2020-08-18, Exam in

Turbulence modeling, MTF270: Answers

1

a) **Show that the term below is symmetric, dimension-less and trace-less.**

$$T_{ij}^5 = \frac{k^3}{\varepsilon^3}\left(\bar{\Omega}_{ik}\bar{s}_{km}\bar{s}_{mj} - \bar{s}_{im}\bar{s}_{mk}\bar{\Omega}_{kj}\right) \tag{1}$$

Symmetric

Switch index i and j

$$\bar{\Omega}_{jk}\bar{s}_{km}\bar{s}_{mi} - \bar{s}_{jm}\bar{s}_{mk}\bar{\Omega}_{ki}$$

We know that s_{ij} is symmetric and $\bar{\Omega}_{ij}$ anti-symmetric. Hence, we can write

$$-\bar{\Omega}_{kj}\bar{s}_{mk}\bar{s}_{im} + \bar{s}_{mj}\bar{s}_{km}\bar{\Omega}_{ik} \tag{2}$$

Term 1 in Eq. 2 and Term 2 1 are the same. Term 2 in Eq. 2 and Term 1 1 are the same. Hence, T_{ij}^5 is symmetric.

Traceless

Set $i = j$ gives

$$\bar{\Omega}_{ik}\bar{s}_{km}\bar{s}_{mi} - \bar{s}_{im}\bar{s}_{mk}\bar{\Omega}_{ki} = 2\bar{\Omega}_{ik}\bar{s}_{km}\bar{s}_{mi} = 2\bar{\Omega}_{ik}A_{ki} \tag{3}$$

where $A_{ki} = \bar{s}_{km}\bar{s}_{mi}$ is a symmetric tensor. Hence, Eq. 3 is zero because it is an inner product of a symmetric tensor and an anti-symmetric tensor.

b) **The $k-\omega$ SST model includes two transport equations, the k and ω equations. …**

The reason is that in the outer region the coefficient are blended into coefficient of the $k-\varepsilon$ model.

2

Consider the transport equation for the Reynolds stresses ...

Look at Section 9.1 and repeat the derivation. Here, the only velocity gradient is $\partial\bar{v}_3/\partial x_1$. Note that the shear stress is now positive (since $\partial\bar{v}_3/\partial x_1 < 0$.

3

a) **Consider the Reynolds stress model. The slow pressure strain model for the wall effect reads...**

$$\Phi_{ij,1w} = c_{1w}\frac{\varepsilon}{k}\left(\overline{v'_k v'_m} n_{k,w} n_{m,w}\delta_{ij} - \frac{3}{2}\overline{v'_k v'_i} n_{k,w} n_{j,w} - \frac{3}{2}\overline{v'_k v'_j} n_{i,w} n_{k,w}\right) f$$

Since the wall is in the $x_2 - x_3$ plane, we set $n_i = (0,0,1)$. For example, the first term will then read

$$\overline{v'_3 v'_3} n_{3,w} n_{3,w}\delta_{ij} = \overline{v'_3 v'_3}\delta_{ij}$$

b) **Consider a 2D finite volume grid. In scale-similarity we filter twice ...**

Look at Section 18.12.1 in the eBook. Divide the square into four sub-squares with centers at $(I-1/4, J-1/4)$, $(I+1/4, J-1/4)$, $(I-1/4, J+1/4)$, $(I+1/4, J+1/4)$. Then compute the twice-filtering velocity as

$$\bar{\bar{v}} = \frac{1}{4}\left(\bar{v}_{I-1/4,J-1/4} + \bar{v}_{I+1/4,J-1/4} + \bar{v}_{I-1/4,J+1/4} + \bar{v}_{I+1/4,J+1/4}\right)$$

$\bar{v}_{I-1/4,J-1/4}$, for example, is computed as

$$\bar{v}_{I-1/4,J-1/4} = \frac{1}{4}\left(\bar{v}_{I-1/2,J-1/2} + \bar{v}_{I,J-1/2} + \bar{v}_{I,J} + \bar{v}_{I-1/2,J}\right)$$

3

a) **Explain the difference between ε_κ, $\varepsilon(\kappa)$ and ...**

See Fig. 8.2 and the text related to the figure.

b) **Consider a $k-\tau$ turbulence model ...**

Look at Eq. 16.1. in the eBook. Here we have $\tau = k/\varepsilon$. Hence

$$\frac{d\tau}{dt} = \frac{d}{dt}\left(\frac{k}{\varepsilon}\right) = \frac{1}{\varepsilon}\frac{dk}{dt} + k\frac{d(1/\varepsilon}{dt} = \frac{1}{\varepsilon}\frac{dk}{dt} - \frac{k}{\varepsilon^2}\frac{d\varepsilon)}{dt}$$

Then the production of the τ equation, for example, is

$$P^\tau = \frac{1}{\varepsilon}P^k - \frac{k}{\varepsilon^2}P^\varepsilon$$

5

Consider the flow in a boundary layer along a plane located at $x_2 = 0$. ...

See Assignment 2a and 2b.