V

$\mathbf{x}$

$\mathbf{y}$

y

0

1

2

2

$\partial\bar{\theta}/\partial x_3 > 0$

$\partial\rho/\partial x_3 < 0$

$\bar{\theta}_2 > \bar{\theta}_0$

$\bar{\theta}_1 < \bar{\theta}_0$

F

F

$\rho_2 < \rho_0$

$\rho_1 > \rho_0$

x_3

A

B

0

$\overline{v_0(r)}$

$\overline{v_1'^2} \simeq 5\overline{v_2'^2}$

$\bar{v}_1$

x_1

x_2

θ

streamline

Δx

$\widehat{\Delta}$

grid filter

test filter

I

$\bar{v}_i$

$\bar{v}_i$

$\bar{v}(x)$

cut-off, grid filter

test filter

resolved on test filter

resolved on grid filter

SGS test filter scales

SGS grid scales

$\kappa_c = \pi/\Delta$

$\kappa = \pi/\widehat{\Delta}$

Eqs. H.15. H.16 into Eq. H.14 gives

$$\mathcal{L}_{ij} = -2C\left(\widehat{\Delta}^2 |\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2\widehat{|\bar{s}|\bar{s}_{ij}}\right) \tag{H.17}$$

$$Q = (\mathcal{L}_{ij} + 2CM_{ij})^2 \tag{H.18a}$$

$$M_{ij} = \left(\widehat{\Delta}^2 |\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2\widehat{|\bar{s}|\bar{s}_{ij}}\right) \tag{H.18b}$$

Find a Q which best satisfies Eq. H.17 for all i, j

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0, \quad \partial^2 Q/\partial C^2 = 8M_{ij}M_{ij} > 0$$

We get

$$C = -\frac{\mathcal{L}_{ij}M_{ij}}{2M_{ij}M_{ij}}, \quad \text{stability problems: needs smoothing}$$

▶ See Section 18.20.1, RANS vs. LES

▶ Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	central differencing
Time discretization	1st order	2nd order (e.g. C-N)
Turbulence model	> two-equations	zero- or one-equation

▶ **Start** and **end** time averaging. $t_{end} - t_{start} \simeq 100H/\langle\bar{v}\rangle_{center}$

start   end