AA 31447: Learning outcomes for 2020

Lecture 1-2

1. How is the buoyancy term, ρg_i, re-written in incompressible flow?

2. Show the principles how to derive the transport equation for $\overline{v_i' v_j'}$, Eq. 11.11 (see Section 11.2 on p. 111)

3. Derive the transport equation of $\overline{v_i' v_j'}$, Eq. 11.11 (see Section 9)

4. Given the transport equation of $\overline{v_i' v_j'}$, Eq. 11.11, derive the exact k equation (Eq. 11.4)

5. Given the transport equation for the temperature, θ, and the transport equation for $\bar{\theta}$, show the principles (in the same way as is done for the $\overline{v_i' v_j'}$ equation on Section 11.2 on p. 111) how to derive the transport equation for $\overline{v_i' \theta'}$, Eq. 11.22. Discuss the physical meaning of the different terms. Which terms need to be modeled?

6. What is the expression for the total heat flux that appears in the $\bar{\theta}$ equation?

7. Which terms in the $\overline{v_i' v_j'}$ equation need to be modeled? Explain the physical meaning of the different terms in the $\overline{v_i' v_j'}$ equation.

8. Show how the turbulent diffusion (i.e. the term which includes the triple correlation) in the k equation is modeled.

9. How is the production term modeled in the $k - \varepsilon$ model? Show how it can be expressed in $\bar{s}_{ij}$

10. Given the modeled k equation, derive the modeled ε equation.

11. Derive the Boussinesq assumption (see Section 11.6). How is the turbulent heat flux, $\overline{v_i' \theta'}$, modeled in the Boussinesq approach?

12. Discuss and show how the dissipation term, ε_{ij}, is modeled.

13. Use physical reasoning to derive a model for the diagonal components of the pressure-strain term (slow part).

14. Using physical reasoning, the model for the pressure-strain term above is formulated only for the normal streses. Show that if the model is expressed in the principal directions, then a model for the shear stress is also obtained.

15. The slow pressure-strain model reads $\Phi_{ij,1} = -c_1 \rho \frac{\varepsilon}{k}\left(\overline{v_i' v_j'} - \frac{2}{3}\delta_{ij} k\right)$. The anisotropy tensor is defined as $a_{ij} = \frac{\overline{v_i' v_j'}}{k} - \frac{2}{3}\delta_{ij}$. Show that for decaying grid turbulence, the model for the slow pressure-strain model indeed acts as to make the turbulence more isotropic if $c_1 > 1$.

16. Derive the exact Poisson equation for the pressure fluctuation, Eq. 11.65.

Lecture 3.4

1. For a Poisson equation
$$\frac{\partial^2\varphi}{\partial x_j \partial x_j} = f$$
there exists an exact analytical solution
$$\varphi(\mathbf{x}) = -\frac{1}{4\pi}\int_V \frac{f(\mathbf{y})dy_1dy_2dy_3}{|\mathbf{y}-\mathbf{x}|} \tag{AA.1}$$
Use Eqs. 11.65 and AA.1 to derive the exact analytical solution (Eq. 11.68) for the fluctuating pressure. Which are the "slow" and "rapid" terms? Why are they called "slow" and "rapid"?

2. Derive the algebraic stress model (ASM). What main assumption is made?

3. Show the physical reasoning leading to the modeled slow pressure strain term, $\Phi_{22,1w}$, for wall effects. What sign does it have? Give also the expressions for $\Phi_{11,1w}$ and $\Phi_{33,1w}$

4. The modeled slow and rapid pressure strain term read $\Phi_{ij,1} = -c_1\rho\frac{\varepsilon}{k}\left(\overline{v_i'v_j'} - \frac{2}{3}\delta_{ij}k\right)$ and $\Phi_{ij,2} = -c_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P^k\right)$, respectively. Give the expression for the production terms, modeled pressure-strain terms and modeled dissipation terms for a simple boundary layer. In some stress equations there is no production terms nor any dissipation term. How come? Which is the main source term (or sink term) in these equations?

5. Describe the physical effect of stable stratification and unstable stratification on turbulence.

6. Consider buoyancy-dominated flow with x_3 vertically upwards. The production term for the $\overline{v_i'v_j'}$ and the $\overline{v_i'\theta'}$ equations read
$$G_{ij} = -g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}, \quad P_{i\theta} = -\overline{v_i'v_k'}\frac{\partial\bar{\theta}}{\partial x_k}$$
respectively (we assume that the velocity gradient is negligible). Show that the Reynolds stress model dampens and increases the vertical fluctuation in stable and unstable stratification, respectively, as it should. Show also that k in the $k-\varepsilon$ model is affected in the same way.

7. Watch the on-line lecture *Turbulence* (20 minutes into the movie) at
`http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html`

   i. Consider the flow in the channel where the fluid on the top (red) and the bottom (yellow) are separated by a horizontal partition. The two fluids are identical. Study how the two fluids mix downstream of the partition. In the next example, the fluid on the top is hot (yellow) and light, and the one at the bottom (dark blue) is cold (heavy); how do the fluids mix downstream of the partition, better or worse than in the previous example? This flow situation is called *stable stratification*. In the last example, the situation is reversed: cold, heavy fluid (dark blue) is moving on top of hot, light fluid (yellow). How is the mixing affected? This flow situation is called *unstable stratification*. Compare in meteorology where heating of the ground may cause unstable stratification or when *inversion* causes stable stratification.

8. Consider streamline curvature for a streamline formed as a circular arc (convex curvature). Show that the turbulence is dampened if $\partial v_\theta/\partial r > 0$ and that it is enhanced if the sign of $\partial v_\theta/\partial r$ is negative.

9. Streamline curvature: now consider a boundary layer where the streamlines are curved away from the wall (concave curvature). Show that the Reynolds stress model gives an enhanced turbulence production (as it should) because of positive feedback between the production terms. Why is the effect of streamline curvature in the $k - \varepsilon$ model much smaller?

10. Consider stagnation flow. Show that in the Reynolds stress model, there is only a small production of turbulence whereas eddy-viscosity models (such as the $k - \varepsilon$ model) give a large production of turbulence.

11. What is a realizability constraint? There are two main realizability constraints, one on the normal and one on the shear stresses: give the form of these constraints.

12. Show that the Boussinesq assumption may give negative normal stresses. In which coordinate system is the risk largest for negative normal stresses? Derive an expression (2D) how to avoid negative normal stresses by reducing the turbulent viscosity (Eq. 13.12).
    Hint: the eigenvalues, λ_1, λ_2, are obtained from $|\bar{s}_{ij} - \delta_{ij}\lambda| = 0$, $I_2^{2D} = \frac{1}{2}(C_{ii}C_{jj} - C_{ij}C_{ij})$

13. What is a non-linear eddy-viscosity model? When formulating a non-linear model, the anisotropy tensor $a_{ij} = -2\nu_t \bar{s}_{ij}/k$ is often used. The three terms read

$$
\begin{aligned}
c_1\tau^2 &\left(\bar{s}_{ik}\bar{s}_{kj} - \frac{1}{3}\bar{s}_{\ell k}\bar{s}_{\ell k}\delta_{ij}\right) \\
&+ c_2\tau^2\left(\bar{\Omega}_{ik}\bar{s}_{kj} - \bar{s}_{ik}\bar{\Omega}_{kj}\right) \\
+ c_3\tau^2 &\left(\bar{\Omega}_{ik}\bar{\Omega}_{jk} - \frac{1}{3}\bar{\Omega}_{\ell k}\bar{\Omega}_{\ell k}\delta_{ij}\right)
\end{aligned}
$$

    Show that each term has the same properties as a_{ij}, i.e. non-dimensional, traceless and symmetric (see Exam 2017-05-30, Answers, Question T3a).

Week 3

1. Which equations are solved in the V2F model?

2. The transport equation for $\overline{v_2'^2}$ reads (the turbulent diffusion term is modeled)

$$\frac{\partial \rho \bar{v}_1 \overline{v_2'^2}}{\partial x_1} + \frac{\partial \rho \bar{v} \overline{v_2'^2}}{\partial x_2} = \frac{\partial}{\partial x_2}\left[(\mu + \mu_t)\frac{\partial \overline{v_2'^2}}{\partial x_2}\right] \underbrace{-2\overline{v_2' \partial p'/\partial x_2}}_{\rho\Phi_{22}} - \rho\varepsilon_{22}$$

   Show how this equation is re-written in the V2F model.

3. The f equation in the V2F model reads

$$L^2 \frac{\partial^2 f}{\partial x_2^2} - f = -\frac{\Phi_{22}}{k} - \frac{1}{T}\left(\frac{\overline{v_2'^2}}{k} - \frac{2}{3}\right), \quad T \propto \frac{k}{\varepsilon}, \quad L \propto \frac{k^{3/2}}{\varepsilon}$$

   Explain how the magnitude of the right side and L affect f (Fig 15.1). How does f enter into the $\overline{v_2'^2}$ equation? Show that far from the walls, the V2F model (i.e. the f and the $\overline{v_2'^2}$ equation) returns to the $\overline{v_2'^2}$ equation in the Reynolds stress model. In the V2F model, the v^2 equation is solved: what is the difference between $\overline{v_2'^2}$ and v^2 (see the discussion in connection to Eq. 15.9)?

4. What does the acronym SST mean? The SST model is a combination of the $k - \varepsilon$ and the $k - \omega$ model. In which region is each model used and why? How is ω expressed in k and ε?

5. Derive a transport equation for ω from the k and ε transport equations; you only need to do the production, the destruction and the viscous diffusion terms.

6. In the SST model, two blending function, F_1 and F_2, are used; explain what is the object of F_1 or F_2.

7. What is the purpose of the shear stress limiter in the SST model? Show that the eddy-viscosity assumption gives too high a shear stress in APG since $P^k/\varepsilon \gg 1$ (Eq. 16.15).

8. Show the difference between volume averaging (filtering) in LES and time-averaging in RANS. Show that avereging once or twice is different in RANS and LES.

9. Consider the spatial derivative of the pressure in the filtered Navier-Stokes: show that the derivative can be moved outside the filtering integral (it gives an additional second-order term).

10. The filtered non-linear term has the form

$$\overline{\frac{\partial v_i v_j}{\partial x_j}}$$

    Show that it can be re-written as

$$\frac{\partial \bar{v}_i \bar{v}_j}{\partial x_j}$$

giving an additional term

$$-\frac{\partial}{\partial x_j}(\overline{\bar{v}_i \bar{v}_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) = -\frac{\partial \tau_{ij}}{\partial x_j}$$

on the right side.

11. Consider a 1D finite volume grid. Carry out a second filtering of $\bar{v}$ at node I and show that $\bar{\bar{v}}_I \neq \bar{v}_I$.

12. Consider the energy spectrum. Show the three different regions (the large energy-containing scales, the $-5/3$ range and the dissipating scales). Where should the cut-off be located? Show where the SGS scales, grid (i.e resolved) scales and the cut-off, κ_c are located in the spectrum.

13. Show how a sinus wave $\sin(\kappa_c x)$ corresponding to cut-off is represented on a grid with two and four cells, respectively. How is κ_c related to the grid size Δx for these cases?

14. Watch the on-line lecture *Turbulence* at http://www.tfd.chalmers.se/˜lada/MoF/flow_viz.html

Week 5

1. Taking guidance from the RANS k equation, formulate the one-equation k_{sgs} equation
2. Consider the energy spectrum and discuss the physical meaning of $P_{k_{sgs}}$ and ε_{sgs}.
3. Discuss the energy path in connection to the source and sink terms in the $\bar{k}$, $\bar{K}$ and the k_{sgs} equations, see Figs. Q.4 and Q.5. How are $\bar{k}$ and k_{sgs} computed from the energy spectrum?
4. What is a test filter? Grid and test filter Navier-Stokes equation and derive the relation
$$\widehat{\bar{v}_i\bar{v}_j} - \widehat{\bar{v}}_i\widehat{\bar{v}}_j + \widehat{\tau}_{ij} = \mathcal{L}_{ij} + \widehat{\tau}_{ij} = T_{ij} \tag{AA.2}$$
Draw an energy spectrum and show which wavenumber range $\bar{k}$, k_{sgs}, $k_{sgs,test}$ cover.
5. Formulate the Smagorinsky model for the grid filter SGS stress, τ_{ij}, and the test filter SGS stress, T_{ij}. Use Eq. AA.2 and derive the relation [12]
$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} = -2C\left(\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \widehat{\Delta^2|\bar{s}|\bar{s}_{ij}}\right)$$
6. The equation you derived above is a tensor equation for C. Use this relation and derive the final expression for the dynamic coefficient, C, Eq. 18.41.
7. Show that when a first-order upwind schemes is used for the convection term, an additional diffusion term and dissipation terms appear because of a numerical SGS viscosity

[12] note that the test filter here covers Δ^2 (which is correct); Eq. 18.38 is not correct

Week 6

1. What are the five main differences between a RANS finite volume CFD code and a LES finite volume CFD code? What do you need to consider in LES when you want to compute time-averaged quantities? (see Fig. 18.12). How can the integral time scale, T_{int}, be used?(see Section M.3)

2. Derive the Smagorinsky model in two different ways (Sections 18.6 and 18.22)

3. When doing LES, how fine does the mesh need to be in the near-wall region? Why does it need to be that fine?

4. Describe URANS. How is the instantaneous velocity decomposed? What turbulence models are used? What is scale separation?

5. Mention four different ways to estimate the resolution of an LES that you have made; see Section 18.26 Which method is good/bad? Which is best?

6. What is DES? The destruction term in the RANS S-A model reads $\left(\frac{\tilde{\nu}_t}{d}\right)^2$; how is it computed in the S-A DES model?

7. How is the length scale computed in a $k - \varepsilon$ two-equation DES model? Where in a boundary layer does the DES model switch from RANS to LES?

8. The modified (reduced) length scale in two-equation DES models can be introduced in different ways. It it usually introduced in one transport equation. Which one and which term? Apart from this transport equation, it is sometimes used in a another equation. Which one? What is the effect on the modeled, turbulent quantities?

9. What is DDES? Why was it invented?

10. Describe hybrid LES-RANS based on a one-equation model.

11. What is the physical meaning of f_k in PANS?

12. The PANS equations are given in Eqs. 23.19. Assume that $f_\varepsilon = 1$. Consider the destruction term in the ε equation and the coefficient $C^*_{\varepsilon 2}$. Explain what happens if f_k is reduced from 1 (RANS mode) to $f_k = 0.4$ (LES mode).

13. Describe the SAS model. How is the von Kármán length scale defined? An additional source term is introduced in the ω equation: what is the form of this term? Describe how this source reduces the turbulent viscosity. When is the term large and small, respectively?

Week 7

1. Give a short description of the method to generate synthetic turbulent inlet fluctuations; see Chapter 27 and first 13 slides in
http://www.tfd.chalmers.se/~lada/slides/slides_inlet.pdf
What form on the spectrum is assumed? How is the wavenumber, κ_e, for the energy-containing eddies, determined? How are the maximum and minimum wavelengths, κ_{max}, κ_{min}, determined? With this method, the generated shear stress is zero: why? How is the correlation in time achieved?

2. What is embedded LES? Give an example when we may use it.

3. Consider the PANS model: in the derivation, we assumed that f_k is constant. Relaxing that condition is useful when presccribing k and ε prescribed at inlet and at embedded RANS/LES interfaces. Derive the additional term in the k equation (Eq. 23.24). Show that the the sign of the gradient at the inlet (and embedded interface) does indeed give a reduction of k as it should.

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AB.1 On-line Lecture 1

¶See Section 11.1.1, Flow equations

▶Boussinesq approximation: density variation only in gravitation (buoyancy) term

$$\frac{\partial \rho_0 \bar{v}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\rho_0 \bar{v}_i \bar{v}_j\right) = -\frac{\partial \bar{p}}{\partial x_i}+\mu\frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j}-\frac{\partial \tau_{ij}}{\partial x_j}-\rho_0\beta(\bar{\theta}-\theta_0$$

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$\bar{p}$ is hydrodynamic pressure: $\rho f_i \rightarrow (\rho-\rho_0)g_i$

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If we let density depend on pressure and temperature, differentiation gives

$$d\rho = \left(\frac{\partial \rho}{\partial \theta}\right)_p d\theta + \left(\frac{\partial \rho}{\partial p}\right)_\theta dp$$

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Incompressible flow: $\Rightarrow \partial\rho/\partial p=0$

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If we let density depend on pressure and temperature, differentiation gives

$$d\rho=\left(\frac{\partial \rho}{\partial \theta}\right)_p d\theta+\left(\frac{\partial \rho}{\partial p}\right)_\theta dp$$

Incompressible flow: $\Rightarrow \partial \rho / \partial p=0$

$$\beta=-\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial \theta}\right)_p \Rightarrow$$

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