What about the viscous terms: where do they play an important role? Which terms are non-zero *at* the wall? (you can easily show that with paper and pen).

So far we have looked at the $\bar{v}_1$-momentum equation. The database corresponds to a two-dimensional flow. Now let's think of the forces as vectors. The gradient of the normal stresses in the $x_1 - x_2$ plane represent the force vector (see Eq. L.1)

$$\mathbf{F}_N = \left(-\frac{\partial \overline{v_1'^2}}{\partial x_1}, -\frac{\partial \overline{v_2'^2}}{\partial x_2}\right) \tag{L.2}$$

and the corresponding force vector due to the shear stresses reads (see Eq. L.1)

$$\mathbf{F}_S = \left(-\frac{\partial \overline{v_1'v_2'}}{\partial x_2}, -\frac{\partial \overline{v_1'v_2'}}{\partial x_1}\right) \tag{L.3}$$

Find the first term in Eqs. L.2 and L.3 in the $\bar{v}_1$ momentum equation, Eq. L.1. Consider the second line in Eq. L.1 which is the $\bar{v}_2$ equation and find the other two terms in Eqs. L.2 and L.3. Note that $\mathbf{F}_N$ and $\mathbf{F}_S$ are forces per unit volume ($[N/m^3]$).

Assignment 1.3. The left-hand side can be formulated in three different ways (steady flow), $d\bar{v}_i/dt$, $\bar{v}_j\partial\bar{v}_i/\partial x_j$ and $\partial/\partial x_j(\bar{v}_i\bar{v}_j)$, see Section 2.4. Consider the first form, $d\bar{v}_i/dt$. This is the Lagrangian form, i.e. we follow a particle, see Fig. 1.1. Now, pick a vertical grid line at x_1 and try to estimate $d\bar{v}_1/dt$. Then you want to estimate v_1, Δt seconds later. During this time the fluid particle has moved the vector distance $(\Delta x_1, \Delta x_2)$ to the next vertical grid line with the speed $V = (v_1^2 + v_2^2)^{1/2}$ so that $\Delta t = \Delta s/V$ where $\Delta s = ((\Delta x_1)^2 + (\Delta x_2)^2)^{1/2}$

Compare this way of computing the left-hand side with $\partial/\partial x_j(\bar{v}_1\bar{v}_j)$.

Note that we have now estimated $d\bar{v}_1/dt$ using the the discrete approximation of the chain rule in Eq. 1.1, i.e.

$$\frac{d\bar{v}_1}{dt} \simeq \frac{\Delta\bar{v}_1}{\Delta t} = V\frac{\Delta\bar{v}_1}{\Delta s} \simeq \bar{v}_1\frac{\partial\bar{v}_1}{\partial x_1} + \bar{v}_2\frac{\partial\bar{v}_1}{\partial x_2} \tag{L.4}$$

Assignment 1.4. Plot the vector field $\mathbf{F}_N$ to learn something about its properties. When $\overline{v_2'^2}$ reaches a maximum or a minimum along a grid line normal to the wall, what happens with the vector field $\mathbf{F}_N$? Zoom-in on interesting regions.

Assignment 1.5. Plot also vector fields of the shear stress, $\mathbf{F}_S$ (see Eq. L.3), the pressure gradient and the viscous terms. Zoom up in interesting regions. Anything interesting?

L.1.3 The turbulent kinetic energy equation

The exact transport equation for the turbulent kinetic energy, k, reads

$$\begin{aligned}\frac{\partial}{\partial x_j}(\bar{v}_j k) &= \nu\frac{\partial^2 k}{\partial x_j \partial x_j} + P^k + D_k - \varepsilon\\ P^k &= -\overline{v_i'v_j'}\frac{\partial\bar{v}_i}{\partial x_j}\end{aligned} \tag{L.5}$$