HYBRID LES-RANS: ESTIMATION OF RESOLUTION USING TWO-POINT CORRELATIONS, ENERGY SPECTRA AND DISSIPATION SPECTRA IN RE-CIRCULATING FLOW

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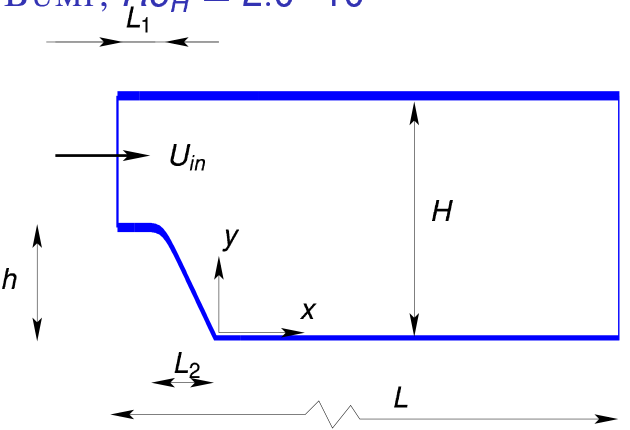
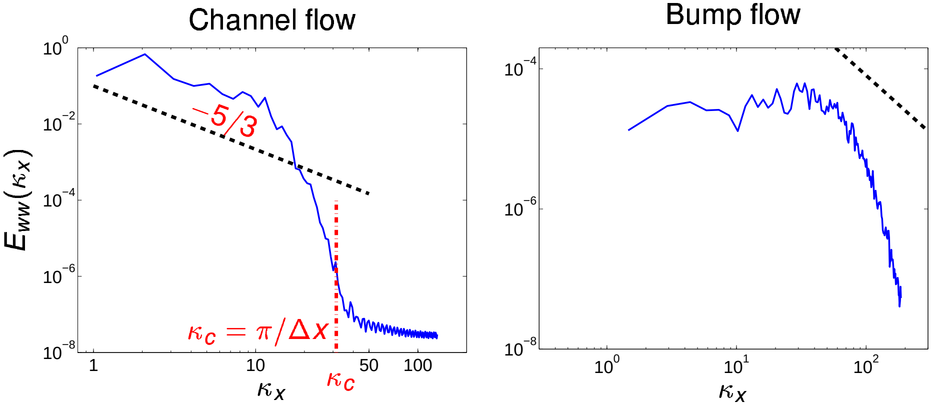
ONERA BUMP, $Re_H = 2.0 \cdot 10^6$



- $\delta_{in}/H = 0.043$, $h/H = 0.46$, Mesh: $221 \times 122 \times 32/64/128$
- $W/H = 1.67$ in expts. Here: $W_{slice}/H = 0.61$ (no side walls)
- $\Delta x/\delta_{in} = 0.33$, $\Delta z/\delta_{in} = 0.44/0.22/0.11$. $\Delta x^+ = 1300$ and $\Delta z^+ = 1800/900/450$.

Energy Spectra from Time Series

- From frequency to wavenumber $\kappa_x = 2\pi f / U$



$Re_\tau = 4000, Re_H = 2 \cdot 10^5$
$\Delta x/\delta = 0.1, \Delta x^+ = 400$

$Re_H = 2 \cdot 10^6, x/H = -1$
$\Delta x/\delta_{in} = 0.33, \Delta x^+ = 900.$

Energy Spectra from Time Series

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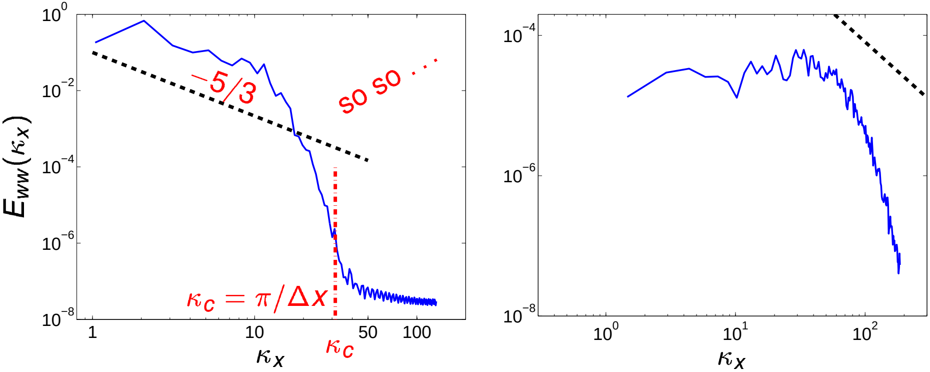
Channel flow — Bump flow



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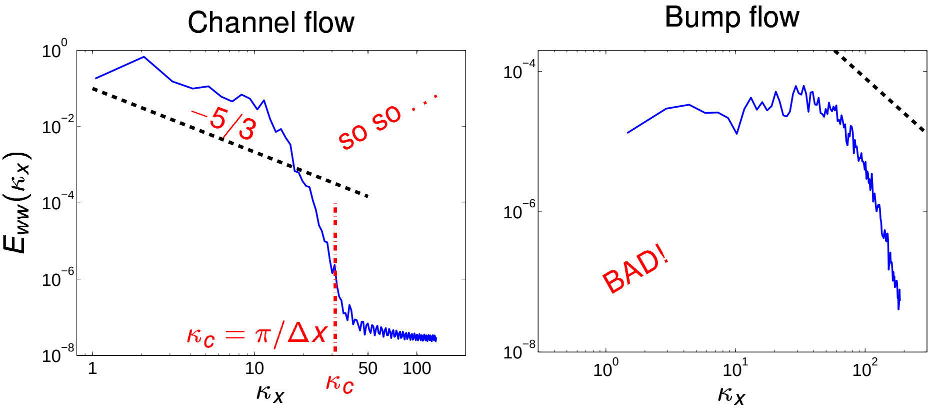

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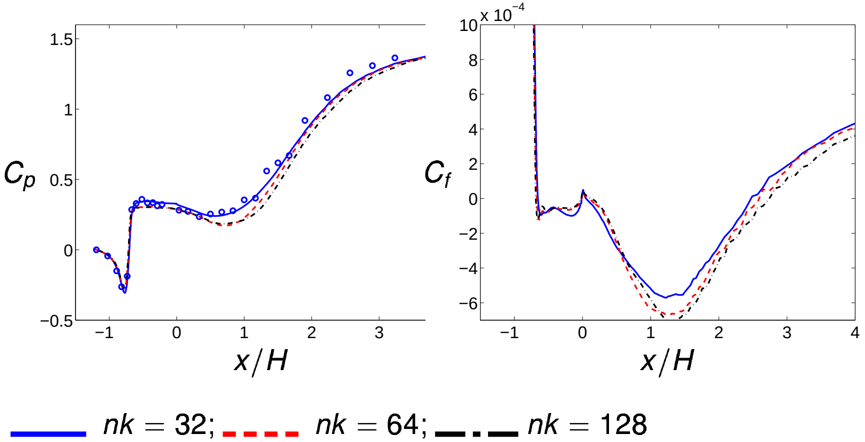
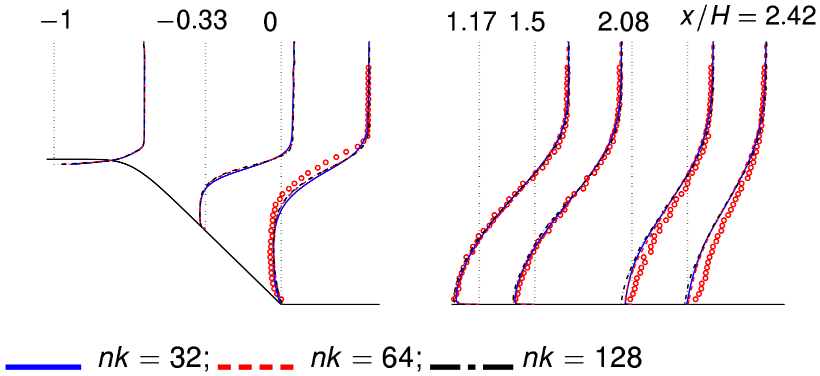
Computational Method

- Finite volume with central differencing in space and time (Crank-Nicolson)
- Fractional step
- Hybrid LES-RANS: a one-equation k_{sgs} model in both regions with machting along a fixed grid line
- Mesh: $221 \times 122 \times 32/64/128$, $CFL_{max} \simeq 2$.

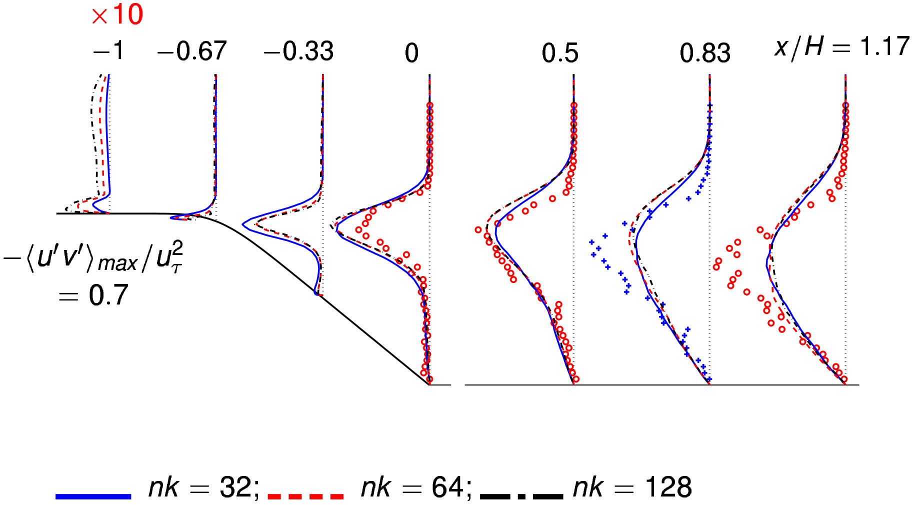
Results: Pressure and Skin Friction



Results: Velocities



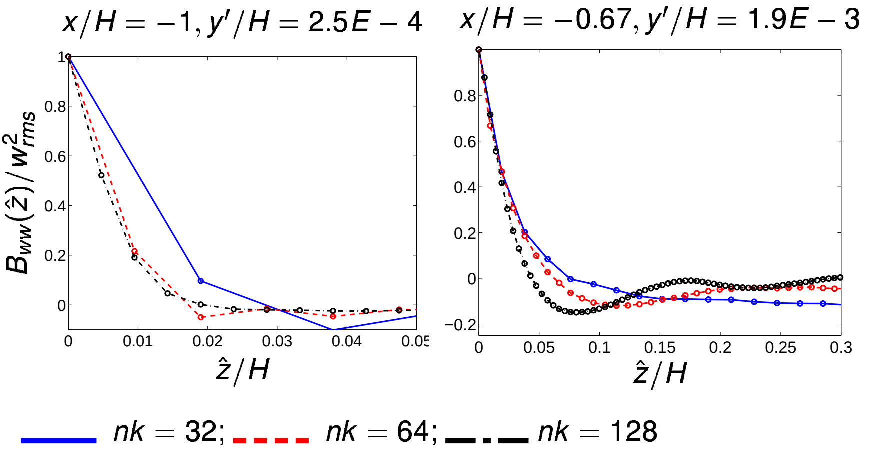
Results: Resolved Shear Stresses



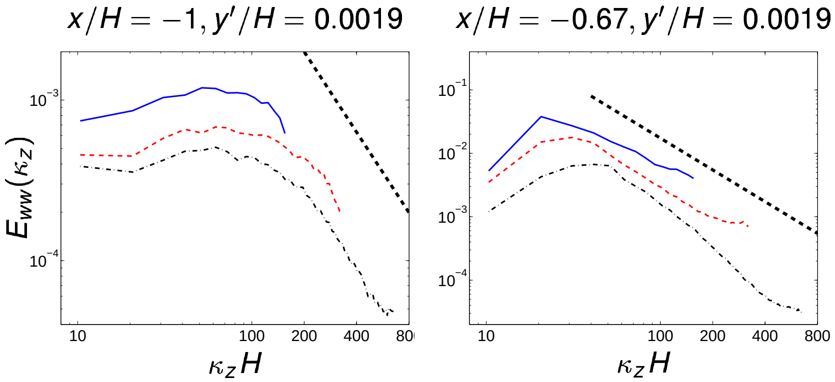
$nk = 32$; $nk = 64$; $nk = 128$

- At $-0.5 < x/H < 3$, $u_{rms,peak} > 0.3 U_{out}$.

Normalized two-point correlation, $B_{ww}(\hat{z})/w^2_{rms}$



ENERGY SPECTRA $E_{ww}(\kappa_z)$.



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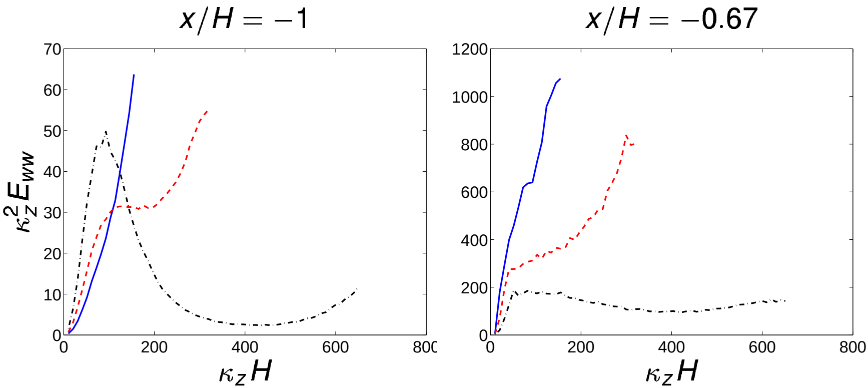
Estimated Dissipation vs. κ_z From $E(\kappa_z)$

The dissipation ε_{wz} can – in theory – be obtained from

$$\varepsilon^E_{wz} = 2\nu \left\langle \left(\frac{\partial w'}{\partial z} \right)^2 \right\rangle = 2\nu \frac{\partial^2 B_{ww}(\hat{z})}{\partial \hat{z}^2}\bigg|_{\hat{z}=0} = 2\nu \sum_{k=1}^{N} \kappa_z^2 E_{ww}(k)$$



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$\varepsilon^E_{wz} \neq \varepsilon^{FV}_{wz}$

Exact Dissipation vs. κ_Z

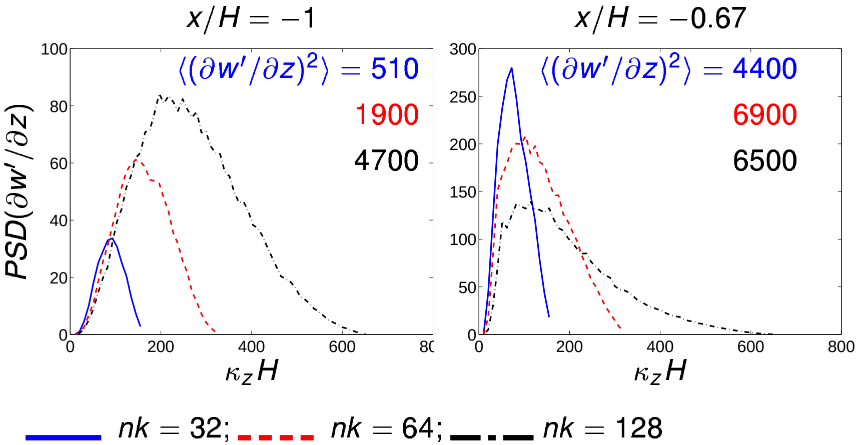
Instead, form a DFT of $\partial w'/\partial z$ as

$$\hat{W}_z(k) = \frac{1}{N}\sum_{n=1}^{N} \frac{\partial w'(n)}{\partial z} \exp\left(\frac{-\imath 2\pi(n-1)(k-1)}{N}\right)$$

where $\hat{W}$ are the Fourier coefficients of $\partial w'/\partial z$ and then create the Power Spectral Density, i.e. $\hat{W}_z * \hat{W}_z^*$. Then indeed

$$\varepsilon_{wz}^{FV} = 2\nu \sum_{k=1}^{N} \hat{W}_z * \hat{W}_z^* = 2\nu \sum_{k=1}^{N} PSD(\partial w'/\partial z)$$

RESULTS: EXACT DISSIPATION VS. κ_z



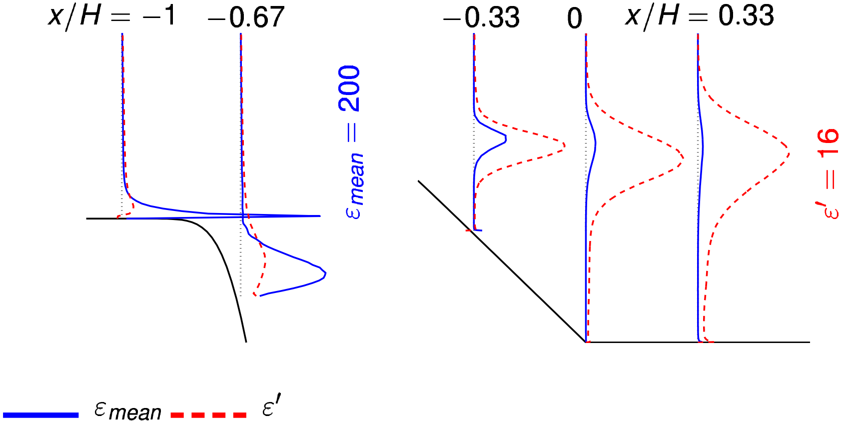
- Note that dissipation takes place at rather small wave numbers

RESULTS: SGS DISSIPATIONS. $nk = 128$

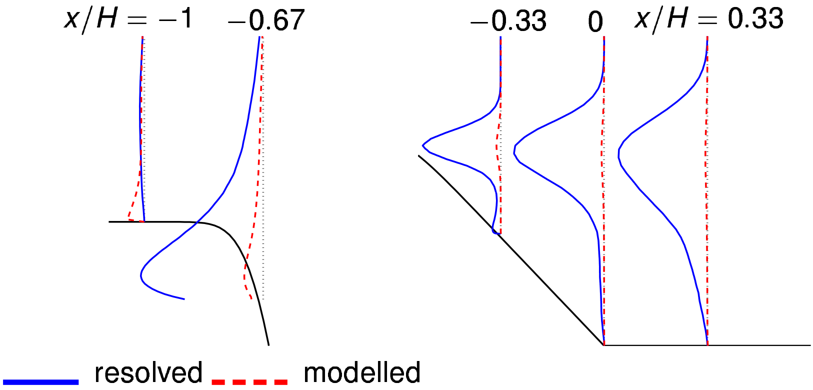
$\varepsilon = 2\langle \nu_T \bar{s}_{ij}\bar{s}_{ij}\rangle$ SGS dissipation in $\langle \bar{u}_i\bar{u}_i\rangle/2$-eq.

$\varepsilon_{mean} = 2\langle \nu_T\rangle\langle \bar{s}_{ij}\rangle\langle \bar{s}_{ij}\rangle$ SGS dissipation in $\langle \bar{u}_i\rangle\langle \bar{u}_i\rangle/2$-eq.

$\varepsilon' = \varepsilon - \varepsilon_{mean}$ SGS dissipation in $\langle u'_i u'_i\rangle/2$-eq.



Resolved and Modelled Shear Stresses. $nk = 32$



Conclusions: Estimating Resolution

- Useful quantities
  - Two-point correlations
  - Ratio $\varepsilon'_{SGS}/\varepsilon_{SGS,mean}$
  - Ratio of resolved and modelled stresses?

Conclusions: Estimating Resolution

- Useful quantities
  - Two-point correlations
  - Ratio $\varepsilon'_{SGS}/\varepsilon_{SGS,mean}$
  - Ratio of resolved and modelled stresses?
- Not Useful quantities
  - Energy spectra (resolution is often over-estimated)
  - The power spectral density of the resolved velocity gradients
  - The dissipation component $\varepsilon_{wz} = \kappa_z^2 E_{ww}$