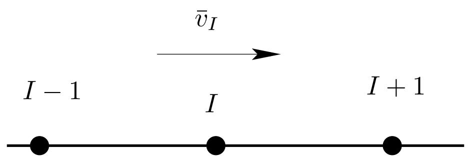

Figure 18.9: Numerical dissipation.

18.14 Numerical dissipation

The main function of an SGS model is to dissipate (i.e. to dampen) resolved turbulent fluctuations. The SGS model is – hopefully – designed to give a proper amount of dissipation. This is the reason why in LES we should use a central differencing scheme, because this class of schemes does not give any *numerical* dissipation. All upwind schemes give numerical dissipation in addition to the modeled SGS dissipation. Indeed, there are LES-methods in which upwind schemes are used to create dissipation and where no SGS model is used at all (e.g. MILES [64]). However, here we focus on ensuring proper dissipation through an SGS model rather than via upwind differencing. It can be shown using Neumann stability analysis that all upwind schemes are dissipative (see *Further reading* at
`http://www.tfd.chalmers.se/~lada/comp_turb_model/`). Below it is shown that first-order upwind schemes are dissipative.

The first-derivative in the convective term is estimated by first-order upwind differencing as (finite difference, see Fig. 18.9)

$$\bar{v}\left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}_I\left(\frac{\bar{v}_I - \bar{v}_{I-1}}{\Delta x} + \mathcal{O}\left(\Delta x\right)\right) \tag{18.34}$$

where we have assumed $\bar{v}_I > 0$. Taylor expansion gives

$$\bar{v}_{I-1} = \bar{v}_I - \Delta x \frac{\partial \bar{v}}{\partial x} + \frac{1}{2}(\Delta x)^2 \frac{\partial^2 \bar{v}}{\partial x^2} + \mathcal{O}\left((\Delta x)^3\right)$$

so that

$$\frac{\bar{v}_I - \bar{v}_{I-1}}{\Delta x} = \frac{\partial \bar{v}}{\partial x} - \frac{1}{2}\Delta x \frac{\partial^2 \bar{v}}{\partial x^2} + \mathcal{O}\left((\Delta x)^2\right)$$

Insert this into Eq. 18.34

$$\bar{v}\left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}\frac{\partial \bar{v}}{\partial x} - \underbrace{\frac{1}{2}\Delta x \bar{v}\frac{\partial^2 \bar{v}}{\partial x^2}}_{\mathcal{O}(\Delta x)} + \bar{v}\mathcal{O}\left((\Delta x)^2\right)$$

where the second term on the right side corresponds to the error term in Eq. 18.34. When this expression is inserted into the LES momentum equations, the second term on the right-hand side will act as an additional (numerical) diffusion term. The total diffusion term will have the form

$$\text{diffusion term} = \frac{\partial}{\partial x}\left\{(\nu + \nu_{sgs} + \nu_{num})\frac{\partial \bar{v}}{\partial x}\right\} \tag{18.35}$$

where the additional numerical viscosity, $\nu_{num} \simeq 0.5|\bar{v}|\Delta x$. This means that the total dissipation due to SGS viscosity and numerical viscosity is (cf. Eq. 18.17)

$$\varepsilon_{sgs+num} = 2(\nu_{sgs} + \nu_{num})\bar{s}_{ij}\bar{s}_{ij}$$