2016-05-31, Exam in

Turbulence modeling, MTF270

- **Time:** 14.00-18.00 **Location:** M
- **Teacher:** Lars Davidson, phone 772 1404, 0730-791 161
- **Aids**
  - Formula sheet, 2 pages, appended to the exam
- **Checking the evaluation and results of your written exam**: at 12-13 on 22 and 23 June in room Navier
- **Grading**: 20-29p: 3, 30-39: 4, 40-50: 5.

T1. a) Show the principles how to derive the transport equation for $\overline{v_i' v_j'}$. Explain the physical meaning of the different terms (see Formula sheet). (5p)

b) Show how the production term is modeled in the $k-\varepsilon$ model (Boussinesq)? Show how it can be expressed in $\bar{s}_{ij}$ (5p)

T2. a) The slow pressure-strain model reads $\Phi_{ij,1} = -c_1 \rho \frac{\varepsilon}{k}\left(\overline{v_i' v_j'} - \frac{2}{3}\delta_{ij} k\right)$. The anisotropy tensor is defined as $a_{ij} = \frac{\overline{v_i' v_j'}}{k} - \frac{2}{3}\delta_{ij}$. Show that for decaying grid turbulence, the model for the slow pressure-strain model indeed acts as to make the turbulence more isotropic if $c_1 > 1$. (5p)

b) Consider streamline curvature for a streamline formed as a circular arc (convex curvature). Show that the turbulence is dampened if $\partial v_\theta / \partial r > 0$ and that it is enhanced if the sign of $\partial v_\theta / \partial r$ is negative. (5p)

Hint: The simplified momentum equation reads $\frac{\rho v_\theta^2}{r} - \frac{\partial p}{\partial r}$.

T3. a) The f equation in the V2F model reads (5p)

$$L^2 \frac{\partial^2 f}{\partial x_2^2} - f = -\frac{\Phi_{22}}{k} - \frac{1}{T}\left(\frac{\overline{v_2'^2}}{k} - \frac{2}{3}\right), \quad T \propto \frac{k}{\varepsilon}, \quad L \propto \frac{k^{3/2}}{\varepsilon}$$

In the V2F model, the v^2 equation is solved: what is the difference between $\overline{v_2'^2}$ and v^2? Explain in which term the difference between $\overline{v_2'^2}$ and v^2 is important.

b) Discuss the energy path in connection to the source and sink terms in the $\bar{k} = 0.5\langle \bar{v}_i' \bar{v}_i' \rangle$, $\bar{K} = 0.5\langle \bar{v}_i \rangle \langle \bar{v}_i \rangle$ and the k_{sgs} equations. How are $\bar{k}$ and k_{sgs} computed from the energy spectrum? (5p)

T4. a) What is a test filter? Grid and test filter Navier-Stokes equation and derive the relation (5p)

$$\widehat{\overline{v}_i \overline{v}_j} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j + \widehat{\tau}_{ij} = \mathcal{L}_{ij} + \widehat{\tau}_{ij} = T_{ij}$$

b) What is the difference between DES and DDES? Why was DDES invented? (5p)

T5. a) Give a short description of the method to generate synthetic turbulent inlet fluctuations. What form on the spectrum is assumed? How are the maximum and minimum wavelengths, κ_{max}, κ_{min}, determined? (5p)

b) Consider the SAS model. It includes the von Kármán length scale (5p)

$$L_{vK,3D} = \kappa \frac{|\bar{s}|}{|U''|}$$

where $|\bar{s}|$ and U'' denote the generic first and second derivative, respectively, of the velocity. An additional term, P_{SAS}, is added to the ω equation

$$P_{SAS} = \tilde{\zeta}_2 \kappa |\bar{s}|^2 \frac{L_t}{L_{vK,3D}}, \quad L_t = \frac{k^{1/2}}{\omega c_\mu^{1/4}}.$$

What is the object of this additional source term? Describe how this source reduces the turbulent viscosity. When is the term large and small, respectively?

MTF270 Turbulence modeling: Formula sheet

May 30, 2016

The continuity, Navier-Stokes and temperature equations for incompressible flow with constant viscosity read (*conservative* form)

$$\frac{\partial v_i}{\partial x_i} = 0$$
$$\rho_0 \frac{\partial v_i}{\partial t} + \rho_0 \frac{\partial v_i v_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_j} - \rho_0 \beta(\theta - \theta_0) g_i$$
$$\frac{\partial \theta}{\partial t} + \frac{\partial v_i \theta}{\partial x_i} = \alpha \frac{\partial^2 \theta}{\partial x_i \partial x_i}$$

▶The Navier-Stokes equation for incompressible flow with constant viscosity read (*non-conservative* form, p denotes the hydrostatic pressure, i.e. $p = 0$ if $v_i = 0$)

$$\rho_0 \frac{\partial v_i}{\partial t} + \rho_0 v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_j}$$

The *time averaged* continuity equation, Navier-Stokes equation temperature equations read

$$\begin{aligned}
\frac{\partial \bar{v}_i}{\partial x_i} &= 0 \\
\rho_0 \frac{\partial \bar{v}_i \bar{v}_j}{\partial x_j} &= -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \bar{v}_i}{\partial x_j} - \rho_0 \overline{v_i' v_j'}\right) - \rho_0 \beta(\bar{\theta} - \theta_0) g_i \\
\frac{\partial \bar{v}_i \bar{\theta}}{\partial x_i} &= \alpha \frac{\partial^2 \bar{\theta}}{\partial x_i \partial x_i} - \frac{\partial \overline{v_i' \theta'}}{\partial x_i}
\end{aligned}$$

The Boussinesq assumption reads

$$\overline{v_i' v_j'} = -\nu_t \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) + \frac{2}{3}\delta_{ij} k = -2\nu_t \bar{s}_{ij} + \frac{2}{3}\delta_{ij} k$$

The modeled $\overline{v_i' v_j'}$ equation with IP model reads

$$\begin{aligned}
\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k} = & \quad \text{(convection)} \\
-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k} & \quad \text{(production)} \\
-c_1 \frac{\varepsilon}{k}\left(\overline{v_i' v_j'} - \frac{2}{3}\delta_{ij} k\right) & \quad \text{(slow part)} \\
-c_2\left(P_{ij} - \frac{2}{3}\delta_{ij} P^k\right) & \quad \text{(rapid part)} \\
+c_{1w}\rho_0 \frac{\varepsilon}{k}\,[\,\overline{v_k' v_m'} n_k n_m \delta_{ij} - \frac{3}{2}\overline{v_i' v_k'} n_k n_j & \\
-\frac{3}{2}\overline{v_j' v_k'} n_k n_i\,]\, f\left[\frac{\ell_t}{x_n}\right] & \quad \text{(wall, slow part)} \\
+c_{2w}\,[\,\Phi_{km,2} n_k n_m \delta_{ij} - \frac{3}{2}\Phi_{ik,2} n_k n_j & \\
-\frac{3}{2}\Phi_{jk,2} n_k n_i\,]\, f\left[\frac{\ell_t}{x_n}\right] & \quad \text{(wall, rapid part)} \\
+\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k} & \quad \text{(viscous diffusion)} \\
+\frac{\partial}{\partial x_k}\left[c_k\, \overline{v_k' v_m'}\, \frac{k}{\varepsilon} \frac{\partial \overline{v_i' v_j'}}{\partial x_m}\right] & \quad \text{(turbulent diffusion)} \\
-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'} & \quad \text{(buoyancy production)} \\
-\frac{2}{3}\varepsilon \delta_{ij} & \quad \text{(dissipation)}
\end{aligned}$$

▶The exact transport equation for turbulent heat heat flux vector $\overline{v_i'\theta'}$ reads

$$\frac{\partial \overline{v_i'\theta'}}{\partial t} + \frac{\partial}{\partial x_k}\bar{v}_k\overline{v_i'\theta'} = \underbrace{-\overline{v_i'v_k'}\frac{\partial \bar{\theta}}{\partial x_k} - \overline{v_k'\theta'}\frac{\partial \bar{v}_i}{\partial x_k}}_{P_{i\theta}} \underbrace{-\overline{\frac{\theta'}{\rho}\frac{\partial p'}{\partial x_i}}}_{\Pi_{i\theta}} \underbrace{-\frac{\partial}{\partial x_k}\overline{v_k'v_i'\theta'}}_{D_{i\theta,t}}$$
$$\underbrace{+\alpha\overline{\frac{\partial}{\partial x_k}\left(v_i'\frac{\partial \theta'}{\partial x_k}\right)} + \nu\overline{\frac{\partial}{\partial x_k}\left(\theta'\frac{\partial v_i'}{\partial x_k}\right)}}_{D_{i\theta,\nu}} \underbrace{-(\nu+\alpha)\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial \theta'}{\partial x_k}}}_{\varepsilon_{i\theta}} \underbrace{-g_i\beta\overline{\theta'^2}}_{G_{i\theta}}$$

▶The exact k equation reads

$$\frac{\partial k}{\partial t} + \frac{\partial \bar{v}_j k}{\partial x_j} = -\overline{v_i'v_j'}\frac{\partial \bar{v}_i}{\partial x_j} - \frac{\partial}{\partial x_j}\left[\frac{1}{\rho}\overline{v_j'p'} + \frac{1}{2}\overline{v_j'v_i'v_i'} - \nu\frac{\partial k}{\partial x_j}\right] - \nu\overline{\frac{\partial v_i'}{\partial x_j}\frac{\partial v_i'}{\partial x_j}}$$

▶The exact $\overline{v_i'v_j'}$ equation reads

$$\frac{\partial \overline{v_i'v_j'}}{\partial t} + \frac{\partial}{\partial x_k}(\bar{v}_k\overline{v_i'v_j'}) = -\overline{v_j'v_k'}\frac{\partial \bar{v}_i}{\partial x_k} - \overline{v_i'v_k'}\frac{\partial \bar{v}_j}{\partial x_k}$$
$$-\frac{\partial}{\partial x_k}\left(\overline{v_i'v_j'v_k'} + \frac{1}{\rho}\delta_{jk}\overline{v_i'p'} + \frac{1}{\rho}\delta_{ik}\overline{v_j'p'} - \nu\frac{\partial \overline{v_i'v_j'}}{\partial x_k}\right)$$
$$+\frac{1}{\rho}\overline{p'\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - 2\nu\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}$$

▶The modeled k and ε equations

$$\frac{\partial k}{\partial t} + \bar{v}_j\frac{\partial k}{\partial x_j} = \nu_t\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\frac{\partial \bar{v}_i}{\partial x_j} + g_i\beta\frac{\nu_t}{\sigma_\theta}\frac{\partial \bar{\theta}}{\partial x_i}$$
$$-\varepsilon + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{\partial \varepsilon}{\partial t} + \bar{v}_j\frac{\partial \varepsilon}{\partial x_j} = \frac{\varepsilon}{k}c_{\varepsilon 1}\nu_t\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\frac{\partial \bar{v}_i}{\partial x_j}$$
$$+ c_{\varepsilon 1}g_i\frac{\varepsilon}{k}\frac{\nu_t}{\sigma_\theta}\frac{\partial \bar{\theta}}{\partial x_i} - c_{\varepsilon 2}\frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]$$