^4, i.e.

$$F_{v_1'} = \frac{1}{v_{rms}^4} \int_{-\infty}^{\infty} v_1'^4 f_{v'1}(v) dv$$

Using time integration is reads $F_{v_1'} = \overline{v_1'^4}/v_{1,rms}^4$. For a Gaussian distribution

$$f_g(v') = \frac{1}{v_{rms}} \exp \left(-\frac{v' - r_{rms}}{2v_{rms}^2} \right) = -3$$

F.4   Mean flow

All data in the data files below have been stored every $10th$ time step.

Download the file `uvw_inst_small.dat`, `y.dat` and the Matlab file `pl_vel.m` which reads the data files. The data file includes v_1, v_2 and v_3 from the same DNS as above, but now you are given the time history of all x_2 nodes at one chosen x_1 and x_3 node. There are $nj = 98$ nodes in the x_2 direction; node 1 and nj are located at the lower and upper wall, respectively.

Your data are instantaneous. Compute the mean velocity. Plot it both as linear-linear plot and a log-linear plot (cf. Fig. 6.4).

In the log-linear plot, use x_2^+ for the wall distance. Include the linear law, $v_1^+ = x_2^+$, and the log law, $v_1^+ = \kappa^{-1} \ln x_2^+ + B$ ($\kappa = 0.41$ is the von Kármán constant and $B = 5.2$). How far out from the wall does the velocity profile follow the linear law? At what x_2^+ does it start to follow the log-law?

Compute the bulk velocity

$$V_{1,b} = \frac{1}{2h} \int_0^{2h} \bar{v}_1 dx_2 \tag{F.2}$$

(recall that h denote half the channel width) What is the Reynolds number based on $V_{1,b}$ and centerline velocity, $V_{1,c}$, respectively?

F.5   The time-averaged momentum equation

Let us time average the streamwise momentum equation. Since the flow is fully developed and two dimensional we get

$$0 = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_1} + \nu \frac{\partial^2 \bar{v}_1}{\partial x_2^2} - \frac{\partial \overline{v_1' v_2'}}{\partial x_2} \tag{F.3}$$

This equation is very similar to fully developed laminar flow which you studied in Assignment 2 in *TME075 Mechanics of solids and fluids*, see Eq. 3.24; the difference is that we now have an additional Reynolds stress term. Recall that all terms in the equation above represent *forces* (per unit volume). Let us investigate how these forces

(the pressure gradient, the viscous term and the Reynolds stress term) affect fluid particles located at different x_2 locations. Compute and plot the three terms. (the file uvw_inst_small.dat does not include $\bar{p}$; set $\partial \bar{p}/\partial x = -1$.)

If a term is positive it means that it pushes the fluid particle in the positive x_1 direction. What about the viscous term? Is it always negative? Where is it largest? At that point, which term balances it? How large is the third term? Why? The pressure term should be a *driving* force. The Reynolds stress term, where is it positive and where is it negative?

F.6 Wall shear stress

Compute the wall shear stress at both walls. They should be exactly equal. Are they?

F.7 Resolved stresses

In Section F.4 you computed the mean velocities. From the instantaneous and the mean velocity, you can compute the fluctuations as

$$v_i' = v_i - \bar{v}_i \tag{F.4}$$

Now you can easily compute all stresses $\overline{v_i' v_j'}$. Plot the normal stresses in one figure and the shear stresses in one figure (plot the stresses over the entire channel, i.e. from $x_2 = 0$ to $x_2 = 2h$). Which shear stresses are zero?

F.8 Fluctuating wall shear stress

In the same way as the velocity, the wall shear stress can be decomposed into a mean value and a fluctuation. In general, any fluctuating variable, ϕ, can be decomposed into a mean and fluctuation as $\phi = \bar{\phi} + \phi'$. The root-mean-square (RMS) is then defined as

$$\phi_{rms} = \left(\overline{\phi'^2}\right)^{1/2} \tag{F.5}$$

Compute the RMS of the wall shear stress. This is a measure of the fluctuating tangential force on the wall due to turbulence. If heat transfer is involved, the fluctuating temperature at the wall inducing fluctuating heat transfer may be damaging to the material of the walls causing material fatigue. This is probably the most common form of fluid-solid interaction.

F.9 Production terms

In order to understand why a stress is large, it is useful to look at its transport equation, see Eq. 9.12. Usually, a stress is large when its production term, P_{ij}, is large (there may be exceptions when other terms, such as the diffusion term, are largest). Plot the production terms for all non-zero stresses across the entire channel. Which ones are zero (or close to)? Why? Does any production term change sign at the centerline? If so, what about the sign of the corresponding shear stress plotted in Section F.7?

F.10   Pressure-strain terms

The pressure-strain term reads (see Eq. 9.14)

$$\Pi_{ij} = \overline{\frac{p'}{\rho} \left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i} \right)} \tag{F.6}$$

Our data are obtained from incompressible simulations, in which the pressure may vary unphysically in time ($\partial p/\partial t$ does not appear in the equations). Hence, we prefer to compute the velocity-pressure gradient term

$$\Pi_{ij}^p = -\overline{\frac{v_i'}{\rho} \frac{\partial p'}{\partial x_j}} - \overline{\frac{v_j'}{\rho} \frac{\partial p'}{\partial x_i}}, \tag{F.7}$$

see the second line in Eq. 9.3. The pressure diffusion term in the $\overline{v_2'^2}$ equation – which is the difference between Eqs. F.6 and F.7 (the two first terms in Eq. 9.8) – is small except very close to the wall (see Figs. 9.2 and 9.3). Hence, the difference between Π_{ij}^p and Π_{ij} is small.

Download the data file `p_inst_small.mat` and the Matlab file `pl_press_strain.m` which reads the data file. The time histories of the pressure along five x_2 lines [(x_1, x_2, x_3), $(x_1 \pm \Delta x_1, x_2, x_3)$ and $(x_1, x_2, x_3 \pm \Delta x_3)$] are stored in this file. This allows you to compute all the three spatial derivatives of p'. Using the velocities stored in `uvw_inst_small.dat` (see Section F.4), you can compute all the terms in Eq. F.7.

Plot the pressure strain, Π_{ij}^p, for the three normal stresses and the shear stress across the channel. For which stresses is it negative and positive? Why?

Which term do you think is the largest source and sink term, respectively, in Eq. 9.12?

F.11   Dissipation

The dissipation of turbulent fluctuations (i.e. transformation of turbulent kinetic energy into internal energy, i.e. increased temperature) reads (see Eq. 9.12)

$$\varepsilon_{ij} = \nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}} \tag{F.8}$$

Download the file `diss_inst.mat` and the Matlab file `pl_diss.m` which reads it. The data file includes the time history of the velocities along five x_2 lines [(x_1, x_2, x_3), $(x_1 \pm \Delta x_1, x_2, x_3)$ and $(x_1, x_2, x_3 \pm \Delta x_3)$] so that you can compute all spatial derivatives. The data cover only the lower half of the channel. Compute and plot all components of ε_{ij}. Which is largest and which are close to zero (they would be identically zero had I had provided more samples, i.e. longer time histories)? Plot also $\varepsilon = 0.5\varepsilon_{ii}$, i.e.

$$\varepsilon = \nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_i'}{\partial x_k}} \tag{F.9}$$

Where is it largest and why? In which equation does this quantity appear?

Let us now consider the equations for the mean kinetic energy, $K = \bar{v}_i \bar{v}_i / 2$ (Eq. 8.29) and turbulent kinetic energy, $k = \overline{v_i' v_i'}/2$ (Eq. 8.13). The dissipation in the K equation reads

$$\varepsilon_{mean} = \nu \frac{\partial \bar{v}_i}{\partial x_k} \frac{\partial \bar{v}_i}{\partial x_k} \tag{F.10}$$

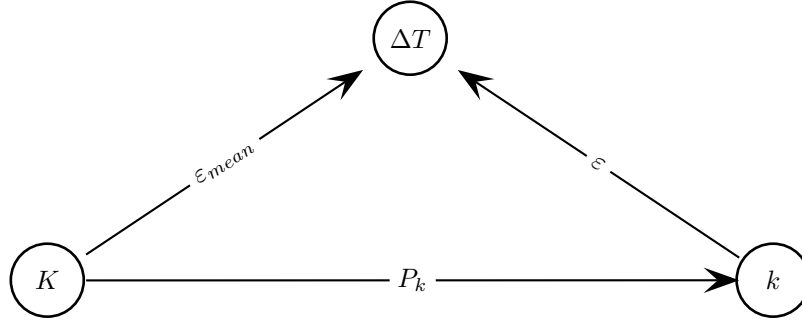Figure F.1: Transfer of energy between mean kinetic energy (K), turbulent kinetic energy (k) and internal energy (denoted as an increase in temperature, ΔT). $K = \frac{1}{2}\bar{v}_i\bar{v}_i$ and $k = \frac{1}{2}\overline{v'_i v'_i}$. The dissipations, ε and ε_{mean}, are defined in Eqs. F.9 and F.10, respectively.

The flow of kinetic energy between K, k and ΔT is illustrated in Fig. F.1. Compute and plot also ε_{mean} and P_k. Which is large and which is small? How is the major part of the kinetic energy transformed from K to ΔT? Is it transformed via k or directly from K to ΔT?

F.12 Two-point correlations

Two-point correlations are used to investigate over how large spatial distance the turbulence is correlated, see Section 10.1. Usually it is normalized so that the two-point correlation varies between one and minus one. Here we will compute the two-point correlations of v'_3 in the x_3 direction in which the flow (and turbulence) is *homogeneous* (this means that the derivative with respect to this spatial coordinate of all *time-averaged* quantities is zero, i.e. $\partial \bar{v}_1/\partial x_3 = 0$, $\partial \overline{v'_i v'_j}/\partial x_3 = 0$, $\partial \varepsilon/\partial x_3 = 0$ etc).

Download the data file w_inst_k.mat and y_half_every_forth.dat, and the Matlab file pl_twocorr.m which reads the data files. The data file includes the time history of v'_3 in the lower half of the channel and only for every fourth grid point in the x_2 direction. For every x_2 grid point the values along the x_3 are included (only every second point).

Now we will compute the two-point correlation between the $x_3 = 0$ node and all other x_3 nodes. For this case the normalized two-point correlation of v'_3 reads

$$
\begin{aligned}
B_{v'_3 v'_3}^{norm}(x_2, \hat{x}_3) &= \frac{1}{v_{3,rms}^2}\overline{v'_3(x_2, x_3 = 0)v'_3(x_2, x_3)} \\
&= \frac{1}{v_{3,rms}^2}\overline{v'_3(x_2)v'_3(x_2, \hat{x}_3)}
\end{aligned}
\tag{F.11}
$$

where $\hat{x}_3 = x_3 - 0 = x_3$ is the separation distance between the two point. Compute and plot $B_{v'_3 v'_3}^{norm}(x_2, \hat{x}_3)$ for different x_2 locations. Does it increase or decrease for increasing x_2? Explain why.

From the two-point correlation, the integral length scale can be computed, see Eq. 10.5. Compute the integral length scale and plot it versus x_2. The upper limit in this integral is infinity (∞). You can either carry out the integration using all available data in the x_3 direction; in general this may not be optimal and another option

is to terminate the integration at the point where the two-point correlation becomes negative. Compare these two options.

When plotting two-point correlations, it is no point showing both symmetric parts; show only half of it (cf. the two-point correlations in Section 10.1).

The integral length scale is – as the word implies – the integral (i.e. an average) of the size (length) of all eddies. It is mathematically equal to the area below the B^{norm} function. How large is it compared to half the channel width (which corresponds to the boundary layer thickness)? How does the integral length compare to the wall distance? Can it be larger than x_2? Can an eddy be larger than the wall distance?

F.13 Do something fun!

You have been provided with a lot of data which you have analyzed in many ways. Now think of some other way to analyze the data. There are many interesting things yet to be analyzed!