2020-06-02, Exam in

Turbulence modeling, MTF270: Answers

1

a) **Show that the term below in the non-linear model is symmetric and traceless.**

$$\bar{\Omega}_{i\ell}\bar{\Omega}_{\ell m}\bar{s}_{mj} + \bar{s}_{i\ell}\bar{\Omega}_{\ell m}\bar{\Omega}_{mj} - \frac{2}{3}\bar{\Omega}_{mn}\bar{\Omega}_{n\ell}\bar{s}_{\ell m}\delta_{ij} \tag{1}$$

Symmetric

Switch index i and j

$$\bar{\Omega}_{j\ell}\bar{\Omega}_{\ell m}\bar{s}_{mi} + \bar{s}_{j\ell}\bar{\Omega}_{\ell m}\bar{\Omega}_{mi} - \frac{2}{3}\bar{\Omega}_{mn}\bar{\Omega}_{n\ell}\bar{s}_{\ell m}\delta_{ji} \tag{2}$$

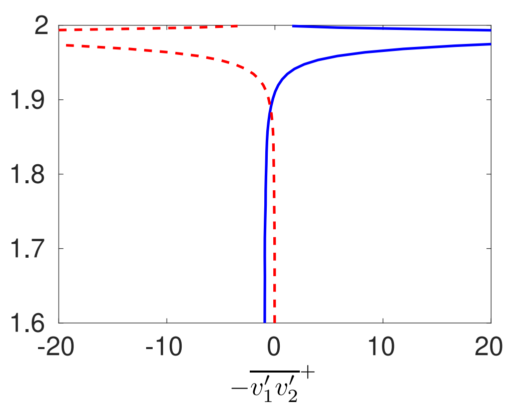
We know that $\delta_{ij} = \delta_{ji}$. Hence, Term 3 in Eqs. 1 and 2 are the same.

Furthermore, $\bar{s}_{ij} = \bar{s}_{ji}$ and $\bar{\Omega}_{ij} = -\bar{\Omega}_{ji}$.

Term 1 in Eq. 2: switch index for all variables: $\bar{\Omega}_{j\ell}\bar{\Omega}_{\ell m}\bar{s}_{mi} = \bar{\Omega}_{\ell j}\bar{\Omega}_{m\ell}\bar{s}_{im} = \bar{s}_{im}\bar{\Omega}_{m\ell}\bar{\Omega}_{\ell j}$ which is the same as Term 2 in Eq. 1 (different dummy indices)

Term 2 in Eq. 2: switch index for all variables: $\bar{s}_{j\ell}\bar{\Omega}_{\ell m}\bar{\Omega}_{mi} = \bar{s}_{\ell j}\bar{\Omega}_{m\ell}\bar{\Omega}_{im} = \bar{\Omega}_{im}\bar{\Omega}_{m\ell}\bar{s}_{\ell j}$ which is the same as Term 1 in Eq. 1 (different dummy indices)

b) **Consider the flow in the upper half of a fully-developed channel flow ...**



Blue solid line: diffusion term by resolved turbulence; red dashed line: viscous diffusion term; $\partial\bar{p}/\partial x \equiv 1$. Far from the wall, the diffusion term by resolved turbulence and the pressure gradient balance each other. The non-zero terms at the wall are the viscous diffusion term and the pressure gradient.

2

a) **Consider the transport equation for the Reynolds stresses . . .**

Fig. 9-6 show the terms in the $\overline{v_1'v_2'}$ equation in the lower half of the channel where $\overline{v_1'v_2'} < 0$. In the upper half, the $\overline{v_1'v_2'}$ is mirrored (i.e. it is positive). Hence the source terms in Fig. 9.6b will have the same form in the upper half as in the lower half but with opposite sign (where x_2^+ denotes the distance from the upper wall). At the wall the pressure strain and the pressure diffusion are non-zero (as in Fig. 9-6a).

b) **Streamline curvature: consider a boundary layer, $\delta(x_1)$, . . .**

Look at Section 12.2 in the eBook. The streamline bends upwards in Fig. 12.3 and hence $\partial \bar{v}_2/\partial x_1 > 0$. Here the streamline bends downwards (it follows the upper surface) and hence $\partial \bar{v}_2/\partial x_1 < 0$. You should show Eqs. 12.6a-c and do the same discussion as below Eqs. 12.6a-c but vice-versa (i.e. the magnitude of P_{12} decreases etc).

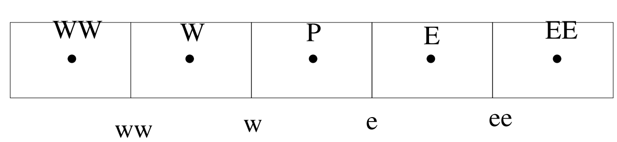
3

a) **Consider the V2F model. Explain the difference between v^2 and $\overline{v_2'^2}$. . . .**

In Eq. 15.9 in the eBook the equations in the V2F model are shown. The key term is the production term P_k in Φ_{22} which is the source term in the f equation. kf is in term the main source term (corresponding to the pressure-strain term) in the v^2 equation.

- In a boundary layer along a $x_1 - x_3$ plane the dominating velocity gradient is $\partial \bar{v}_1/\partial x_2$ or $\partial \bar{v}_3/\partial x_2$. That corresponds to the rapid pressure-strain term in the $\overline{v_2'^2}$ equation in Section 11.9, Hence $v^2 = \overline{v_2'^2}$.
- In a boundary layer along a $x_2 - x_3$ plane the dominating velocity gradient is $\partial \bar{v}_2/\partial x_1$ or $\partial \bar{v}_3/\partial x_1$. That corresponds to the rapid pressure-strain term in the $\overline{v_2'^2}$ equation. Hence $v^2 = \overline{v_1'^2}$.
- In a boundary layer along a $x_1 - x_2$ plane the dominating velocity gradient is $\partial \bar{v}_1/\partial x_3$ or $\partial \bar{v}_2/\partial x_3$. That corresponds to the rapid pressure-strain term in the $\overline{v_3'^2}$ equation. Hence $v^2 = \overline{v_3'^2}$.

b) **Consider a 1D finite volume grid. In SGS models based on the dynamic procedure we use test filtering. . . .**



Our cell is at P (see figure above). Now we should integrate over four control volumes, i.e. from node WW to node EE (cf. Section 12.2 in the eBook). We split it into four integrals

$$
\begin{aligned}
\widehat{\bar{v}_P} &= \frac{1}{4\Delta x}\int_{WW}^{EE}\bar{v}dx = \frac{1}{4\Delta x}\left(\int_{WW}^{W}\bar{v}dx + \int_{W}^{P}\bar{v}dx + \int_{P}^{E}\bar{v}dx + \int_{E}^{EE}\bar{v}dx\right)\\
&= \frac{1}{4\Delta x}\left(\bar{v}_{ww}\Delta x + \bar{v}_w\Delta x + \bar{v}_e\Delta x + \bar{v}_{ee}\Delta x\right)\\
&= \frac{1}{4}\left(\frac{\bar{v}_{WW}+\bar{v}_W}{2} + \frac{\bar{v}_W+\bar{v}_P}{2} + \frac{\bar{v}_P+\bar{v}_E}{2} + \frac{\bar{v}_E+\bar{v}_{EE}}{2}\right)\\
&= \frac{1}{8}\left(\bar{v}_{WW} + 2\bar{v}_W + 2\bar{v}_P + 2\bar{v}_E + \bar{v}_{EE}\right)
\end{aligned}
$$

4

a) **Explain the energy transfer in the figure below.**

- $\bar{K}$ is the mean kinetic energy $\langle\bar{v}_i\rangle\langle\bar{v}_i\rangle/2$
- $-\left\langle\bar{v}'_i\bar{v}'_j\right\rangle\dfrac{\partial\langle\bar{v}_i\rangle}{\partial x_j}$ is the production term of $\bar{k} = \langle\bar{v}'\bar{v}'\rangle/2$
- $\langle k_{sgs}\rangle$ is the SGS kinetic energy
- $2\langle\nu_{sgs}\rangle\langle\bar{s}_{ij}\rangle\langle\bar{s}_{ij}\rangle$ is the SGS dissipation of $\bar{K}$
- ΔT is the temperatue increase due to viscous dissipation
- $\nu\dfrac{\partial\langle\bar{v}_i\rangle}{\partial x_j}\dfrac{\partial\langle\bar{v}_i\rangle}{\partial x_j}$ is the viscous dissipation by mean flow of $\bar{K}$
- ε is the viscous dissipation of k_{sgs}
- $\nu\left\langle\dfrac{\partial\bar{v}'_i}{\partial x_j}\dfrac{\partial\bar{v}'_i}{\partial x_j}\right\rangle$ is the viscous dissipation of $\bar{k}$ by resolved turbulence
- ε'_{sgs} is viscous dissipation of $\bar{k}$

b) **Consider the energy spectrum . . .**

To turn the model into an DES model we will modify the dissipation term, see Eq. 20.4 in the eVook. Here, the dissipation term reads

$$C_k\frac{k^2}{\tilde{\nu}_t}$$

By compering with Eq. 20.4 we get

$$\max\left(C_\varepsilon\frac{k^{3/2}}{\Delta}, C_k\frac{k^2}{\tilde{\nu}_t}\right)$$

5

Consider the flow in a boundary later along a plane located at $x_2 = 0$.

See Assignments 2a and 2b.