Equation T. 3 corresponds to the original PANS model. Recall that the turbulent diffusion in, for example, the $k$ equation reads

$$
\begin{equation*}
\frac{\partial}{\partial x_{j}}\left(\frac{\nu_{t}}{\sigma_{k}} \frac{\partial k}{\partial x_{j}}\right) \tag{T.4}
\end{equation*}
$$

Since $f_{k}=0.4$, it means that the turbulent diffusion in the $k$ and $\varepsilon$ equations are $1 / 0.4^{2} \simeq 6$ times larger in [173] than in [207]. The consequence is that peaks in $k$ and $\varepsilon\left(\right.$ and also $\left.\nu_{t}\right)$ are reduced in the former case compared to the latter (this is the physical role played by diffusion: it transports $k$ from regions of high $k$ to regions of low $k$ ). This explains why the peaks of $k$ are much larger in [207] compared to in [173].

Hence, in the original PANS model (Eq T.3), the RANS turbulent viscosity appears in the turbulent diffusion of $k$ (and $\varepsilon$ ), because the turbulent diffusion term reads (recall that $f_{k}=k / k_{t o t a l}=k / k_{\text {RANS }}$ where $k_{\text {RANS }}$ denotes the turbulent kinetic energy in a RANS simulation)

$$
\begin{align*}
\frac{\partial}{\partial x_{j}}\left(\frac{\nu_{t}}{f_{k}^{2} \sigma_{k}} \frac{\partial k}{\partial x_{j}}\right) & =\frac{\partial}{\partial x_{j}}\left(\frac{c_{\mu} k^{2}}{\varepsilon f_{k}^{2} \sigma_{k}} \frac{\partial k}{\partial x_{j}}\right)  \tag{T.5}\\
=\frac{\partial}{\partial x_{j}}\left(\frac{c_{\mu} k_{R A N S}^{2}}{\varepsilon \sigma_{k}} \frac{\partial k}{\partial x_{j}}\right) & =\frac{\partial}{\partial x_{j}}\left(\frac{\nu_{t, R A N S}}{\sigma_{k}} \frac{\partial k}{\partial x_{j}}\right)
\end{align*}
$$

cf. Eqs. 18 and 19 in [145]. Thus the total (i.e. RANS) viscosity is responsible for the transport of the modeled turbulent kinetic energy.

## T. 4 Location of interface

The results analyzed above were from LES simulations [173, 207] (i.e. the PANS model was used in LES mode). Now we will analyze results from PANS where $f_{k}$ is computed. In $[169,218] f_{k}$ is computed based on the DES model. We will use data obtained from this model but on a finer mesh and larger domain than in $[169,218]$. Here we call the model D-PANS.

Run the file pl_vect_hump_fine.py (Python) or pl_vect_hump_fine.m (MAtlab/Octave) which loads the file vectz_fine.dat, xy_hump_fine. dat, and $x 065$ _off. dat. This mesh has $649 \times 110 \times 32$ cells with $Z_{\max }=0.2$ (the mesh is plotted in pl_vect_hump_fine). Recall that $\Delta z=0.2 / 32$. Start by finding where the PANS model predicts the switch from RANS to LES (i.e where $f_{k}$ goes from one down to, say, 0.4). $f_{k}$ is computed in Eq. 16 in [169]. Plot location of the switch (the wall distance) versus $x$.

Plot $f_{k}$ also at a couple of $x_{1}$ locations ( $0.65 \ldots 1.30$ ). Is it bigger or smaller than the prescribed values ( 0.4 and 1 )? Compare it also with the definition of $f_{k}=k / k_{\text {total }}$ (cf. Fig. 26 in [165]).

In [219], $f_{k}$ is computed as

$$
\begin{equation*}
f_{k}=c_{\mu}^{-1 / 2}\left(\frac{\Delta}{L_{t}}\right)^{2 / 3}, \quad L_{t}=\frac{k_{\text {total }}^{3 / 2}}{\langle\varepsilon\rangle} \tag{T.6}
\end{equation*}
$$

Compare this $f_{k}$ with $f_{k}$ with D-PANS.

## T. 5 Location of interface in DES and DDES

Let's compare D-PANS with DES and DDES. In SA-DES, the interface is defined as the location where the wall distance is equal to $C_{D E S} \Delta$ where $\Delta=\max \left\{\Delta_{x}, \Delta_{y}, \Delta_{z}\right\}$, see Eq. 20.3. How does this compare with switching locating defined by D-PANS?

