Eqs. H.15. H.16 into Eq. H.14 gives

$$\begin{aligned}
&\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} = -2C\left(\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2\widehat{|\bar{s}|\bar{s}_{ij}}\right) \Rightarrow \\
&\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C\underbrace{\left(\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2\widehat{|\bar{s}|\bar{s}_{ij}}\right)}_{M_{ij}} \\
&= \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} = 0
\end{aligned} \tag{H.17}$$

$$Q = \left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij}\right)^2$$

Find a Q which best satisfies Eq. H.17 for all i, j

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0, \quad \partial^2 Q/\partial C^2 = 8M_{ij}M_{ij} > 0$$

We get

$$C = -\frac{\mathcal{L}_{ij}M_{ij}}{2M_{ij}M_{ij}}, \quad \text{stability problems: needs smoothing}$$

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	central differencing
Time discretization	1st order	2nd order (e.g. C-N)
Turbulence model	$>$ two-equations	zero- or one-equation

▶**Start** and **end** time averaging. $t_{end} - t_{start} \simeq 100H/\langle\bar{v}\rangle_{center}$