Data on Case 7.2, submitted to ERCOFTAC/IAHR Turbulence Modeling Workshop -98

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Case 7.2: Two-Dimensional Flow and Heat Transfer over a Smooth Wall Roughened with Square Sectioned Ribs

The prediction of the flow field and heat transfer was made on a domestic finite volume code, CALC-BFC, [2]. The code uses co-located grid with SIMPLEC pressure correction. The differencing schemes were either hybrid, QUICK, or van Leer.
Two different mesh were used: $ni \times nj = 82 \times 70$ and $ni \times nj = 82 \times 120$ (stream-wise and wall normal respectively). The meshes were refined (with tanh) both at the upper and lower wall and on each surface of the rib.
Periodic boundary condition was used in the stream-wise ($x-$) direction. For all variables this was set as: $\phi(1) = \phi(ni-1)$ and $\phi(ni) = \phi(2)$.
To simulate the friction force a source term was included in the x-momentum equation:

$$-\frac{d\overline{p}}{dx} \cdot \mathrm{Vol} \tag{1}$$

The mean pressure gradient, had to be iteratively found.
The periodic temperature could be simulated by introducing a source term in the energy equation:

$$\rho U \frac{d\overline{T}}{dx} \cdot \mathrm{Vol} \tag{2}$$

The mean temperature gradient can be found from global conservation of heat, according to:

$$\frac{d\overline{T}}{dx} = \frac{q}{c_p \rho \overline{U} H} \tag{3}$$

Turbulence Models

Two different eddy viscosity turbulence models were used, a two-layer $k-\varepsilon$ model and a $k-\omega$ model.
The two-layer $k-\varepsilon$ model by Chen and Patel [1] uses the standard $k-\varepsilon$-model [3] in the main flow and close to the wall only the k-equation, and fixing the dissipation rate according to:

$$\begin{aligned} \varepsilon &= \frac{k^{3/2}}{l_\varepsilon} \\ l_\varepsilon &= C_l y (1 - \exp(-R_y/A_\varepsilon)) \end{aligned} \tag{4}$$

with the eddy viscosity set as:

$$\begin{aligned} \nu_t &= C_\mu \sqrt{k} l_\mu \\ l_\mu &= C_l y (1 - \exp(-R_y / A_\mu)) \end{aligned} \tag{5}$$

The involved parameters are:

$$R_y = \sqrt{k} y / \nu, \qquad A_\varepsilon = 2C_l, \qquad A_\mu = 70, \qquad C_l = \kappa C_\mu^{-3/4} \tag{6}$$

In the code the program switches to the one-equation model at a pre-fixed cell-node which approximately equals $R_y = 250$ or $y^+ = 50$.
The $k - \omega$ model by Peng *et al* [5] is:

$$\begin{aligned} \frac{Dk}{Dt} &= 2\nu_t S_{ij} S_{ij} - c_k f_k k\omega + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \\ f_k &= 1 - 0.722 \exp\left[-\left(\frac{R_t}{10}\right)^4\right] \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{D\omega}{Dt} &= C_{\omega 1}\frac{\omega}{k} 2\nu_t S_{ij} S_{ij} - C_{\omega 2}\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + C_\omega \frac{\nu_t}{k}\left(\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}\right) \\ f_\omega &= 1 + 4.3 \exp\left[-\left(\frac{R_t}{1.5}\right)^{1/2}\right] \end{aligned} \tag{8}$$

with the eddy-viscosity set as:

$$\begin{aligned} \nu_t &= C_\mu f_\mu \frac{k}{\omega} \\ f_\mu &= 0.025 + \left[1 - \exp\left\{-\left(\frac{R_t}{10}\right)^{3/4}\right\}\right]\left[0.975 + \frac{0.001}{R_t}\exp\left\{-\left(\frac{R_t}{200}\right)^2\right\}\right] \end{aligned} \tag{9}$$

The constants in this model are:

$$\begin{aligned} &C_\mu = 1.0, \quad C_k = 0.09, \quad \sigma_\omega = 1.35, \quad \sigma_k = 0.8 \quad C_\omega = 0.75 \\ &C_{\omega 1} = 0.42, \quad C_{\omega 2} = 0.075 \end{aligned} \tag{10}$$

In this case due to the infinite value of ω at the wall the first two cells have a fixed value according to:

$$\omega = \frac{6\nu}{C_{\omega 2} y^2} \tag{11}$$

Heat Transfer Model

The model used for heat transfer is the standard Boussinesq approximation:

$$\overline{u_i T} = -\frac{\nu_t}{Pr_t}\frac{\partial T}{\partial x_i} \tag{12}$$

The turbulent Prandtl number was made dependent on the wall distance, according to Kays and Crawford [4]:

$$Pr_t = \left[0.5882 + 0.228\left(\frac{\nu_t}{\nu}\right) - 0.0441\left(\frac{\nu_t}{\nu}\right)^2\left\{1 - \exp\left(\frac{-5.165}{\nu_t/\nu}\right)\right\}\right]^{-1} \tag{13}$$

The viscosity was fixed, and were not allowed to changed with temperature (eg through Sutherland's Law). The Prandtl number was fixed to $Pr = 0.72$ (air). A constant heat-flux, q was fixed along the ribbed wall, with the heat-flux on each rib-wall was set to $q/3$. The non-ribbed wall was adiabatic. The predicted Nusselt-number for $Re = 12600$ was scaled with the Nusselt-number for smooth channel, according to Dittus-Boelter:

$$Nu_s = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \tag{14}$$

References

[1] H.C. Chen and V.C. Patel. Near-wall turbulence models for complex flows including separation. *AIAA Journal*, 26:641–649, 1988.

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[3] W.P. Jones and B.E. Launder. The prediction of laminarization with a two-equation model of turbulence. *Int. J. Heat and Mass Transfer*, 15:301–314, 1972.

[4] W.M. Kays. Turbulent Prandtl number - where are we? *J. Heat Transfer*, 116:284–295, 1994.

[5] S-H. Peng, L. Davidson, and S. Holmberg. A modified low-Reynolds-number $k-\omega$ model for recirculating flows. *J. Fluid Engineering*, 119:867–875, 1997.